# On Causality and Predictivity

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**Abstract**

Certain approaches to quantum gravity, such as the one based on the concept of purely virtual particles (fakeons), sacrifice the cause-effect relation at very small scales to reconcile renormalizability with unitarity. Other developments have also urged caution regarding the idea of causality as a fundamental principle. In this paper, we examine the problem from multiple perspectives, including locality and predictivity, and extend the existing skepticism in several directions. Emphasizing the impact of unruly "disruptors", we point out that the illusory arrow of time associated with causality and predictivity is inherently statistical. This renders the cause-effect relation strained at the microscopic level. We also show that causation is a borderline concept that demands belief in entities which can act on nature without being part of it. Ultimately, not only is renouncing microcausality a reasonable price to pay for a consistent and predictive theory of quantum gravity (as is the one based on the fakeon idea), but the very notion of causality is misleading. Resting as it does on metaphysical assumptions, it should therefore be abandoned in fundamental physics.

# 1 Introduction

Among the foundational principles of Quantum Field Theory (QFT) are locality, renormalizability, and unitarity. While unitarity is a self-consistency requirement, essential for probability conservation, locality and renormalizability are more pragmatic principles that have successfully guided the construction of theories describing three of the four fundamental interactions of nature. Microcausality, often closely linked to locality and analyticity, has historically played an important, though perhaps less examined, role. However, for various reasons, the necessity of strict causality is increasingly being questioned in fundamental physics, particularly in relation with quantum gravity (QG).

Although it is generally anticipated that quantum gravity may demand the sacrifice of certain fundamental principles, this necessity does not automatically mandate the abandonment of the entire QFT framework, which has proven so successful in describing the other three interactions. This position stands in contrast to the assumptions underlying approaches such as String Theory, Loop Quantum Gravity, and Holography, which necessitate radical foundational shifts. Such broad departures from QFT frequently lack predictive power and may reflect an undue pessimism regarding the flexibility of the standard QFT approach.

It is clear that tackling quantum gravity from within the QFT framework presents a reduced space of maneuver. On the other hand, it is precisely this powerful constraint on available options that can lead to a predictive outcome. Should the necessary theoretical renunciation prove to be minimal, the approach's effectiveness would be even more convincing.

Among the principles that may be readily relaxed are locality, causality, and analyticity, provided they are not abandoned entirely but merely modified to the minimum indispensable extent. While analyticity may be viewed as partially dispensable, given its status as a pragmatic requirement, causality and, to some extent, locality, may point to more fundamental principles that some may not be willing to renounce so quickly, not even at the microscopic level.

What is a "cause"? This seemingly innocent question conceals more complexity than is customarily assumed. Since Hume [1], the answer should not be taken for granted. Is causation a practical shortcut, as Hume claims, or a fundamental principle of nature? This is not a minor problem, as elevating a practical shortcut to the status of a physical law — or even a principle — carries the risk of misdirection, potentially obscuring promising candidate solutions to existing open problems.

Nonrelativistic and relativistic mechanics do not prompt significant suspicion about

causality. Quantum mechanics starts to plant a seed of doubt, because its standard interpretation includes the non-causal (probabilistic and instantaneous) collapse of the wave function upon measurement, fundamentally breaking the deterministic evolution governed by the Schrödinger equation. Quantum field theory raises more concerns, since it lacks a convincing and broadly accepted definition of microcausality. Despite this conceptual ambiguity, QFT has achieved remarkable success and predictive power. Given that substantial progress can be made without resolving the issue, the strict necessity of the causality concept is, at the very least, redundant for fundamental physics. Quantum gravity marks a step ahead. In many approaches, strict causality is not expected to be crucial at or below the Planck length. Some frameworks predict that causality may be broken at much larger scales.

For example, in quantum gravity with fakeons [2, 3] (i.e., purely virtual particles, or particles that can never be on the mass shell [4, 5, 6]), the scale of causality violation is $1/m_\chi$, where $m_\chi$ is the mass of the "gravifakeon", a spin-2 purely virtual particle that belongs to the fundamental triplet of the theory (graviton, inflaton and fakeon), necessary to ensure consistency through unitarity and renormalizability. On cosmological grounds [7], $m_\chi$ is constrained to be larger than $m_\phi/4$, where $m_\phi$ denotes the mass of the inflaton (approximately $10^{13}$GeV according to the results on primordial spectra of scalar fluctuations [8]). Ultimately, the fakeon approach allows for violations of causality that extend up to six orders of magnitude above the Planck length.

Another approach for building QG models that are both renormalizable and unitary relies on removing the ghosts of higher-derivative local theories by inserting appropriate nonpolynomial form factors into the propagators [9]. The resulting theories are nonlocal and violate microcausality, albeit in different ways.

Nonlocal QFT has been attracting attention for a long time, from the pioneering works of Pais and Uhlenbeck [10] and Efimov [11]. Interest in this area has been revived more recently by many authors [9, 12]. The main problem with a plain nonlocal approach is that it entails an infinite degree of arbitrariness, and no physical principle is currently known to single out a unique theory. This contrasts sharply with the fakeon framework, which yields a unique strictly renormalizable QG model [2]. It has been argued [13] that if a nonlocal unitary theory has a regular local limit, that limit must be a model containing fakeons. Then the local limit provides the missing criterion for selecting the "right theory" within the infinite space of nonlocal theories.

Acausal behaviors and related phenomena are encountered in several other contexts. Among these, we mention the Lee-Wick models [14], in which the "abnormal particles"

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rapidly decay. Further examples are approaches based on propagators with complex poles [15], analogies with QCD [16], antilinear symmetries [17] and unstable ghosts [18]. It is worth noting that a violation of causality is not expected in the Stelle theory [19], which is quadratic gravity with a spin-2 massive ghost. The primary trouble with that theory, however, is its lack of unitarity.

In this paper, we critically examine the concept of causality within both deterministic and non-deterministic frameworks. We argue that the notion of cause is only tenable when it refers to entities that are genuinely external to the system under observation. Crucially, these external entities must not be subject to the laws of physics. Otherwise, they would merely constitute internal components of larger systems, thereby losing their essence as causes. Because the universe contains no truly external entity capable of acting upon the universe itself, we conclude that there are no true causes within nature.

The idea of cause, therefore, is a borderline concept. First, it belongs to our description of nature rather than being an intrinsic feature of nature itself. However, just as the laws of physics are invariant under changes of coordinates and reference frames, it is evident that nature does not depend on our ways to formulate it. Second, describing nature in terms of causes treads into metaphysics, or "the supernatural", since it involves entities that are outside nature, but can act on it.

The cause-effect relation affords us the illusion of having control over the future by changing the course of events. Once we accept that this is, in fact, an illusion, we could settle for simply *predicting* the future. Motivated by this consideration, we critically examine the issue of predictivity, and argue that true predictions are also impossible as a matter of principle. Instead, we can only make statements that can be verified retrospectively (prepostdictions). This limitation arises because physical systems can never be perfectly isolated, nor can their initial conditions be fixed completely. The possibility that unruly "disruptors" might emerge from regions of spacetime beyond our control and alter the final outcome in unforeseen ways can never be rigorously excluded, although it can be practically dismissed on statistical grounds. This very fact, however, confirms that the illusory arrow of time attributed to causality is an effect of statistics, emerging at macroscopic scales. This renders microcausality disposable, even in the presence of external forces.

In theories with fakeons the maximum we can achieve is delayed prepostdictions. However, the delay expected in quantum gravity is so short ($1/m_\chi \lesssim 10^{-37}$ seconds) that it does not significantly worsen the fundamental limits on our predictive capabilities.

One may object that, for everyday purposes and indeed many established scientific applications, these are distinctions without a practical impact, in the sense that we can

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still understand one another by talking about causes and effects, and predictions. The problems we discuss here gain significance when causality is elevated to a fundamental principle. If we want to decipher quantum gravity, or anyway explore the unknown, what is sufficient for practical utility may well be insufficient for fundamental investigation. This caution should not come as a surprise, since quantum theory has already taught us to be wary of taking anything for granted.

Causality often overlaps with non-superluminality, the property that signals cannot propagate faster than light in Special Relativity. Yet non-superluminality is not inherent in the notion of causality. If one assumes that relations of cause and effect do exist, then non-superluminality merely restricts them to events lying within the past and future light cones. On the other hand, if we accept, as we demonstrate in this paper, that relations of cause and effect lack a fundamental meaning, we are not thereby forced to accept superluminal propagation.

The arguments advanced in this paper should not be construed as contentious, as there is little need to convince the majority of the physics community on the opportunity of relaxing the requirement of microcausality, especially in quantum gravity. Apart from specific, perhaps dogmatic, segments within String Theory and Loop Quantum Gravity, which appear to maintain an a-priori, unexamined commitment to strict causality, skepticism about causation as a fundamental principle is widespread. Moreover, the broad community response has been encouragingly receptive and open-minded since the introduction of the fakeon framework in 2017. The present work, therefore, aims not at widespread persuasion, but rather to sharpen the articulation of points that many are already well-disposed to accept, emphasizing that if the price for achieving a consistent and, crucially, experimentally testable theory of quantum gravity is to relax the constraint of strict microcausality, as the fakeon theory requires, it is a trade-off they are willing to accept. At the same time, we caution upfront that we do not stop there, but push the claim much further, suggesting that the notion of causality is inherently misleading (insofar as it relies on metaphysical assumptions) and should therefore be completely abandoned in fundamental physics.

The paper is organized as follows. In Section 2, we demonstrate that the notion of cause is untenable in a deterministic theory. In section 3, we show that it hints at the existence of truly external entities, and we point out the conceptual concerns and contradictions implied by this necessity. Section 4 examines the issue of predictivity in "causal" theories. This discussion is extended to theories incorporating fakeons in Section 5. In Section 6, we comment on the difficulties posed by defining causality in quantum field theory and quantum gravity, while in Section 7 we discuss the relationship with nonlocality. Section

8 contains the conclusions.

## 2 Determinism and causality

While aspects of causality were questioned earlier, the Scottish empiricist David Hume was the first to articulate a rigorous philosophical skepticism about it. Here are his words, from *An Enquiry Concerning Human Understanding* [1].

"All events seem entirely loose and separate. One event follows another; but we never can observe any tie between them. They seem *conjoined*, but never *connected*. And as we can have no idea of any thing which never appeared to our sense or inward sentiment, the necessary conclusion *seems* to be that we have no idea of connexion or power at all, and that these words are absolutely without any meaning, when employed either in philosophical reasonings or common life." [Section VII, Part II, p. 76]

"When we look about us towards external objects, and consider the operation of causes, we are never able, in a single instance, to discover any power or necessary connexion; any quality, which binds the effect to the cause, and renders the one an infallible consequence of the other. We only find, that the one does actually, in fact, follow the other." [Section VII, Part I, p. 64]

"It appears, then, that this idea of necessary connexion among events arises from a number of similar instances which occur of the constant conjunction of these events; nor can that idea ever be suggested by any one of these instances, surveyed in all possible lights and positions. But there is nothing in a number of instances, different from every single instance, which is supposed to be exactly similar; except only, that after a repetition of similar instances, the mind is carried by habit, upon the appearance of one event, to expect its usual attendant, and to believe that it will exist. This connexion, therefore, which we *feel* in the mind, this customary transition of the imagination from one object to its usual attendant, is the sentiment or impression from which we form the idea of power or necessary connexion." [Section VII, Part II, p. 77]

"Suitably to this experience, therefore, we may define a cause to be *an object, followed by another, and where all the objects similar to the first are followed by objects similar to the second.* Or in other words *where, if the first object had not been, the second never had existed.* The appearance of a cause always conveys the mind, by a customary transition, to the idea of the effect" [Section VII, Part II, p. 79]

"All inferences from experience, therefore, are effects of custom, not of reasoning. Custom, then, is the great guide of human life. It is that principle alone, which renders our

experience useful to us, and makes us expect, for the future, a similar train of events with those which have appeared in the past." [Section V, Part I, pp. 44-45]

In other words, Hume posited that causation is a psychological habit, not an intrinsic feature of reality. As he argued, there are no causes and no effects, but merely "trains of events in succession". The first observation that comes to mind is that this "train of events" more precisely describes determinism, rather than causality. The distinction between the two concepts is crucial for our analysis.

In a deterministic world the present is uniquely determined by both the past and the future. Hence, it is redundant to claim that the present is "caused" by the past, given that no alternative was available. Events are merely parts of a fixed chain, and we interpret them as causes only because of how we experience time. We say, for example: "exposing yourself to fresh air today causes you to be sick tomorrow". But we could equally well say: "you will be sick tomorrow, because it is already written, and for that reason you exposed yourself to fresh air today." The direction of the cause-effect relationship is thus arbitrary, contrary to what causation is supposed to be: a chronological ordering equipped with an arrow signifying a necessary link from the past, through the present, to the future. It is our narration of the universe that equips the flow of events with an apparent arrow.

The fundamental laws of physics (aside from the T violation predicted by the Standard Model of elementary particles) are symmetric under time reversal. In classical electrodynamics, standard boundary conditions at infinity force the use of the retarded potentials over the advanced ones. This choice, however, has no inherent connection to causation. In other situations, such as inside a cavity with reflecting walls, or describing how smartphones emit and receive, we need combinations of both retarded and advanced potentials. These combinations also have no relation with causality, nor do they imply its violation. They simply reflect the fact that the electromagnetic field is mathematically described as a superposition of incoming and outgoing waves.

For definiteness, consider a pointlike oscillating dipole $\mathbf{d}$ placed at the origin. The "source"

$$J^\mu(t, \mathbf{r}) = -(\cos(\omega t)\mathbf{d} \cdot \nabla \delta^{(3)}(\mathbf{r}), \omega \mathbf{d} \sin(\omega t)\delta^{(3)}(\mathbf{r})), \qquad \partial_\mu J^\mu = 0, \tag{2.1}$$

gives the vector potential

$$A_-^\mu(t, \mathbf{r}) = -\frac{1}{4\pi}\left(\nabla \cdot \frac{\mathbf{d}\cos(\omega(t-r))}{r}, \frac{\omega\mathbf{d}\sin(\omega(t-r))}{r}\right), \tag{2.2}$$

upon solving $\Box A^\mu = J^\mu$ in the Lorenz gauge $\partial_\mu A^\mu = 0$, where $r = |\mathbf{r}|$. However, the same

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current (2.1) also gives the solution[1]

$$A^\mu_+(t, \mathbf{r}) = -\frac{1}{4\pi} \left( \nabla \cdot \frac{\mathbf{d}\cos(\omega(t+r))}{r}, \frac{\omega\mathbf{d}\sin(\omega(t+r))}{r} \right), \qquad (2.3)$$

in which case it is not a "source", but a "sink".

Thus, is (2.1) the "cause" or the "end", the emitter or the receiver? If (2.1) were the cause, it would suffice to cause the electromagnetic field encoded in the vector potential (2.2). Instead, the choice between (2.2) and (2.3) rests on the boundary conditions at infinity. Hence, the current (2.1) *per se* is not a cause.

Remarks along these lines, including the irrelevance of causality for physics, were articulated more than a century ago by philosophers such as Bertrand Russell and his followers. Here are some excerpts from Russell's essay "On the notion of cause" [20]. 1) "In the motions of mutually gravitating bodies, there is nothing that can be called a cause, and nothing that can be called an effect; there is merely a formula. Certain differential equations can be found, which hold at every instant for every particle of the system, and which, given the configuration and velocities at one instant, or the configurations at two instants, render the configuration at any other earlier or later instant theoretically calculable. That is to say, the configuration at any instant is a function of that instant and the configurations at two given instants. This statement holds throughout physics, and not only in the special case of gravitation." 2) "All philosophers, of every school, imagine that causation is one of the fundamental axioms or postulates of science, yet, oddly enough, in advanced sciences such as gravitational astronomy, the word 'cause' never appears." 3) "The reason why physics has ceased to look for causes is that, in fact, there are no such things. The law of causality, I believe, like much that passes muster among philosophers, is a relic of a bygone age, surviving, like the monarchy, only because it is erroneously supposed to do no harm." 4) "The word 'cause' is so inextricably bound up with misleading associations as to make its complete extrusion from the philosophical vocabulary desirable."

Unfortunately, Russell's arguments failed to open a breach in the methodology of physics. This failure resulted in the continued adoption of *ad hoc* definitions for the sake of preserving the notion of cause, rather than challenging the concept itself at its core. We believe that it is about time to settle the matter and eliminate causality from the foundations of physical sciences.

As noted, the essence of determinism is the lack of alternatives. What if alternatives were available? Then perhaps one could make sense of concepts such as cause and effect, because it would be possible to "change the course of events". Nevertheless, the only realm

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[1]The transformation $t \to -t$, $\mathbf{r} \to -\mathbf{r}$, $\mathbf{d} \to -\mathbf{d}$ implies $J^\mu \to J^\mu$, $\partial_\mu \to -\partial_\mu$, $\Box \to \Box$, $A^\mu_\pm \to A^\mu_\mp$.

where multiple outcomes may follow the same initial conditions in physics is quantum theory. There, the selection among several possibilities is determined by chance, i.e., it occurs without cause. Although the course of events is not written in advance, it is not possible to control it or guide it. We must therefore concede that quantum theory is not a promising candidate to resurrect the notion of causality.

It is worth stressing that time does not possess an arrow in quantum mechanics either. It is true that the choice of a particular future among many options (e.g., the choice between left and right in the Stern-Gerlach experiment) is created *ex nihilo*. At first sight, this uncaused selection may appear to endow time with a direction. However, this is not true, because we can mirror the statement by looking backwards in time: the present may originate from different pasts, and it is impossible to uniquely trace back the past that led to a particular present.

Consider an electron that is prepared with spin $+1/2$ along the $z$ direction. If we then plan to measure its spin along the $x$ direction, we cannot predict whether the result will be $+1/2$ or $-1/2$. This unpredictability is due to the state being a superposition of $x$-spin eigenstates. Symmetrically, if we find that the spin is, say, $-1/2$ along the $x$ direction, that knowledge alone does not allow us to uniquely infer its preceding state: it is impossible to determine whether the particle was preceded by spin $+1/2$ or $-1/2$ along the $z$ direction. This perfect symmetry in the uncertainty of determination – both forward in time (prediction) and backward in time (retrodiction) – highlights the acausal nature of quantum measurement without an inherent arrow of time.

# 3    Causes as truly external entities

Some light is shed on our pondering by noticing that talking about causes and effects can only make sense if there is a clear distinction between the system under observation and something *external* to the system, such as a source or a force. To better explain this point, we consider two systems that involve modifications to the second Principle of Dynamics, $ma = F$, and are supposed to violate causality. One is Dirac's method to remove runaway solutions in classical electrodynamics [24, 21]. The other is the case of fakeons, or purely virtual particles, which stand as promising tools for a variety of applications.

It is useful to compare these two systems face to face to highlight their differences and commonalities. While fakeons are claimed to be fundamental entities, Dirac's theory is an effective description of the "friction" and energy loss due to the emission of radiation by an accelerating charged particle. Furthermore, in Dirac's case time possesses an arrow

(a detail which is irrelevant to our main argument, but which might otherwise inject a source of confusion into the discussion), whereas fakeons are invariant under time reversal. Other differences (e.g., Dirac's treatment is purely classical, while fakeons, which are due to unitarity in quantum field theory, do not have a classical counterpart, yet affect ordinary particles indirectly) are not crucial for the discussion of this paper: both systems are valuable for illustrating the conceptual points we wish to make.

Let us start from $ma(t) = F(t)$, where $F(t)$ is an external force. We may assert that $F$ is the cause and the acceleration $a = \ddot{x}$ is the effect. The equation is deemed causal, because the trajectory, given by

$$x(t) = \frac{1}{m} \int_0^t (t - t')F(t')\mathrm{d}t' + v_0 t + x_0,$$

clearly demonstrates that $x(t)$ at any time $t > 0$ is solely determined by the force $F(t')$ at earlier or equal times $t' \leqslant t$. To determine the trajectory $x(t)$ in the future up to a time $t_+ > t$, we must control, or otherwise know in advance, the force $F(t)$ until that time. Since this requirement seems to raise no objection, at least in principle, we conclude that the system is causal.

Instead of $ma = F$, in Dirac's case [24, 21] we encounter the equation

$$m\ddot{x}(t) = \frac{1}{\tau} \int_t^\infty \mathrm{d}t' \, \mathrm{e}^{\frac{t-t'}{\tau}} F(t'), \tag{3.1}$$

where $\tau$ is a constant with the dimension of time. This formula is generated by the parent local, higher-derivative equation

$$m\ddot{x}(t) - m\tau \, \dddot{x}(t) = F(t), \tag{3.2}$$

upon inversion of the operator $1 - \tau(\mathrm{d}/\mathrm{d}t)$ through "Diarc's prescription", which requires perturbativity in $\tau$. The correction appearing in the left-hand side of (3.2) is the Abraham-Lorentz force, responsible for the infamous runaway solution, which solves (3.2) but does not solve (3.1). Thus, Dirac's equation (3.1) removes the runaway solution of (3.2).

We may assert that equation (3.1) violates causality. Indeed, the solution

$$x(t) = \frac{1}{m\tau} \int_0^t \mathrm{d}t_1 \int_0^{t_1} \mathrm{d}t_2 \int_{t_2}^\infty \mathrm{d}t_3 \, \mathrm{e}^{\frac{t_2-t_3}{\tau}} F(t_3) + v_0 t + x_0$$

shows that, in order to predict $x(t)$, it is not sufficient to know $F(t')$ at times $t' \leqslant t$. Due to the damping factor $\mathrm{e}^{(t_2-t_3)/\tau}$, to accurately predict the trajectory up to time $t$, we require knowledge of $F(t_3)$ up to times $t_3 \simeq t_2 + \tau$, with $t \geqslant t_1 \geqslant t_2$. Ultimately, this implies that we need $F(t')$ up to times $t' \simeq t + \tau$.

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In other words, if we want to predict the future, we have to anticipate the external force in a little bit *more* future. Crucially, what happens within an interval of time of order $\tau$ is *out of our predictive control.*

Now we highlight the crucial point of our argument: the notion of causality, as well as its violation, is meaningful only because we label $F$ as an *external* force. We cannot apply the same reasoning if the force is internal, such as one arising from self-interactions.

To illustrate what we mean, consider an elastic force $F = -m\omega^2 x$. Then equation (3.1) becomes

$$\ddot{x}(t) = -\frac{\omega^2}{\tau} \int_t^\infty dt' \, e^{\frac{t-t'}{\tau}} x(t'),\tag{3.3}$$

and the solution is given by the damped oscillations [21]

$$x(t) = c e^{\lambda t} + c^* e^{\lambda^* t},\tag{3.4}$$

where $c$ is a complex constant, while

$$\lambda = \frac{1}{3\tau}\left[1 - \frac{1}{(-1)^{1/3}W} - (-1)^{1/3}W\right],$$

$$W = \left(1 + \frac{27}{2}\omega^2\tau^2 - \frac{\Upsilon}{2}\right)^{1/3}, \qquad \Upsilon = 3\sqrt{3}\,\omega\tau\sqrt{4 + 27\omega^2\tau^2}.$$

Let us pay attention to the following fact: the force on the right-hand side of (3.3) at a time $t$ is determined by the trajectory $x(t')$ at later times $t' > t$. This might suggest that causality is still violated, because "it is impossible to know the future in advance". However, the system is deterministic, so the future *is*, in fact, known in advance. More accurately, it is predetermined at all times! Ultimately, the solution depends on two initial conditions, such as the position $x_0 = c + c^*$ and the velocity $v_0 = c\lambda + c^*\lambda^*$ at $t = 0$.

Consequently, the model presents no violation of causality when $F$ is internal to the system. We would not be able to say this if $F$ were external. The difficulty suggested by the unusual form (3.3) of the equation of motion is only apparent, an illusion of acausality that is unfounded.

We often think that nature is described by the equations of motion (or the "physical laws") rather than by their solutions. Instead, the equations belong to our description of nature. Therefore, it is of little consequence that perfectly valid trajectories like (3.4) are generated by equations, such as (3.3), that appear acausal to us. In this respect, there is no crucial difference between equation (3.3) and a "causal" equation. In Hume's words, both lead to consistent "trains of events".

This ultimately confirms that the notion of causality loses its meaning in a deterministic system. It may retain some significance when $F$ is external, and only as long as such an $F$

is regarded as exempt from the constraints of determinism. Having noted that quantum theory cannot come to the rescue here, we need to postulate an $F$ that is not bound to obey the laws of physics! The question then becomes: what is such an $F$?

We reach similar conclusions in the case of fakeons. The main difference is that the fakeon equations are supposed to describe fundamental properties of nature, which means that the challenge they pose to our understanding is robust, whereas the challenge posed by an effective theory, such as Dirac's one, can be more easily dismissed as merely apparent. A further key distinction is that the fakeon equations are symmetric under time reversal, thus avoiding the need to burden our discourse with an irrelevant arrow of time.

A typical fakeon equation [21] is

$$m\ddot{x}(t) = \frac{1}{2\tau} \int_{-\infty}^{\infty} dt' \sin\left(\frac{|t - t'|}{\tau}\right) F(t'). \tag{3.5}$$

To solve it, knowledge of the external force $F(t')$ is required at all times $t'$. However, due to the oscillating behavior of the sine function, at the practical level it is sufficient to know $F(t')$ only in a neighborhood $|t - t'| \lesssim \tau$ of the time $t$ of interest. Since this interval involves some future, we say that (3.5) is acausal.

The parent local, higher-derivative equation is

$$m\ddot{x}(t) + m\tau^2 \, \ddddot{x}(t) = F(t), \tag{3.6}$$

which gives (3.5) upon inversion of the operator $1 + \tau^2(d^2/dt^2)$ through the fakeon prescription [21]. Equation (3.6) has four solutions, of which only two satisfy (3.5).

If we replace the external force with an internal one, such as the elastic force $F = -m\omega^2 x$, the fakeon equation (3.5) becomes

$$\ddot{x} = -\frac{\omega^2}{2\tau} \int_{-\infty}^{\infty} dt' \sin\left(\frac{|t - t'|}{\tau}\right) x(t'), \tag{3.7}$$

which has the solution [21]

$$x(t) = ce^{i\Omega t} + c^* e^{-i\Omega t}, \qquad \Omega = \frac{1}{\tau\sqrt{2}}\sqrt{1 - \sqrt{1 - 4\omega^2\tau^2}}, \tag{3.8}$$

for $\omega < 1/(2\tau)$, uniquely fixed by the initial position and velocity. The proper reduction of the set of degrees of freedom to the physical ones can be proved in fakeon equations with generic self-interactions [21].

Again, we see that the right-hand side of equation (3.7) may suggest acausality, since it implies that the force "causing" the acceleration on the left-hand side must be known at all

times. However, the force in question is not external, but a self-interaction. Consequently, it is known at all times "for free" by solving equation (3.7) self-consistently. We thus conclude that no actual violation of causation is present in this system.

We learn that if we want to introduce a meaningful notion of causality, the system cannot be independent of its exterior. The idea of cause therefore does not make sense in an isolated system. It strictly requires the presence of some truly external entity. Since, however, that entity is necessarily internal to a larger system, we conclude that causation has no fundamental meaning in nature.

A popular definition of causality is the requirement that closed timelike curves (CTCs) are forbidden [25], on the grounds that such curves would permit a person to return to their own past by moving forward in time, thereby gaining the ability to change history. However, as previously established, if the system is deterministic, changing the past is impossible, and a CTC provides no help in this respect [26]. As said, one can legitimately claim that the future causes the past, regardless of whether CTCs exist or not. Moreover, the assertion that "one can change one's own past" necessarily presupposes a transcendental power capable of interfering with nature. Then, however, one need not return to the past to alter it: it is sufficient to change the future, since this deterministically affects the past. Again, there is no need of CTCs for this.

Thus, it is evident that CTCs do not violate causality. Unfortunately, the debate on causality is often burdened by ad hoc definitions and overlaps with side concepts, leading to claims of causality violations based solely on breaches of those definitions.

Ultimately, the concept of cause is the result of a sequence of conceptual missteps and misunderstandings. First, one must erroneously assume that the world is neither deterministic nor quantum, i.e., that alternative outcomes are available, but their selection is not governed by quantum chance. Second, one must postulate something external to nature and endow it with the "superpower" to make choices concerning nature. That entity, which would then be designated the "cause", would be deemingly "responsible" for the effects, in a clear reflection of the social notions of guilt and responsibility. Third, that entity acting upon nature would, by definition (because it is supposed to "exist"), have to be part of nature itself. It would then immediately cease to be a cause, because it would not be external to nature.

Once again, we observe that the idea of causation is the side effect (!) of our narrative about nature, but fundamentally does not belong to nature. Even worse: it belongs to the supernatural, because it postulates the existence of something external to nature, not subject to the laws of nature, which can nevertheless act upon nature!

One might counter with the following statement: "I do not know whether I will have a cold tomorrow, yet I can expose myself to fresh air, if I decide to (or am foolish enough): the cold is the effect, and my action is the cause". The weak point of this narration is the hidden assumption that the "I" is an entity not subject to the laws of physics, endowed with the mysterious superpower of acting on nature without being part of it, such as the power often referred to as *free will*. Once the "I" is included into a larger system with the rest of nature (we are made of atoms...), the idea of cause loses its meaning, both at the classical and quantum levels, for the reasons explained previously.

## 4 Prepostdictivity

Causality is commonly believed to grant us the power of controlling and shaping the future. Having demonstrated that this is a myth, we might still cultivate the illusion of possessing the less powerful ability of at least predicting the future. By downgrading the pretense of causality to that of predictivity, we avoid the necessity of introducing supernatural entities that are "external to nature yet capable of acting upon it," and thus exempt from obeying the laws of physics. The claim that we can predict nature is less easy to dismiss. However, as we show in this section, that too is an illusion. For the moment, we adhere to "causal" equations of motions $ma = F$ (with $F$ external, whatever that might imply). We will relax this assumption later.

The relevant question for a physicist, then, is: can we make predictions about the future, relying only upon the present and the past?

Our goal is to demonstrate that there exists no physical situation where we can truly make predictions, that is to say, definitively anticipate the future within a specified region of space. We first conduct our analysis within the framework of Special Relativity (SR), and subsequently extend it to Galilean Relativity (GaR). For simplicity, we restrict our discussion to one-dimensional systems.

Consider "us" as a point particle with trajectory $x(t)$, illustrated in fig. 1. At time $t = 0$ we are located at point O and wish to make a prediction concerning the point P at time $t_+ > 0$.

The maximum knowledge we may possess at O is concentrated in the past light-cone with tip at O, which we denote by Ô. It is important to stress that we may only have knowledge of objects that we have seen or detected somehow. We cannot know much about dark objects (entities that do not emit electromagnetic radiation), nor objects that have been obscured by others. Moreover, we have no knowledge of objects that have never
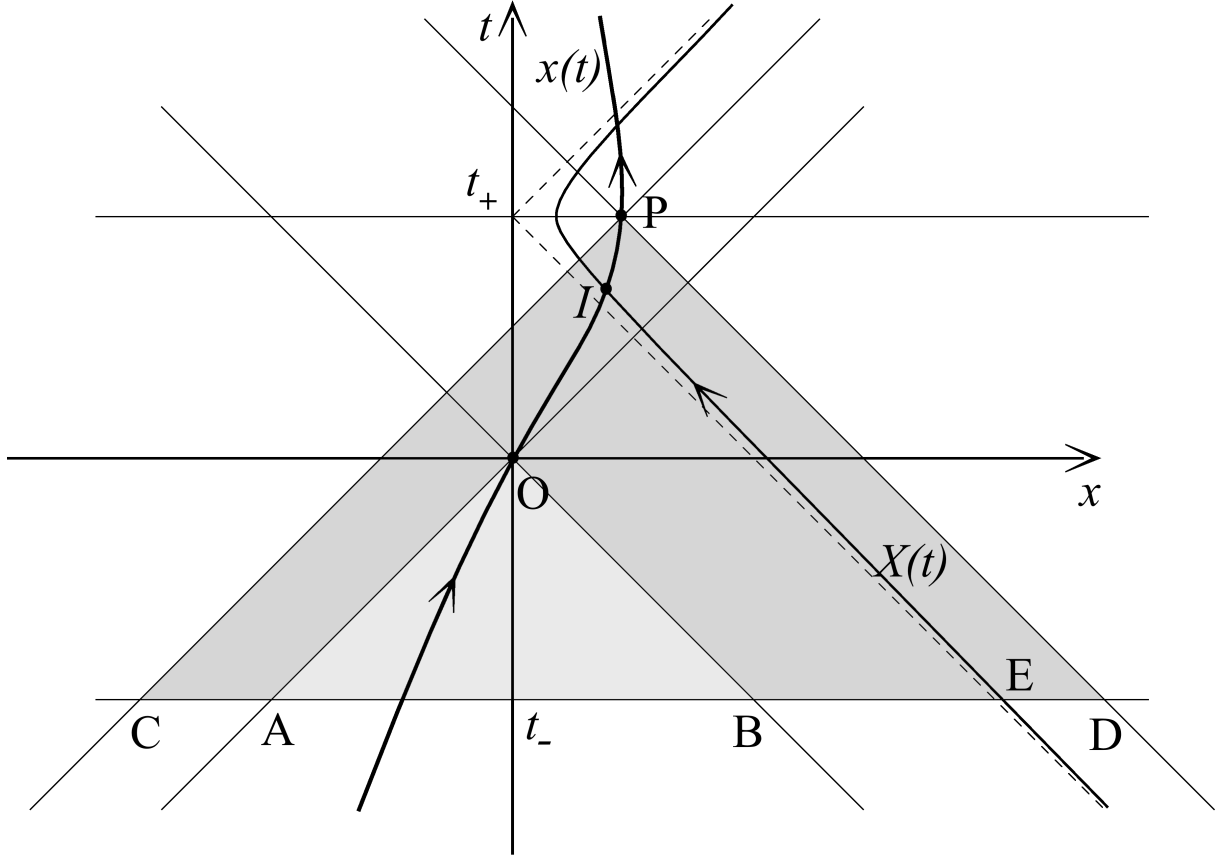
Figure 1: Prepostidictions

crossed Ô in their past histories, but cross the past light cone of an event of our future trajectory, an example being P̂, the past light cone with tip at P.

Consider a body moving along the hyperbolic motion defined by the trajectory $X(t)$ shown in the figure:

$$x^2 - (t - t_+)^2 = r^2, \qquad x \geqslant r.$$

Its past $(t < 0)$ does not intersect Ô (our past light cone at $t = 0$). Its future $(t > 0)$ crosses P̂ (the past light cone at the hypothetical future event P). From O we can have no knowledge of the object in question. Yet, it can disrupt the future of our system at $I$, the point of intersection between $x(t)$ and $X(t)$.

We call such an object a "disruptor". When we want to stress that it has no intersection with the past light cone Ô, we call it an "extradisruptor".

We conclude that, even assuming we live in a deterministic universe, at time $t = 0$ we cannot be certain that we will end at a point P at time $t_+$. This uncertainty exists despite the trajectory predicted by the equations of motion in the absence of disruptors. The

possibility of extradisruptors, objects unknowable from our past light cone $\hat{O}$, but capable of influencing our future trajectory, fundamentally implies that we cannot make definitive predictions about the future.

Assume that we started collecting data at some time $t_- < 0$. Our maximum knowledge at $t = 0$ relies on what intersected $\hat{O}$. Extradisruptors are, by definition, out of reach, as a matter of principle. In practice we cannot even exclude "intradisruptors", such as dark or shadowed objects that intersected $\hat{O}$ but did not send any or enough signals towards us.

Therefore, how can we fix the initial conditions at, say, time $t_-$? The past light cone $\hat{O}$ is a proper subset of $\hat{P}$. Consequently, its intersection with the horizontal constant-time slice $t = t_-$ spans a smaller segment (AB) compared to the intersection of $\hat{P}$ with the same slice, which spans a larger segment (CD). We may have knowledge of objects that intersected $\hat{O}$, sent signals to us, then exited $\hat{O}$ and subsequently reached the segment AB or CD. However, because we lack any knowledge of disruptors, we have no way to fix complete initial conditions in CA and BD at times $t \leqslant 0$. The conclusion is that we cannot predict what is going to happen to "us", or our system, at a later time $t_+$.

For the sake of completeness, let us consider the limit $t_- \to -\infty$. We have the following paradox: every $x$ appears to be in $\hat{O}$, because for every $x$ there is a remote time before which $x$ belongs to $\hat{O}$. Nevertheless, extradisruptors have no intersection with $\hat{O}$, which means that points $x$ outside $\hat{O}$ (i.e., those belonging to the complement of $\hat{O}$ inside $\hat{P}$) must exist. What is important here is that not even taking $t_- = -\infty$ allows us to gather enough initial conditions to predict the future beyond O. Even in the best-case scenario, extradisruptors cannot be excluded.

Let us examine potential ways to circumvent this difficulty. An option is to assume that the system is isolated. As common as this assumption is, it is not realistic, as we cannot build walls or shields sufficiently robust to ensure that no external object, like the one with trajectory $X(t)$, can disrupt the experiment. Another possibility is to assume that nothing else exists in the universe besides the point particle with trajectory $x(t)$. This, however, is an unjustified idealization, and a metaphysical stretch, since it amounts to making assumptions about regions of spacetime that are fundamentally inaccessible to us without invoking superluminality.

Ultimately, the best we can do is make our "prediction", hope that no disruptions, like the ones previously described, occur in the meantime, and check a posteriori that the outcome in P is the predicted one. Then, and only then, can we assert that *we were able* to predict it. However, this retrospective verification process is fundamentally a postdiction, which is why our initial statement was not a prediction, but rather a "prepostdiction".

At first, one might assume that the limitations just described are inherent to Special Relativity, hoping that they are absent in nonrelativistic mechanics. This is incorrect, as analogous difficulties persist in the limit $c \to \infty$, where SR reduces to Galilean Relativity. In this limit, the past light cones depicted in the figure become half-planes, yet the knowledge available at O is still insufficient to determine the future at P. In GaR, macroscopic bodies can move at arbitrarily high speeds, making it impossible to exclude that a heavy item could traverse a very large distance during the time interval between $t = 0$ and $t_+$, thereby disrupting the outcome at P, regardless of the robustness of any wall built to supposedly isolate the system. To fully exclude such a possibility, we would need to specify the initial conditions of every object in the universe, which is patently impossible. Once again, we are left to hope for the best, bet that no disruption occurs in the meantime, and check a posteriori at P whether we got away with it or not. Thus, even in the nonrelativistic limit, we can only make prepostdictions.

## 4.1 Disruptors, incoming waves and statistics

An objection might be raised: why has no unfortunate situation like those feared here ever disrupted any experiment[2]? If disruptions are so statistically disfavoured, why should we bother? Isn't discarding such occurrences equivalent to assuming standard boundary conditions at infinity in classical electrodynamics (i.e., postulating that there are no sources at infinity sending signals towards us)? Otherwise we would not be able to trust solutions that rely solely on the retarded potentials. Isn't it paranoid to suspect that nature is conspiring from far away to mess with us by disrupting our experiments "on purpose"?

The core point is that we are questioning a candidate *physical principle* here: the ability to truly predict in physics. If a statement holds true only barring extremely rare and unlikely situations, it cannot be elevated to the rank of a fundamental principle. While conceptual shortcuts are more than sufficient for everyday life and many common physical situations, they are not guaranteed to be adequate when inquiring about quantum gravity, the microscopic world, or, more generally, the unknown. This distinction constitutes the rationale of our investigation. We are questioning whether the methodology usually adopted is strategically sound for research on fundamental interactions. Our suggestion is that it is not.

Returning to the common, and apparently innocuous, assumption of "no incoming radiation from infinity", which dictates the use of the retarded potentials in classical elec-

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[2]A physical scenario that may illustrate the failure of prediction is the phenomenon of rogue waves (which may be seen as intradisruptors) and possibly tsunamis (extradisruptors) in a stormy sea.

trodynamics, this choice concerns regions of spacetime that are inaccessible to us, even in theory. Consequently, the adoption of the retarded solution, such as (2.2), over the advanced one, such as (2.3), relies on an unprovable assumption: in effect, a "no-disruptors" bet, and a metaphysical stretch.

Every light signal perceived by our eyes is a potential disruptor. Indeed, 1) it originates from regions of spacetime that were inaccessible to us prior to the moment of perception; and 2) its source may have remained outside our past light cone until that very moment. Is it possible to gather enough knowledge to predict with absolute certainty, at least in theory, what we will see in a minute from now? The answer is no, and not merely for practical reasons, but as a matter of principle. All we can do is wait and see.

Under "normal" circumstances, the retarded choice is natural at the macroscopic level. It is statistically implausible that, once an oscillating dipole such as (2.1) is turned on, a coherent influx of waves coming from infinity would conspire to transform it into a "sink". This is the key point of our argument: the choice between (2.2) and (2.3) is dictated by statistics; it is not a matter of cause and effect.

Let us dig more into this. In our universe, the retarded pair (2.1)-(2.2) describes emission, whereas the advanced pair (2.1)-(2.3) describes reception. Upon our input, a smartphone generates a motion of charged particles, as in (2.1), to encode, for instance, a speaking voice. This produces a field like (2.2), that is, the emission of a signal for communication. Conversely, an incoming field such as (2.3) reaches the smartphone from an external source, independent of any input from us, and induces a motion like (2.1) of charged particles within the antenna. The device then translates this signal into data, a voice, or a message. The call is a potential disruptor, since it originates from a region of spacetime that remained inaccessible until the very moment the phone began to ring.

Now, consider a "mirror" universe characterized by inverted boundary conditions at infinity and a statistical framework entirely reciprocal to our own, where highly improbable events manifest consistently, coincidence after coincidence, over vast timescales[3]. For instance, an incoming field such as (2.3) would be coupled with a deliberate action, such as the oscillation (2.1) of charged particles triggered by our command; notably, this process would result in a net gain of energy rather than its expenditure. Conversely, whenever an outgoing field like (2.2) occurs, it corresponds to a sudden, unplanned oscillation like (2.1), triggered by thermal noise.

Should we conclude that (2.3) is the cause of (2.1)? If so, it would imply that our

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[3]The mirror scenario we are describing could just be an extraordinary statistical fluctuation within our own universe.

will is determined by the incoming field. In this view, *free* will would not exist: human beings would be governed by nature like automata[4]. Alternatively, one could insist that "(2.1) is the cause of (2.3)", that is, not only do we possess free will, but it is so potent that it deterministically alters the past. From a purely physical standpoint, the choice of narrative is indifferent.

It must be emphasized that this argument hinges on the presence of a "subject". Otherwise we would be unable to distinguish "planned" or "wanted" actions from "unplanned" ones, rendering the discourse even more tenuous. Thus, the conclusion remains that notions such as cause, or free will, cannot be sustained unless one invokes either metaphysics (the subject), or statistics (the law of large numbers, or the boundary conditions at infinity).

The important point is that (in our universe), the retarded choice (as well as the advanced one, when it applies) is driven by statistical properties, inherent to the macroscopic world. It is not a fundamental property of nature. Hence, extending that choice, or an equivalent one, to the microscopic world is unjustified. There is no reason why the universe at infinitesimal scales should resemble the universe at large scales (which means: a desert expanse, where the assumptions of no incoming radiation and no disruptors feel natural), instead of, say, a turbulent, stormy sea. For example, fakeons, through equations like (3.5), contemplate a superposition of incoming and outgoing waves at the microscopic level, precisely recalling a stormy sea.

The conclusion is that the arrow of time resulting from causality (possibly downgraded to predictivity) resembles the thermodynamic arrow of time, in the sense that both rely on the law of large numbers. In the same way as statistical irreversibility does not hold at the microscopic level (given that the law of large numbers breaks down with small numbers), causation and predictivity lose meaning at small distances (even in the presence of external forces), because assumptions barring disruptors or incoming waves, which rely on statistics, cannot be trusted there. These arguments point towards causation and predictivity as emerging properties of (our description of) the universe rather than fundamental ones, thereby making the abandonment of microcausality and micropredictivity not only

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[4]Note that we would have no possibility to "change our mind" after gaining knowledge about an incoming wave. Specifically, assume that Bob is positioned at $r > 0$ and perceives an incoming wave at time $t - r$ converging towards the origin. Bob then knows that someone there, say Alice, will activate $J^\mu$ at time $t$ (given our assumption of inverted statistical laws). However, Bob cannot reach Alice in time to intervene, as the crucial information would need to travel faster than light to get to her before she enacts her intention. In other words, the incoming wave acts as an extradisruptor for Alice. Alice and Bob can only meet later and share their story, at which point Alice would discover that she has no free will, since Bob knew her plans in advance.

acceptable, but also necessary.

In passing, it is intriguing to note that fakeon theories imply that the universe is endowed with a "radial arrow" pointing from the microscopic "stormy sea" to the macroscopic "desert expanse". If fakeon models describe reality, this is a rigorous arrow, not just an approximate one.

## 4.2   Remarks

We end this section by addressing other, minor issues on prepostdictivity. Disruptors are part of the "external forces" $F$ that enter the equations of motion $ma = F$. Hence, it may be objected that we can still describe the disruptions, whenever they occur, by means of ordinary physical laws. The key point is that we can only describe them a posteriori, but we cannot anticipate them a priori. External forces like these cannot be arranged as we wish, controlled or predicted. Was not the ability to control nature the rationale for postulating causes and "external" forces in the first place?

A possibility is to downgrade the predictions to statistical ones, in the following sense. If the same experiment is repeated several times, the disruptions will not be the same, which allows us to filter their effects away, so to speak. Yet, how do we correctly average over them? What exactly constitutes an "average prepostdiction"? How can we quantify the statistics of disruptions without making assumptions about unreachable regions of spacetime? Besides, when we resort to statistical predictions, as we do in quantum mechanics, it is precisely because we cannot predict the outcome of a single experiment.

Summarizing, when we downgrade causation to mere chronological ordering, we are essentially demoting the major purpose of controlling, or shaping, the future evolution of a system to the more modest goal of predicting it. However, as demonstrated, it is unjustified to elevate predictivity to a fundamental principle, or extend it to the microscopic world. A fortiori, the same conclusion applies to causality.

The impossibility of predicting the future cannot be entirely mirrored onto the impossibility of tracing back the past. It is true that if we begin collecting data at some time $t = t_+$ at point P, we cannot know whether intradisruptors have changed the course of events at $t < t_+$. However, we cannot mirror extradisruptors, since they always intersect past light cones lying in the future. This means that we may have knowledge of them when we trace back the past, at least in principle. To some extent, this introduces a sort of arrow pointing from the present to the future. However, this is still not a fundamental property of the universe. It merely reflects our ability to keep memory of events, a possibility that arises when macroscopic or non isolated systems are involved. At the microscopic level we

have perfect symmetry, as shown in the example of Section 2 concerning spin measurements along the $x$ and $z$ directions. Besides, if we want the system to be visible to us, it cannot be isolated, since it must radiate towards us.

# 5   Delayed prepostdictivity

In the previous section we argued in the realm of "causal" equations of motion $ma = F$. What about prepostdictivity in fakeon equations like (3.5), or Dirac's equation, (3.1)?

In "plain" prepostdictivity (the one associated with $ma = F$), we need to know the external force $F$ up to the moment $t_+$ of our "prediction". We have established that, since $F$ is beyond our control for the reasons explained before, we cannot really predict. We can at most cross our fingers and prepostdict. Yet, we do not need to know $F$ *beyond* the time $t_+$.

On the other hand, in systems governed by nonlocal equations like (3.5) and (3.1), we need to know $F$ in a little bit more future, which is approximately $t_+ + \tau$, where $\tau$ is the characteristic scale of nonlocality. Thus, at time $t_+$ we cannot postdict what happened till then. We can at most postdict what happened till $t_+ - \tau$. To postdict till $t_+$, we have to be more patient and wait till $t_+ + \tau$. This means that if we are making a statement at time $t = 0$ that aims to reach as far as $t_+$, it is not enough to cross our fingers till $t_+$. We need to cross our fingers till $t_+ + \tau$.

The outcome is that in fakeon systems, as well as in Dirac's one, we do not have plain prepostdictivity, but rather a *delayed* prepostdictivity, with the delay being quantitatively defined by the characteristic time scale $\tau$.

We can illustrate this fact more explicitly under the assumption that $\tau$ is small. Then equations (3.1) and (3.5) can be written as

$$\text{Dirac:} \qquad m\ddot{x}(t) = F(t + \tau) + \mathcal{O}(\tau^2) = F(t) + \Delta_\tau F(t) + \mathcal{O}(\tau^2),$$
$$\text{fakeon:} \qquad m\ddot{x}(t) = F(t) - \Delta_\tau^2 F(t) + \mathcal{O}(\tau^4),$$

respectively, where $\Delta_\tau F(t) = F(t + \tau) - F(t)$ is the forward difference, while $\Delta_\tau^2 F(t) = F(t+\tau) - 2F(t) + F(t-\tau)$ denotes the second central difference. We see that the right-hand sides explicitly involve the force in the future time $t + \tau$.

In passing, we note that a truncated equation such as $\ddot{x}(t) = -\omega^2 x(t + \tau)$ (a harmonic oscillator with a delayed elastic force) admits infinitely many solutions, as is generally expected of nonlocal formulations. Instead, the Dirac and fakeon equations, while nonlocal, are of special types, being derived from parent local equations. Their solution space is just the physically expected one [21].

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Coming back to our problem, a possible way out is to interpret $\tau$ as the minimum time resolution, that is to say, to postulate that we cannot experimentally distinguish events that are separated by time intervals shorter than $\tau$. The "present" is therefore defined as "present within an interval of time of order $\tau$". In this scenario, $t$ and $t+\tau$ can be regarded as "the same time", and the equations may be considered causal.

Even when we postulate a truly external force $F$, the violation of causality fades away as soon as we interpret $\tau$ as the minimum time resolution. This means that, strictly speaking, we cannot definitely claim that the Dirac and fakeon equations imply the violation of causality, not even in the presence of external entities: they may merely imply the existence of a fundamental limit on temporal resolution.

The crucial difference is that Dirac's equation is effective, so the minimum time resolution $\tau$ it contains cannot hint to a fundamental property of nature. In the fakeon case, $\tau$ may suggest a fundamental impossibility of distinguishing events separated by time intervals shorter than $\tau$. However, as we now demonstrate, neither is this true.

It should be noted that the interpretation of $\tau$ as the minimum time resolution makes sense only if the force $F$ is truly external. If $F$ is a self interaction, there is no problem in interpreting solutions such as (3.4) and (3.8) for arbitrarily short intervals of time. Given that any external force is internal to a larger system, the fakeon models do not predict a *fundamental* time resolution $\tau$ in the universe. Neither does Dirac's system, because it is inherently an effective model.

Thus, even if we adhere to the premise that there exist entities external to nature that can act on nature, it is still not true that fakeon theories violate causality, unless we assume that we can experimentally measure (and thereby give physical sense to) arbitrarily short intervals of time. Indeed, as soon as we accept the possibility that time in nature is equipped with a minimum time resolution $\tau$ (applicable only to those external entities), discussing violations of causality below $\tau$ becomes meaningless.

Ultimately, we reach a situation where the limitations represented by $\tau$ concern the external force, and not nature itself. Even if we hypothesize the existence of entities external to nature but endowed with the superpower of acting upon it, quantum gravity with fakeons compels us to accept limitations on their power (whether it is a delay of prepostdictivity or a minimum time resolution). In other words, nature, i.e., an "infrapower", is able to place limits on a supposed "superpower" transcending nature. A physical limitation on something that does not even exist physically, but merely encodes our pretense of controlling nature, is not a huge price to pay, especially if the reward is a testable theory of quantum gravity.

# 6  Causality in quantum field theory

The two main definitions of causality in quantum field theory are Bogoliubov's condition and the Lehmann-Simanzik-Zimmermann (LSZ) formulation [27].

Bogoliubov's condition [28] applies to diagrams and off-shell correlation functions. In simplified terms, it requires that if a spacetime point $x_1$ is in the timelike future of $x_2$, diagrams involving both $x_1$ and $x_2$ can be arranged so that energy flows only forward in time, from $x_2$ to $x_1$.

The problem with this definition is that one cannot accurately assign spacetime points to on-shell particles, because relativistic wave packets spread, thus obscuring the notion of a definite particle position. Hence, Bogoliubov causality cannot be formulated as a constraint on the S matrix itself, but merely applies to off-shell correlation functions. These are, in general, gauge dependent. Although they can be made gauge invariant (even for insertions of elementary fields, by working with physical degrees of freedom only), this may come with the price of introducing nonlocalities (see next section), which obscure a direct spacetime cause $\rightarrow$ effect picture.

The LSZ definition [29] of causality is enforced via field commutators, which are required to vanish for spacelike separation. When fields are expressed in terms of diagrams, this condition reduces precisely to Bogoliubov's energy-flow condition. From our perspective, working with fields is no different than working with off-shell correlation functions. Consequently, the conceptual difficulties associated with the gauge dependence and the obscurity of a direct spacetime cause $\rightarrow$ effect picture remain fully relevant even under the LSZ formalism.

The main weakness of these conditions, and of alternatives that have been proposed in the literature, is that they sound merely technical, or even artificial. It is hard to relate them to an intuitive notion of causation, which is itself problematic for the reasons explained in the previous sections. Aside from the risk of overlapping causality with non superluminality, the proposed notions often become entangled with locality and analitycity, whereas one needs to keep such properties distinct from one another, especially because analyticity is not a physical principle and locality is mostly a convenient requirement. There is no doubt that the Bogoliubov and LSZ conditions imply constraints on physical quantities, but they offer no clear motivation why nature should conform to those. Ultimately, causality in QFT sounds like a condition on the mathematical scaffolding, not on the physical predictions.

In quantum gravity with fakeons, the Bogoliuobov and LSZ conditions are expected to be violated at the microscopic level. These violations are the quantum counterparts of the

requirement that the external force $F(t)$ be known in a little bit more future, according to the classicized equation (3.5), as discussed in the past section. The outcome, which we reiterate here, is that if fakeons participate in the description of nature in any way, they put limits on the illusory superpower of controlling, or even just predicting, nature. The apparent paradox that an infrapower can constrain a superpower evaporates as soon as we accept the obvious fact that nothing can act on nature without being part of it, thereby collapsing the Platonic illusion of a world of ideas existing "above" or even just "outside" nature, yet capable of acting on it (as "souls", for example).

One could say that quantum gravity cuts to the chase and tolls the death knell for causation, but one could also argue that the QFT framework of the Standard Model of particle physics had already planted the seed of doubt so deeply that quantum gravity does not actually add much. Both positions sound well motivated, with the caveat that fakeons may have a stronger impact in prompting one to pause and delve into the issue profoundly enough to settle the matter once and for all.

# 7   Nonlocality in gauge theories, gravity and fakeon models

The other unusual feature of equations like (3.1) and (3.5) is their nonlocality, which may motivate some criticism by those who believe in some form of locality as a fundamental principle.

It should be noted that, more than a principle, the requirement that the classical Lagrangian should be local in quantum field theory is merely a recipe that has worked successfully so far, especially in conjunction with renormalizability. Minor tweaks to this assumption are not anticipated to pose dramatic risks.

We distinguish between hard nonlocality and soft nonlocality. Hard nonlocality refers to theories, such as those pursued by Krasnikov, Kuz'min, Tomboulis, Modesto and others [9], where the classical Lagrangian is genuinely nonlocal. Soft nonlocality refers to the nonlocality encountered in fakeon theories. Crucially, in both cases, the nonlocality is sufficiently restricted to ensure that the equations of motion are not burdened with the need to specify infinitely many initial conditions: the standard physical initial conditions suffice [21].

The fakeon models do not possess a true classical Lagrangian. One can derive a "classicized" Lagrangian (which is nonlocal) from a parent, local Lagrangian by integrating out the fakeon fields with the appropriate prescription. An example of classicized Lagrangian

is (7.2) below, derived from (7.1).

The descent is a projection that recalls (with due differences) the projection of the Lagrangian in gauge and gravity theories, obtained upon elimination of the unphysical degrees of freedom (which are: the temporal and longitudinal modes of the gauge fields; those of the fluctuation of the metric around flat space; the trace of the fluctuation of the spatial metric; and the Faddeev-Popov ghosts). The result is nonlocal in those familiar cases as well, even in the classical limit, although this fact mostly goes unnoticed.

Consider, for example, classical electrodynamics. The Lagrangian

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu + ieA_\mu) - m)]\psi$$

is local, but contains fields that do not correspond to physical degrees of freedom. If we choose a simple gauge such as $A_L = 0$, for some longitudinal component $A_L$, we obtain

$$\mathcal{L}'_{\text{QED}} = \frac{1}{2}(\partial_\mu \mathbf{A}_\perp)(\partial^\mu \mathbf{A}_\perp) + \frac{1}{2}(\partial_\perp \cdot \mathbf{A}_\perp)^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \mathbf{J}_\perp \cdot \mathbf{A}_\perp$$
$$-\frac{1}{2}A_0\Delta A_0 - (\partial_\perp \cdot \mathbf{A}_\perp)(\partial_0 A_0) - \rho A_0,$$

which is still local (at least in a suitable frame, such as $A_L = A_3$), but explicitly contains the unphysical (nonpropagating) component $A_0$ of the vector potential, besides the transverse components $\mathbf{A}_\perp$. Here $\rho = e\psi^\dagger\psi$ is the charge density and $\mathbf{J}_\perp = e\bar{\psi}\gamma_\perp\psi$ is the transverse current.

Integrating out $A_0$ gives the truly nonlocal, classical Lagrangian

$$\mathcal{L}''_{\text{QED}} = \frac{1}{2}(\partial_\mu \mathbf{A}_\perp)(\partial^\mu \mathbf{A}_\perp) + \frac{1}{2}(\partial_\perp \cdot \mathbf{A}_\perp)^2 + \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + \mathbf{J}_\perp \cdot \mathbf{A}_\perp$$
$$+\frac{1}{2}(\partial_\perp \cdot \dot{\mathbf{A}}_\perp - \rho)\frac{1}{\Delta}(\partial_\perp \cdot \dot{\mathbf{A}}_\perp - \rho),$$

which contains the physical degrees of freedom only and demonstrates the necessity of nonlocal terms at the classical level when unphysical modes are eliminated.

Similarly, in theories with fakeons the classicized, nonlocal Lagrangian is the projection of a parent local one by means of the fakeon prescription. For example, consider a simple model with the parent local Lagrangian [21]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{1}{2M^2}\phi\left[(\Box + m^2)^2 + M^4\right]\phi - \frac{g}{2}\phi\varphi^2, \qquad (7.1)$$

where $\varphi$ is a physical field and $\phi$ is an extra field that we want to quantize as a fakeon. Integrating out $\phi$ in the appropriate way, we obtain the classicized, nonlocal Lagrangian

$$\mathcal{L}_{\text{cl}} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{m^2}{2}\varphi^2 + \frac{g^2}{8}\varphi^2 \left.\frac{M^2}{(\Box + m^2)^2 + M^4}\right|_{\text{f}}\varphi^2, \qquad (7.2)$$

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where the subscript "f" stands for the fakeon Green function (see [21]).

This part of the discourse does not extend *verbatim* to Dirac's case, where a local Lagrangian for (3.2) is not available. Yet, the manipulations described earlier at the level of equations of motion (3.1) and (3.2) are sufficient to convey the intended idea.

The other point is that, although there exist local, gauge invariant observables in gauge and gravity theories, they do not form a basis. A basis must include nonlocal observables. For example, in QED we have local observables such as $F_{\mu\nu}$, $\bar{\psi}\psi$, $F^{\mu\nu}F_{\mu\nu}$, etc., but no local "electron observable". Yet, we know that we can observe the electron. To explain this, we need to switch to nonlocal observables, such as

$$\exp\left(-ie\frac{1}{\Delta}(\nabla \cdot \mathbf{A})\right)\psi, \tag{7.3}$$

a trick again suggested by Dirac [22]. More generally (as discussed in refs. [23]), we can consider expressions such as

$$\exp\left(ie\left.\frac{1}{\Box}\right|_{\mathrm{f}}(\partial^\mu A_\mu)\right)\psi, \tag{7.4}$$

where the fakeon prescription is used to invert the D'Alembertian. The reason for preferring (7.4) over (7.3) is that (7.3) explicitly breaks Lorentz invariance. At the same time, simply replacing the Laplacian $\Delta$ with minus the D'Alembertian $\Box$ (and $\nabla \cdot \mathbf{A}$ with $\partial^\mu A_\mu$) would introduce unphysical degrees of freedom if a different prescription were used for the $\Box^{-1}$ operator. The fakeon approach to gauge-invariant observables can be generalized to non-Abelian theories and gravity (see [23] again).

Thus, we can assert that gauge and gravity theories are inherently nonlocal. This realization makes it harder to dismiss the extra nonlocality brought in by fakeons. The two types of nonlocality are actually in the same class, since both descend from a parent locality through projection.

We recall that, while unitarity in gauge and gravity theories is ensured by the Ward–Takahashi–Slavnov–Taylor identities following from the local symmetries, unitarity in fakeon models follows from a modified diagrammatics [5], based on non-time-ordered correlations functions [6]. The lack of time ordering at high energies is another (expected) facet of the issues discussed here.

# 8  Conclusions

The debate on causality in physics is a typical example of a false start, i.e., taking the concept for granted without adequately investigating it first, and spinning the discussion

around it endlessly. In this paper we have considerably extended the widespread skepticism surrounding the notion of cause, driven by the developments brought about by quantum gravity.

The key question is: what constitutes a "cause"? We have argued that causation can only have a meaning when it refers to entities that are external to the physical system under observation. By this we mean external in an extreme sense: "external to physics", since they are not allowed to obey physical laws, otherwise they would be internal to a larger system and would consequently lose their essence as "causes". Examples are concepts like free will or soul. Pushing this argument further, it becomes clear that the idea of cause belongs to metaphysics, or the transcendental, precisely like the notions of free will and soul, hence it should be abandoned in fundamental science.

First quantum mechanics, then quantum field theory, and, more recently, quantum gravity, have fueled the need to place the notion of cause under close scrutiny. Quantum field theory poses nontrivial challenges to defining causation in a satisfactory way, and quantum gravity, specifically the fakeon framework, cuts to the chase by dismantling the notions of "event" and chronological ordering for sufficiently short intervals of time, even in the presence of hypothetical truly external forces.

If controlling nature and making it do what we want is too much to ask, let us content ourselves with predicting it. Or should we? Hume argued that (conveying his central argument, though not in his exact words) the mere observation that a freely falling body has always fallen to the ground up to the present day does not provide logical certainty that it will do so tomorrow. In other words, the laws of physics are not truly "laws," but rather postdictions (if viewed from the future to the past) and bets (if viewed from the past to the future). We bet that tomorrow a free body will still fall to the ground. If it will not, we will (opportunistically) modify the physical "law" by adding a correction to account for the new effect, whatever it may be. From that moment onward, we will make refined bets by incorporating the correction. There is nothing in this way of proceeding — which is, in fact, how we proceed — that allows us to elevate our findings to "principles" or physical "laws" and be confident that nature will conform to "our" laws tomorrow in the same way as it did so far.

Even in this context, we extended Hume's skepticism significantly. Suppose that the physical laws as we know them are indeed laws that bind nature tomorrow and forever. We have argued that even then we cannot make predictions about the future. Not only, but we have shown that this follows from the supposed "laws" themselves! We can never exclude that disruptors will appear "out of nowhere", so to speak, and change the final

outcome. Systems cannot be sufficiently isolated, nor can initial conditions be completely fixed. These limitations hold even without questioning the future validity of the so-called physical laws.

Consequently, physicists can only make prepostdictions (requiring retrospective verification) at large scales, relying on the law of large numbers to dismiss potential disruptions. This confirms that the illusory arrow of time associated with causality is inherently statistical, and emerges solely at the macroscopic level, thereby rendering microcausality unwarranted and strained.

Not much of this changes when quantum gravity is included, where we can at most make *delayed* prepostdictions. In theories with fakeons, the delay is the reciprocal of the fakeon mass $m_\chi$. If this amount of time is short enough (about $10^{-37}$ seconds for the gravifakeon), it remains consistent with observation [30]. Ultimately, the further renunciation entailed by quantum gravity with fakeons is not overly demanding.

Since the birth of science, physicists have lived under the mirage that they could predict *something.* This is an illusion that has never truly held. The truth is that we just place bets and cross our fingers: a posteriori, we can verify that events unfolded as expected, but we have no way of guaranteeing *a priori* that it will indeed be the case.

To some, these might sound like matters of hairsplitting, but confronting the problem of quantum gravity is precisely the kind of challenge where splitting hairs may truly matter. Others may think that the positions expressed here are "extreme", but we have emphasized that what is truly extreme is postulating the existence of transcendental entities (such as the so-called "causes") that are outside nature, yet endowed with the superpower of acting on nature (setting aside the absurdity of assuming the "existence" of those entities, given that what "exists" is part of nature by definition). Brushing these issues aside and continuing research as before comes with the risk of overlooking profound opportunities. It is better to accept that quantum gravity may require walking on thin ice, and cope with the fact that exploring the unknown may well demand that we challenge even the most basic principles we have long taken for granted.

String theory, for example, is claimed to be strictly causal. One of its many flaws is that in order to reproduce all of its vibrational modes, it effectively contains an infinite tower of particle states. Another is that its standard formulation is inherently restricted to computing on-shell scattering amplitudes between asymptotic states. After decades, no accepted formalism to describe off-shell quantities is available. A third flaw is its lack of predictivity. These are huge prices to pay to perpetuate a controversial "principle" (perhaps even a stereotype) such as causality. And why should we be willing to pay that

price? To maintain the illusion that we are external to nature and possess the supernatural power of controlling it? This is not a physically justified assumption. The moral of the story is that assuming strict causality is unphysical and restricts the range of theories we allow ourselves to explore. Possibly, it excludes the correct solution to the problem of quantum gravity — for instance, the theory of quantum gravity with fakeons, which is predictive and just contains a triplet (the graviton, the "Starobinskion", and the gravifakeon).

These and other facts we have examined expose the notion of causation for what it truly is: a conceptual mirage, a nonexistent solution to a nonexistent problem, a camouflage of Plato's "ideas" (which can act on nature without being part of it). Not only the notion of cause is incompatible with determinism and quantum theory, but it blatantly transgresses into metaphysics.

That said, we have shown that concepts such as purely virtual particles, or fakeons, are not as revolutionary as they might first sound, in the sense that they do not represent a dramatic departure from the previous understanding. Once causality is properly assessed, it becomes clear that fakeons do not bring a true conceptual disruption. The existence of a minimum time resolution $\tau$ (in the presence of external forces) is not difficult to accept, and it is consistent with the data if $\tau$ is sufficiently small. Pre(post)dictivity is just delayed. Finally, a form of nonlocality has always been present in gauge and gravitational theories without causing undue concern. The additional nonlocality introduced by fakeons is only a modest extension of that familiar feature, since both types of nonlocality arise from parent local theories through projection.

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## References

[1] D. Hume, *An Enquiry Concerning Human Understanding*, Chicago, The Open court publishing co.,1900, available at this link.

[2] D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renorm and arXiv: 1704.07728 [hep-th].

[3] D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renorm and arXiv:1806.03605 [hep-th].

[4] D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, 17A1 Renorm and arXiv:1703.04584 [hep-th];

D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141, 18A1 Renorm and arXiv:1801.00915 [hep-th].

[5] D. Anselmi, Diagrammar of physical and fake particles and spectral optical theorem, J. High Energy Phys. 11 (2021) 030, 21A5 Renorm and arXiv: 2109.06889 [hep-th].

[6] D. Anselmi, A new quantization principle from a minimally non time-ordered product, J. High Energy Phys. 12 (2022) 088, 22A5 Renorm and arXiv:2210.14240 [hep-th].

[7] D. Anselmi, E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211, 20A2 Renorm and arXiv:2005.10293 [hep-th].

[8] Planck collaboration, Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211 [astro-ph.CO];

BICEP/Keck XIII: Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season, arXiv:2110.00483 [astro-ph.CO];

BICEP/Keck Collaboration: The Latest Constraints on Inflationary B-modes from the BICEP/Keck Telescopes, arXiv:2203.16556 [astro-ph.CO].

[9] N.V. Krasnikov, Nonlocal gauge theories, Theor. Math. Phys. 73 (1987) 1184 [Teor. Mat. Fiz. 73 (1987) 235];

Yu.V. Kuz'min, The convergent nonlocal gravitation, Sov. J. Nucl. Phys. 50, 1011 (1989) [Yad. Fiz. 50, 1630 (1989)];

E.T. Tomboulis, Super-renormalizable gauge and gravitational theories, arXiv:hep-th/9702146;

L. Modesto, Super-renormalizable quantum gravity, Phys. Rev. D 86 (2012) 044005 and arXiv:1107.2403 [hep-th];

L. Modesto, Finite quantum gravity, arXiv:1305.6741 [hep-th];

F. Briscese, L. Modesto and S. Tsujikawa, Super-renormalizable or finite completion of the Starobinsky theory, Phys. Rev. D89 (2014) 024029 and arXiv:1308.1413 [hep-th].

L. Modesto and L. Rachwał, Universally finite gravitational and gauge theories, Nucl. Phys. B 900 (2015) 147 and arXiv:1503.00261 [hep-th]

L. Modesto and L. Rachwał, Super-renormalizable and finite gravitational theories, Nucl. Phys. B 889 (2014) 228 and arXiv:1407.8036 [hep-th];

L. Modesto, Multidimensional finite quantum gravity, arXiv:1402.6795 [hep-th];

G. Calcagni, B.L. Giacchini, L. Modesto, T. de Paula Netto and L. Rachwał, Renormalizability of nonlocal quantum gravity coupled to matter, arXiv:2306.09416 [hep-th].

[10] A. Pais and G.E. Uhlenbeck, On field theories with non-localized action, Phys. Rev. 79 (1950) 145.

[11] G.V. Efimov, Non-local quantum theory of the scalar field, Commun. Math. Phys. 5 (1967) 42;

G.V. Efimov, Quantization of non-local field theory, Int. J. Theor. Phys. 10 (1974) 19;

G.V. Efimov, Nonlocal interactions of quantized fields, Nauka, Moscow (1977).

[12] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Towards singularity and ghost free theories of gravity, Phys. Rev. Lett. 108 (2012) 031101 and arXiv:1110.5249 [gr-qc];

T. Biswas, A. Conroy, A. S. Koshelev and A. Mazumdar, Generalized ghost-free quadratic curvature gravity, Class. Quantum Grav. 31 (2014) 015022 and arXiv:1308.2319 [hep-th];

L. Buoninfante, A.S. Koshelev, G. Lambiase and A. Mazumdar, Classical properties of non-local, ghost- and singularity-free gravity, J. Cosmol. Astropart. Phys. 09 (2018) 034 and arXiv:1802.00399 [gr-qc];

A.S. Koshelev and A. Tokareva, Unitarity of Minkowski nonlocal theories made explicit, Phys. Rev. D 104 (2021) 025016 and arXiv:2103.01945 [hep-th];

G. Calcagni, Classical and quantum gravity with fractional operators Class. Quant. Grav. 38 (2021) 165005 (E 169601) and arXiv:2106.15430 [gr-qc];

G. Calcagni and L. Rachwał, Ultraviolet-complete quantum field theories with fractional operators, J. Cosmol. Astropart. Phys. 09 (2023) 003  and arXiv:2210.04914 [hep-th];

F. Briscese, G. Calcagni, L. Modesto and G. Nardelli, Form factors, spectral and Källén-Lehmann representation in nonlocal quantum gravity, J. High Energy Phys. 08 (2024) 204 and arXiv:2405.14056 [hep-th].

[13] D. Anselmi, Quantum gravity with purely virtual particles from asymptotically local quantum field theory, Eur. Phys. J. C 85 (2025) 999, 24A2 Renorm and arXiv:2410.21599 [hep-th].

[14] T.D. Lee and G.C. Wick, Negative metric and the unitarity of the $S$-matrix, Nucl. Phys. B 9 (1969) 209;

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033;

T.D. Lee, Complex pole model with indefinite metric, in *Quanta*, P.G.O. Freund, C.J. Giebek and Y. Nambu (eds.), Chicago University Press, Chicago, U.S.A. (1970), p. 260;

R.E. Cutkosky, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, A non-analytic $S$-matrix, Nucl. Phys. B 12 (1969) 281;

N. Nakanishi, Lorentz noninvariance of the complex-ghost relativistic field theory, Phys. Rev. D 3 (814) 811 1971.

[15] M.J.G. Veltman, Unitarity and causality in a renormalizable field theory with unstable particles, Physica 29 (1963) 186;

H. Yamamoto, Convergent field theory with complex masse, Prog. Theor. Phys. 42 (1969) 707;

H. Yamamoto, Quantum field theory of complex mass, Prog. Theor. Phys. 44 (1970) 272;

N. Nakanishi, Indefinite metric quantum field theory, Prog. Theor. Phys. Suppl. 51 (1972) 1;

P.D. Mannheim, Unitarity of loop diagrams for the ghostlike $1/(k^2 - M_1^2) - 1/(k^2 - M_2^2)$ propagator, Phys. Rev. D 98 (2018) 045014 [arXiv:1801.03220];

L. Buoninfante, Remarks on ghost resonances, JHEP 02 (2025) 186 [arXiv:2501.04097];

J. Liu, L. Modesto and G. Calcagni, Quantum field theory with ghost pairs, JHEP 02 (2023) 140 [arXiv:2208.13536];

A. Tokareva, Background-induced complex mass states of graviton: quantization and tensor power spectrum, arXiv:2405.09527;

M. Asorey, G. Krein and I.L. Shapiro, Normal bound states out of massive complex ghosts degrees of freedom in superrenormalizable quantum gravity theories, arXiv:2408.16514;

D. Anselmi, F. Briscese and G. Calcagni, *Perturbative unitarity of fractional field theories and gravity*, work in progress.

[16] B. Holdom and J. Ren, QCD analogy for quantum gravity, Phys. Rev. D 93 (2016) 124030 [arXiv:1512.05305];

G.P. de Brito, Quadratic gravity in analogy to quantum chromodynamics: Light fermions in its landscape, Phys. Rev. D 109 (2024) 086005 [arXiv:2309.03838].

[17] P.D. Mannheim, Ghost problems from Pauli–Villars to fourth-order quantum gravity and their resolution, Int. J. Mod. Phys. D 29 (2020) 2043009 [ arXiv:2004.00376].

[18] J.F. Donoghue and G. Menezes, Unitarity, stability and loops of unstable ghosts, Phys. Rev. D 100 (2019) 105006 [arXiv:1908.02416].

[19] K.S. Stelle, Renormalization of higher derivative quantum gravity, Phys. Rev. D 16 (1977) 953.

[20] B. Russell, On the Notion of Cause, Proceedings of the Aristotelian Society, 13 (1913) 1.

[21] D. Anselmi and G. Calcagni, Classicized dynamics and initial conditions in field theories with fakeons, J. High Energy Phys. (to appear), 25A2 Renorm and arXiv:2510.05276 [hep-th].

[22] P. A. M. Dirac, Gauge invariant formulation of quantum electrodynamics, Can. J. Phys. 33 (1955) 650.

[23] D. Anselmi, Quantum field theory of physical and purely virtual particles in a finite time interval on a compact space manifold: diagrams, amplitudes and unitarity, J. High Energy Phys. 07 (2023) 209, 23A1 Renorm and arXiv:2304.07642 [hep-th];

D. Anselmi, Gauge theories and quantum gravity in a finite interval of time, on a compact space manifold, Phys. Rev. D 109 (2024) 025003, 23A3 Renorm and arXiv: 2306.07333 [hep-th].

[24] P.A.M. Dirac, Classical theory of radiating electrons, Proc. Roy. Soc. London A 167 (1938) 148;

J.D. Jackson, *Classical electrodynamics*, Wiley, New York, U.S.A. (1975), chap. 17;

D. Anselmi, Renormalization and causality violations in classical gravity coupled with quantum matter, JHEP 07 01 (2007) 062, 06A1 Renorm and arXiv:hep-th/0605205.

[25] S.W. Hawking, Chronology Protection Conjecture, Phys. Rev. D 46 (1992) 603.

[26] J.R. Friedman, M.S. Morris, I.D. Novikov, K.S. Thorne and U. Yurtsever, Cauchy problem in spacetimes with closed timelike curves, Phys. Rev. D 42 (1990) 1915.

[27] G. 't Hooft and M.J.G. Veltman, *Diagrammar*, CERN report CERN-73-09.

[28] N.N. Bogoliubov and D.V. Shirkov, *Introduction to the Theory of Quantized Fields*, Interscience Publishers,1959, New York, § 20.

[29] H. Lehmann, K. Symanzik and W. Zimmerman, Zur Formulierung quantisierter Feldtheorien, Il Nuovo Cimento 1 (1955) 205.

[30] D. Anselmi and A. Marino, Fakeons and microcausality: light cones, gravitational waves and the Hubble constant, Class. Quantum Gravity 37 (2020) 095003, 19A3 Renorm and arXiv:1909.12873 [gr-qc].