22R3 Renorm

Purely Virtual Particles in Quantum Gravity, Inflationary Cosmology and Collider Physics

Damiano Anselmi

National Institute of Chemical Physics and Biophysics, Rävala 10, Tallinn 10143, Estonia

Dipartimento di Fisica "E.Fermi", Università di Pisa, Largo B.Pontecorvo 3, 56127 Pisa, Italy

INFN, Sezione di Pisa, Largo B. Pontecorvo 3, 56127 Pisa, Italy

damiano.anselmi@unipi.it

Abstract

We review the concept of purely virtual particle and its uses in quantum gravity, primordial cosmology and collider physics. The fake particle, or "fakeon", which mediates interactions without appearing among the incoming and outgoing states, can be introduced by means of a new diagrammatics. The renormalization coincides with the one of the parent Euclidean diagrammatics, while unitarity follows from spectral optical identities, which can be derived by means of algebraic operations. The classical limit of a theory of physical particles and fakeons is described by an ordinary Lagrangian plus Hermitian, micro acausal and micro nonlocal self-interactions. Quantum gravity propagates the graviton, a massive scalar field (the inflaton) and a massive spin-2 fakeon, and leads to a constrained primordial cosmology, which predicts the tensor-to-scalar ratio r in the window $0.4 \lesssim 1000r \lesssim 3.5$. The interpretation of inflation as a cosmic RG flow allows us to calculate the perturbation spectra to high orders in the presence of the Weyl squared term. In models of new physics beyond the standard model, fakeons evade various phenomenological bounds, because they are less constrained than normal particles. The resummation of self-energies reveals that it is impossible to get too close to the fakeon peak. The related peak uncertainty, equal to the fakeon width divided by 2, is expected to be observable.

1 Introduction

Nature "is written in that great book which ever is before our eyes – I mean the universe – but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are triangles, circles and other geometrical figures..." Since Galileo's time, the language of the book of nature has evolved considerably. For some time, the power of infinitesimal calculus gave us the illusion of the continuum and determinism. Then, unexpectedly, quantum mechanics turned everything upside down, by injecting uncertainty into the laws of physics. Mathematics successfully made room for the new concepts, but several problems remained unresolved, or so it appeared to us. With the advent of quantum field theory (renormalization, the challenges of perturbation theory and the impossibility to move beyond the perturbative expansion in a systematic way), the mathematization of physical phenomena became more challenging.

As far as we know today, the language spoken by the elementary particles is diagrammatic. And consequently, perturbative. Beyond that we have hints, but no satisfactory formal setup. A nonperturbative language might even not exist.

Strictly speaking, there is no reason why nature should be mathematizable by one of the living species it generates around the universe. In the end, we are just clumps of atoms and logic is a net of brain connections among memorized, mainly acoustic perceptions, which are our words (hence *the* word, verb, or logos), shaped by experience through repetition, custom and mental habit (à la Hume), rather than having an existence *per se*, although the idea of logic existing "before nature" is one of those which hardly die and periodically come back under different spells. Actually, our size and the relative scales involved in the phenomena of the universe suggest that the logicization of nature is most likely impossible beyond certain limits. The question is whether we reached ours or there is still room for improvement.

The challenge of quantum gravity inspired many to call everything into question again (this time, for free) and suggest thoroughly new approaches "beyond" quantum field theory, despite the lack of data pointing to such a turmoil. Probably, the underlying assumption was, again, that logic is not just a tool, a language, but pre-exists nature (*in principio erat verbum*), so we should be able to grasp the theory (of something, or even everything) without or with very little experimental data, by overthinking (banking on a special connection with divinities?) or following personal or social tastes ("string theory is so beautiful that it can only be true").

"It's the diagrammatics, stupid": what if, instead, quantum gravity were just a step away from the standard model? Just a fairly guessable missing piece of the puzzle? The diagrammatic approach has worked very well so far for the elementary particles and the standard model. Yet, despite decades of efforts, there is still a lot to understand about it and the basic principles on which it is founded, which are locality, unitarity and renormalizability. Quantum field theory never ceases to surprise, so to speak.

A new concept in the world of diagrammatics is the concept of purely virtual particle, which we review in this paper together with its applications. Purely virtual particles, or fake particles, or "fakeons", are "non particles", or particles that have no classical limit. That is to say, they have a purely quantum nature. Their effects reach the classical limit as effective interactions among the physical particles. Normally, we take for granted that everything belonging to the quantum world should be obtained by quantizing something classical, but if we view the matter the other way around, we can easily make room for entities that are classically hidden and exist only at the quantum level.

The diagrammatics of fakeons [1] is obtained from the usual one by means of surgical operations that selectively remove degrees of freedom at all energies and preserve the optical theorem. The only requirement is that fake particles should be massive and non tachyonic. The main application of fakeons is the formulation of a consistent theory of quantum gravity [2], which is observationally testable due to its predictions in inflationary cosmology [3]. At the phenomenological level, fakeons evade common constraints that limit the employment of normal particles. Among the other things, they can be used to propose new physics beyond the standard model [4] and solve discrepancies with data [5]. We stress that the fakeon diagrammatics is relatively straightforward, to the extent that it can be implemented in software like FeynCalc, FormCalc, LoopTools and Package-X [6] and used to work out physical predictions. For proofs to all orders, see [7, 1]. It is also possible [8, 9] to avoid certain troubles of the Lee-Wick models [10, 11, 12, 13, 14, 15] by switching to theories of particles and fakeons. Finally, the fakeon prescription can be used to give sense to higher-spin massive multiplets [16]. Coupled to gravity, higher-spin massive multiples change the ultraviolet behavior and open the way to asymptotic freedom [17].

The paper is organized as follows. In section 2 we review some key concepts concerning unitarity. In section 3 we introduce the fakeon diagrammatics. In section 4 we briefly recall how the quantization of gravity works by means of fakeons. In section 5 we discuss the main predictions of quantum gravity with fakeons in primordial cosmology. In section 6 we present some ways to use fakeons in phenomenology. In section 7 we discuss the main two new features of the theories with fakeons: the peak uncertainty and the violation of microcausality. Section 8 contains conclusions and outlook.

22R3 Renorm

2 Particles, fakeons and ghosts

Unitarity is the statement that the scattering matrix S is unitary, $S^\dagger S = 1$. Writing $S = 1 + iT$, it is also expressed by the optical theorem

$$iT - iT^\dagger + T^\dagger T = 0, \tag{2.1}$$

which admits a diagrammatic, off-shell version in terms of identities

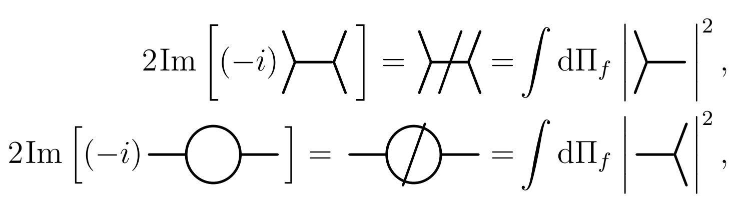
$$G + \bar{G} + \sum_c G_c = 0 \tag{2.2}$$

among cut diagrams [18]. Here G denotes an ordinary (uncut) diagram and stands for iT, $\bar{G}$ is its complex conjugate and stands for $-iT^\dagger$, while G_c are the so-called cut diagrams, obtained by cutting internal lines: they stand for $T^\dagger T$. The vertices and propagators that lie to one side of the cut are the normal ones (as in T), while those that lie to the other side of the cut are the complex conjugate ones (as in $T^\dagger$). The cut propagators give us information about the on-shell content of a particle. The equations (2.2) single out certain analytic properties of the loop integrals, which encode, among the other things, the physical processes where the virtual particles circulating in the loops turn real, which occurs above certain thresholds. A purely virtual particle cannot turn real, by definition, so its cut propagator must vanish.

Denoting the space of physical states by V and inserting a complete set of orthonormal states $|n\rangle \in V$, equation (2.1) implies, in particular,

$$2\mathrm{Im}\langle a|T|a\rangle = \sum_{|n\rangle \in V} |\langle n|T|a\rangle|^2, \tag{2.3}$$

where $|a\rangle \in V$ is an arbitrary state: the total cross section for production of all final states is proportional to the imaginary part of the forward scattering amplitude. The simplest cutting equations are



(2.4)

(2.5)

where the integrals are over the phase spaces Π_f of the final states. In particular, (2.4) implies $\mathrm{Re}[P] \geqslant 0$, if P is the propagator.

Physical particles, ordinary ghosts, Lee-Wick (LW) ghosts and purely virtual particles have propagators

$$\frac{i}{p^2 - m^2 + i\epsilon}, \qquad -\frac{i}{p^2 - m^2 + i\epsilon}, \qquad -\frac{i}{p^2 - m^2 - i\epsilon}, \qquad \pm\mathcal{P}\frac{i}{p^2 - m^2},$$

respectively, where $\mathcal{P}$ denotes the Cauchy principal value. They all satisfy $\mathrm{Re}[P] \geqslant 0$, except for the ordinary ghost, which violates unitarity. The propagators of physical particles and ordinary ghosts can be used "as is" inside Feynman diagrams, which means as they appear in the formulas just written, by integrating on real loop energies and momenta. Instead, the propagators of LW ghosts and purely virtual particles cannot, because the $i\epsilon$ and $-i\epsilon$ prescriptions cannot coexist inside Feynman diagrams without violating unitarity, the locality and Hermiticity of counterterms and stability [19]. These two options need suitable integration prescriptions or, in the case of fakeons, a new diagrammatics.

The removal of degrees of freedom from the incoming and outgoing states is consistent only if it is compatible with unitarity, in which case it is called "projection" and the reduced action is called "projected action". This means that the equation (2.3) holds in a subspace V of the total space W of states one uses to build the theory. Working in an extended space W and projecting to V at the end is normally useful to manipulate simpler Feynman rules, like those of a local theory.

A well-known example of projection is the one concerning the Faddeev-Popov ghosts and the longitudinal/temporal components of the gauge fields in gauge theories. There, the consistency of the projection is ensured by the symmetry. In the case of the LW ghosts, instead, one has to make them unstable, to kick them out of the set of strictly asymptotic states (which are to be taken literally at $t = \pm\infty$): the projection is the very same decay of the LW ghosts. In the case of fakeons, the consistency is ensured by the diagrammatics, so there is no need for giving fakeons nonvanishing widths, dynamically or explicitly. The "width" of a purely virtual particle has a completely different physical interpretation. It is the "peak uncertainty", which measures the impossibility of experimentally approaching the fakeon too closely. The fakeon projection is compatible with unitarity order by order (and diagram by diagram) in the perturbative expansion [1].

3 Purely virtual particles: a new diagrammatics

The simplest way to introduce fakeons is by means of the diagrammatics developed in ref. [1], which is useful for physical particles as well. It is based on the threshold decomposition of ordinary (cut and uncut) diagrams and the suppression of all the thresholds that involve fakeon frequencies. The fakeon procedure works with both signs in front of the propagators (fake particles and fake ghosts), since a sign flip can at most flip the overall signs of the identities (2.2), which encode unitarity, thus keeping them valid. For definiteness, we concentrate on fakeons obtained from physical particles.

At the tree level, we start from the usual Feynman prescription, decompose the propagators by means of the identity

$$\frac{i}{x+i\epsilon} = \mathcal{P}\frac{i}{x} + \pi\delta(x) \tag{3.1}$$

and suppress all the delta functions that refer to fakeons.

Apart from some caveats, this simple recipe can be implemented to all orders. The key ingredient is the possibility of reducing the optical theorem to a set of purely algebraic operations and identities. In brief, the procedure is:

— ignore the integral on the space components of the loop momenta (which defines the *skeleton* diagram);

— perform the integral on the loop energies by means of the residue theorem (which can be viewed as an algebraic operation);

— decompose the result in terms of principal values and delta functions by means of the identity (3.1);

— organize the decomposition properly;

— drop all the deltas that contain fakeon frequencies.

A caveat, which can be appreciated starting from the box diagram, is that the decomposition must be properly organized, due to certain nontrivial identities that are met along the way.

Let us illustrate the procedure on the bubble diagram, which gives the skeleton integral

$$B^s = \int \frac{\mathrm{d}k^0}{2\pi} \prod_{a=1}^{2} \frac{2\omega_a}{(k-p_a)^2 - m_a^2 + i\epsilon_a} = \int \frac{\mathrm{d}k^0}{2\pi} \prod_{a=1}^{2} \frac{2\omega_a}{(k^0-e_a)^2 - \omega_a^2 + i\epsilon_a},$$

where $k^0 = (k^0, \mathbf{k})$ is the loop momentum, $p_a^\mu = (e_a, \mathbf{p}_a)$ are the external momenta (one for each *internal* leg, the redundancy being useful to have more symmetric expressions) and $\omega_a = \sqrt{(\mathbf{k}-\mathbf{p}_a)^2 + m_a^2}$ are the frequencies. For convenience, a product $\prod_{a=1}^{2}(2\omega_a)$ is inserted after dropping the integral on $\mathbf{k}$.

The residue theorem gives

$$B^s = -\frac{i}{e_1 - e_2 - \omega_1 - \omega_2 + i\epsilon} - \frac{i}{e_2 - e_1 - \omega_1 - \omega_2 + i\epsilon}.$$

The threshold decomposition using identity (3.1) gives

$$B^s = -\mathcal{P}\frac{i}{e_1 - e_2 - \omega_1 - \omega_2} - \pi\delta(e_1 - e_2 - \omega_1 - \omega_2)$$
$$-\mathcal{P}\frac{i}{e_2 - e_1 - \omega_1 - \omega_2} - \pi\delta(e_2 - e_1 - \omega_1 - \omega_2). \tag{3.2}$$

Repeating the same procedure with the conjugate diagram and the cut diagrams, we obtain the table

				
—	$-i\hat{\mathcal{P}}^{12}$	$i\hat{\mathcal{P}}^{12}$	0	0
Δ^{12}	-1	-1	0	2
Δ^{21}	-1	-1	2	0

(3.3)

where

$$\mathcal{P}^{ab} = \mathcal{P}\frac{1}{e_a - e_b - \omega_a - \omega_b}, \qquad \hat{\mathcal{P}}^{ab} = \mathcal{P}^{ab} + \mathcal{P}^{ba}, \qquad \Delta^{ab} = \pi\delta(e_a - e_b - \omega_a - \omega_b),$$

and the cut diagram with a tilde is the one where the sides corresponding to T and $T^\dagger$ are interchanged.

Here and below, if C_{ij} denote the entries of the table, a (cut or uncut) diagram G_j is the jth column of the table ($j > 1$), by which we mean the sum

$$G_j \equiv \sum_{i>1} C_{i1} C_{ij}, \tag{3.4}$$

where $C_{21} = 1$. The spectral optical identities are the rows of the table, by which we mean the sums

$$R_i \equiv C_{i1} \sum_{j>1} C_{ij} = 0, \tag{3.5}$$

for $i > 1$, which vanish separately. They decompose the "spectral optical theorem", which is the whole table, i.e., the sum

$$\sum_{j>1} G_j = \sum_{i>1}\sum_{j>1} C_{i1} C_{ij} = 0 \tag{3.6}$$

of all its entries. Finally, the optical theorem is the integral of this identity, divided by $4\omega_1\omega_2$, over the space components $\mathbf{k}$ of the loop momentum, with measure $\mathrm{d}^3\mathbf{k}/(2\pi)^3$.

If an internal leg, say leg 1, is a fakeon, we drop the delta functions containing its frequency from equation (3.2) and so obtain

$$B_{\mathrm{f}}^s = -\mathcal{P}\frac{i}{e_1 - e_2 - \omega_1 - \omega_2} - \mathcal{P}\frac{i}{e_2 - e_1 - \omega_1 - \omega_2}. \tag{3.7}$$

In table (3.3), we drop the rows containing Δ^{12}, which gives

				
—	$-i\hat{\mathcal{P}}^{12}$	$i\hat{\mathcal{P}}^{12}$	0	0

Dropping whole rows preserves the (spectral) optical theorem in an obvious way. Moreover, the last two columns, corresponding to the cut diagrams, disappear as well, since their surviving entries are just zeros. We can understand their disappearance by noting that those diagrams contain a cut fakeon leg and the cut propagator of a fakeon must vanish, because the fakeon cannot be on shell. This leaves us with the table

		
—	$-i\hat{\mathcal{P}}^{12}$	$i\hat{\mathcal{P}}^{12}$

In the case of the skeleton triangle T^s, we can proceed similarly. Without giving details (which can be found in ref. [1]), the decomposition is

$$T^s = -i\mathcal{P}_{\mathrm{T}} - \sum_{\text{perms}} \Delta^{ab}\mathcal{Q}^{ac} + \frac{i}{2}\sum_{\text{perms}} \Delta^{ab}(\Delta^{ac} + \Delta^{cb}), \tag{3.8}$$

where

$$\mathcal{P}_{\mathrm{T}} = \mathcal{P}^{12}\mathcal{P}^{13} + \text{cycl} + (e \to -e), \qquad \mathcal{Q}^{ab} = \mathcal{P}^{ab} - \mathcal{P}\frac{1}{e_a - e_b - \omega_a + \omega_b},$$

and the sums are on $\{a, b, c\}$ equal to the permutations of 1, 2 and 3. The conjugate diagram is $\bar{T}^s$ and the cut diagrams read

$$T^s_{\mathrm{c}} = 2\Delta^{21}(\mathcal{Q}^{23} - i\Delta^{31} - i\Delta^{23}), \qquad \widetilde{T^s_{\mathrm{c}}} = 2\Delta^{12}(\mathcal{Q}^{13} + i\Delta^{13} + i\Delta^{32}), \tag{3.9}$$

plus the ones obtained by cyclically permuting 1, 2 and 3.

If the internal leg 3 is a fakeon, the rows containing Δ^{13}, Δ^{23}, Δ^{31} and Δ^{32} must be suppressed. Then the cut diagrams containing a cut leg 3 become trivial and their columns disappear automatically. We remain with the table

	T^s_{f}	$\bar{T}^s_{\mathrm{f}}$	T^s_{fc}	$\widetilde{T^s_{\mathrm{fc}}}$
—	$-i\mathcal{P}_{\mathrm{T}}$	$i\mathcal{P}_{\mathrm{T}}$	0	0
Δ^{12}	$-\mathcal{Q}^{13}$	$-\mathcal{Q}^{13}$	0	$2\mathcal{Q}^{13}$
Δ^{21}	$-\mathcal{Q}^{23}$	$-\mathcal{Q}^{23}$	$2\mathcal{Q}^{23}$	0

(3.10)

If two internal legs are fakeons, the last two rows disappear, which make the last two columns disappear as well:

	T_{ff}^{s}	$\bar{T}_{\text{ff}}^{s}$
—	$-i\mathcal{P}_{\text{T}}$	$i\mathcal{P}_{\text{T}}$

Other examples (triangle with "diagonal", box, box with diagonal, pentagon, hexagon, etc.) and the proof to all orders can be found in ref. [1]. The threshold decomposition and the fakeon diagrammatics are compatible with gauge invariance and general covariance, through the WTST identities [20]. Indeed, the WTST identities are algebraic relations among the integrands of certain diagrams, so the decomposition and the fakeon projection go through them straightforwardly. Gauge independence is preserved as well, since the thresholds associated with the gauge-trivial modes depend on the gauge-fixing parameters and cannot interfere with the other (physical/fakeon) thresholds, which are gauge invariant and gauge independent.

4 Quantum gravity

Quantum gravity with fakeons propagates the graviton, a scalar field ϕ of mass m_ϕ (the inflaton) and a spin 2 field $\chi_{\mu\nu}$ of mass m_χ. It is formulated starting from the classical action

$$S_{\text{QG}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(2\Lambda + R + \frac{\lambda}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right), \tag{4.1}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, G is the Newton constant, Λ is the cosmological constant and $\lambda = m_\chi^2(3m_\phi^2 + 4\Lambda)/(m_\phi^2(3m_\chi^2 - 2\Lambda))$ is a parameter very close to 1. The theory is renormalizable by power counting [21], since the renormalizability of a theory with fakeons coincides with the one of the Euclidean parent theory.

The three fields can be made explicit by eliminating the higher derivatives as shown in [22]. In particular, the action $S_\chi(g,\phi,\chi)$ of $\chi_{\mu\nu}$ is the sum

$$S_\chi(g,\phi,\chi) = -\frac{\lambda}{8\pi G}S_{\text{PF}}(g,\chi) + S_{\chi\text{int}}(g,\phi,\chi) \tag{4.2}$$

of a term proportional to the nonminimally coupled covariantized Pauli-Fierz action

$$\begin{aligned} S_{\text{PF}}(g,\chi) = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,[&D_\rho\chi_{\mu\nu}D^\rho\chi^{\mu\nu} - D_\rho\chi D^\rho\chi + 2D_\mu\chi^{\mu\nu}D_\nu\chi - 2D_\mu\chi^{\rho\nu}D_\rho\chi_\nu^\mu \\ &-m_\chi^2(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2) + R^{\mu\nu}(\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho)] \end{aligned} \tag{4.3}$$

plus further interactions $S_{\chi\text{int}}(g,\phi,\chi)$, where $\chi = g^{\mu\nu}\chi_{\mu\nu}$ is the trace of $\chi_{\mu\nu}$.

Since Λ is much smaller than m_χ^2, λ is positive, so the $\chi_{\mu\nu}$ kinetic term has the wrong sign. This is the reason why $\chi_{\mu\nu}$ must be quantized as a fakeon. Then $\chi_{\mu\nu}$ is purely virtual and does not belong to the sets of incoming and outgoing states.

It is convenient to postpone the fakeon projection to the very end, to deal with local diagrammatic rules. An early projection forces us to work with rather involved nonlocal vertices. This situation is similar to the one of gauge theories, where it is preferable to work with the local diagrammatic rules of a gauge-fixed action propagating gauge-trivial modes and Faddeev-Popov ghosts and remove them only at the very end.

The projection must also be performed classically. In this sense, the action (4.1) does not describe the true classical limit, because it is unprojected. The true classical action, which is useful to study primordial cosmology, is obtained by "classicizing" quantum gravity [23] and collects the tree diagrams that only have physical particles on the external legs.

5 Inflationary cosmology from quantum gravity

Quantum gravity with fakeons can be used to study primordial cosmology and work out predictions that could even be tested within our lifetime. For this purpose, it is convenient the consider the action (4.1) at $\Lambda = 0$, make the inflaton field ϕ explicit through a field redefinition and keep the fakeon $\chi_{\mu\nu}$ implicit. We obtain the equivalent action

$$S_{\mathrm{QG}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(D_\mu\phi D^\mu\phi - 2V(\phi)\right), \tag{5.1}$$

where

$$V(\phi) = \frac{3m_\phi^2}{32\pi G}\left(1 - \mathrm{e}^{\phi\sqrt{16\pi G/3}}\right)^2 \tag{5.2}$$

is the Starobinsky potential.

As said, the classical limit is not described by either (4.1) or (5.1), which are unprojected. The classicization is nontrivial when the metric is expanded around curved backgrounds rather than flat space. Nevertheless, if the background is the FLRW metric, the degrees of freedom decouple from one another at the quadratic level in the de Sitter limit [3]. Thanks to this fact, the fakeon projection can be perturbatively obtained from the flat-space one.

It can be shown that this procedure works under the consistency condition $m_\chi > m_\phi/4$ [3]. This lower bound on the mass of the fakeon $\chi_{\mu\nu}$ with respect to the mass of the inflaton

ϕ is crucial for the prediction on the tensor-to-scalar ratio r, which is determined within less than an order of magnitude, even before knowing the actual value of m_χ [3].

Note that the theory does not predict other degrees of freedom besides the curvature perturbation $\mathcal{R}$ and the tensor perturbations, when the matter sector is switched off. The fakeon projection eliminates the possibility of having additional scalar and tensor perturbations, as well as vector perturbations.

5.1 Cosmic RG flow

Parametrizing the background metric as $g_{\mu\nu}$ =diag$(1, -a^2, -a^2, -a^2)$, the Friedmann equations and the ϕ equation read

$$\dot{H} = -4\pi G\dot{\phi}^2, \qquad H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + 2V(\phi)\right), \qquad \ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \tag{5.3}$$

where $H = \dot{a}/a$ is the Hubble parameter. For the purposes of this paper, we can assume $\dot{\phi} > 0$. Defining the conformal time

$$\tau = -\int_t^{+\infty} \frac{\mathrm{d}t'}{a(t')} \tag{5.4}$$

and the "coupling"

$$\alpha = \sqrt{\frac{4\pi G}{3}}\frac{\dot{\phi}}{H} = \sqrt{-\frac{\dot{H}}{3H^2}}, \tag{5.5}$$

it is easy to show that α satisfies an equation of the form $\beta_\alpha = \mathrm{d}\alpha/\mathrm{d}\ln|\tau|$, where β_α is a function of α that can be worked out to arbitrarily high orders in α:

$$\beta_\alpha = -2\alpha^2\left[1 + \frac{5}{6}\alpha + \frac{25}{9}\alpha^2 + \frac{383}{27}\alpha^3 + \mathcal{O}(\alpha^4)\right]. \tag{5.6}$$

The interpretation of inflation as a "cosmic RG flow", β_α being the beta function, is predicated on the possibility of viewing the perturbation spectra $\mathcal{P}_T$ and $\mathcal{P}_\mathcal{R}$ of the tensor and scalar fluctuations as correlation functions that satisfy RG evolution equations of the Callan-Symanzik type, in the superhorizon limit [24].

Let us introduce the running coupling $\alpha(x)$, which is the solution of

$$\ln\frac{\tau}{\tau'} = \int_{\alpha(-\tau')}^{\alpha(-\tau)} \frac{\mathrm{d}\alpha}{\beta_\alpha(\alpha)}.$$

For brevity, α will stand for $\alpha(-\tau)$ and α_k for $\alpha(1/k)$, where k is just a constant for now:

$$\ln(-k\tau) = \int_{\alpha_k}^{\alpha} \frac{\mathrm{d}\alpha'}{\beta_\alpha(\alpha')}.$$

At the leading-log level, the running coupling reads

$$\alpha = \frac{\alpha_k}{1 + 2\alpha_k \ln(-k\tau)}. \tag{5.7}$$

Its expression to the next-to-next-to leading log (NNLL) order can be found in [24].

Viewing the spectra as functions of τ and α, their RG evolution equations are

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\ln|\tau|} = \left(\frac{\partial}{\partial \ln|\tau|} + \beta_\alpha(\alpha)\frac{\partial}{\partial\alpha}\right)\mathcal{P} = 0. \tag{5.8}$$

Viewing them as functions of α and α_k, the dependence on α actually drops out and the spectra depend on the momentum k only through the running coupling α_k:

$$\mathcal{P} = \tilde{\mathcal{P}}(\alpha_k), \qquad \frac{\mathrm{d}\tilde{\mathcal{P}}(\alpha_k)}{\mathrm{d}\ln k} = -\beta_\alpha(\alpha_k)\frac{\mathrm{d}\tilde{\mathcal{P}}(\alpha_k)}{\mathrm{d}\alpha_k}. \tag{5.9}$$

Finally, viewing the spectra as functions of k/k_* and $\alpha_* = \alpha(1/k_*)$, where k_* is the pivot scale and α_* is the "pivot coupling", they satisfy

$$\left(\frac{\partial}{\partial \ln k} + \beta_\alpha(\alpha_*)\frac{\partial}{\partial\alpha_*}\right)\mathcal{P}(k/k_*, \alpha_*) = 0. \tag{5.10}$$

The correspondence between the cosmic RG flow and the one of quantum field theory is summarized in table 1.

5.2 Spectra

In high-energy physics, a low-energy effective theory is good enough to make predictions about low energies. In cosmology, it is not so: we must properly treat the high-energy (sub-horizon) limit, even if our purpose is just to make predictions about the low-energy (super-horizon) limit. This is a highly nontrivial problem, since the sub-horizon region is experimentally and observationally inaccessible. We can say something reasonable about it only if the system reduces to one we have experience of around us. This is where fakeons play a crucial role in primordial cosmology.

If $\chi_{\mu\nu}$ is quantized by means of the Feynman prescription instead of the fakeon one, the theory has ghosts and so violates unitarity [21]. From the point of view of primordial cosmology, the problem of ghosts shows up as follows.

On a nontrivial background, the study of the metric fluctuations reduces, in the end, to the problem of harmonic oscillators with time-dependent frequencies. We need to provide a proper quantization condition to study such a system. Normally, the Bunch-Davies

QFT RG flow		Cosmic RG flow
RG flow	$\leftrightarrow$	slow roll
couplings α, λ ...	$\leftrightarrow$	slow-roll parameters ϵ, δ ...
beta functions	$\leftrightarrow$	equations of the background metric
sliding scale μ	$\leftrightarrow$	conformal time τ (or $\eta = -k\tau$)
correlation functions	$\leftrightarrow$	perturbation spectra
Callan-Symanzik equation	$\leftrightarrow$	RG equation at superhorizon scales
RG invariance	$\leftrightarrow$	conservation on superhorizon scales
asymptotic freedom	$\leftrightarrow$	de Sitter limit in the infinite past
subtraction scheme	$\leftrightarrow$	Einstein frame, Jordan frame, etc.
dimensional transmutation	$\rightarrow$	τ drops out from the spectra, "replaced" by k
running coupling	$\rightarrow$	ok
resummation of leading logs	$\rightarrow$	ok
anomalous dimensions	$\rightarrow$	0

Table 1: Correspondence between QFT RG flow and cosmic RG flow

vacuum condition [25] is chosen, which does refer to the sub-horizon limit of the theory, where the problem can be handled because the frequencies of the oscillators becomes time independent. If ghosts are present, no matter how heavy they are, they do not disappear at high energies, but just become massless. A condition like the Bunch-Davies one on ghost oscillators is not robust, even if their frequencies are constant, because we do not have examples of elementary systems of that type that can justify it.

The situation changes in the theory with fakeons. We must ensure that the fakeons are indeed fake at all scales, including the sub-horizon ones. In the low energy regime fakeons disappear for free, because they massive, but in the opposite limit the consistency of the fakeon projection and in particular its classicization [23] on a curved background, requires that we impose a condition, which is the bound $m_\chi > m_\phi/4$ of ref. [3]. In the end, this condition turns out to be rather powerful, because it gives constrained predictions, even if m_χ is still unknown. We see that fakeons provide a second reason, besides the Bunch-Davies vacuum condition, why we must properly treat the high energies to make predictions about the low energies in primordial cosmology.

The spectra of the theory with ghosts are studied in [26] and the comparison with those of the theory with fakeons, which we report below, can be found in [3].

We briefly describe the strategy of the calculation in the theory with fakeons. First, the metric is expanded as

$$\begin{aligned} g_{\mu\nu} = & \, \mathrm{diag}(1, -a^2, -a^2, -a^2) - 2a^2 \left(u\delta_\mu^1\delta_\nu^1 - u\delta_\mu^2\delta_\nu^2 + v\delta_\mu^1\delta_\nu^2 + v\delta_\mu^2\delta_\nu^1 \right), \\ & +2\mathrm{diag}(\Phi, a^2\Psi, a^2\Psi, a^2\Psi) - \delta_\mu^0\delta_\nu^i\partial_i B - \delta_\mu^i\delta_\nu^0\partial_i B \end{aligned} \tag{5.11}$$

in the comoving gauge, where $u = u(t, z)$ and $v = v(t, z)$ are the tensor fluctuations and Ψ, B are the other scalar fluctuations. The ϕ fluctuation $\delta\phi$ is set to zero by a gauge choice, so the curvature perturbation $\mathcal{R}$ coincides with Ψ. For reviews on the parametrizations of the fluctuations, see [27]. Second, the action (5.1) is expanded to the quadratic order in the fluctuations. Third, the higher derivatives are eliminated by introducing extra fields. Forth, the new Lagrangian is diagonalized in the de Sitter limit $\alpha = 0$. Fifth, the fakeon projection is performed, which means that the fakeon fields are integrated out by means of (the classical limit of) the fakeon prescription. Sixth, a number of field redefinitions and time reparametrizations are applied to cast the action into the standard Mukhanov-Sasaki form. Seventh, the equations of motion are solved with the Bunch-Davies vacuum condition. Eighth, all the transformations are undone, to get to the desired two-point functions and the spectra of the fluctuations in the super-horizon limit. For details, see [24].

Thanks to the RG techniques presented above, "RG improved" tensor and scalar power spectra $\mathcal{P}_T$ and $\mathcal{P}_\mathcal{R}$ can be worked out to high orders. This means that, although $\mathcal{P}_T$ and $\mathcal{P}_\mathcal{R}$ are expanded in powers of α_*, the product $\alpha_* \ln(k/k_*)$ is considered of order zero and treated exactly. The results to the NNLL order are

$$\begin{aligned} \mathcal{P}_T(k) = & \frac{4m_\phi^2\zeta G}{\pi}\left[1 - 3\zeta\alpha_k\left(1 + 2\alpha_k\gamma_M + 4\gamma_M^2\alpha_k^2 - \frac{\pi^2\alpha_k^2}{3}\right) + \frac{\zeta^2\alpha_k^2}{8}(94 + 11\xi)\right. \\ & \left. +3\gamma_M\zeta^2\alpha_k^3(14 + \xi) - \frac{\zeta^3\alpha_k^3}{12}(614 + 191\xi + 23\xi^2) + \mathcal{O}(\alpha_k^4)\right], \\ \mathcal{P}_\mathcal{R}(k) = & \frac{Gm_\phi^2}{12\pi\alpha_k^2}\left[1 + (5 - 4\gamma_M)\alpha_k + \left(4\gamma_M^2 - \frac{40}{3}\gamma_M + \frac{7}{3}\pi^2 - \frac{67}{12} - \frac{\xi}{2}F_\mathrm{s}(\xi)\right)\alpha_k^2 + \mathcal{O}(\alpha_k^3)\right] \end{aligned} \tag{5.12}$$

where

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad \gamma_M = \gamma_E + \ln 2,$$

$$F_\mathrm{s}(\xi) = 1 + \frac{\xi}{4} + \frac{\xi^2}{8} + \frac{\xi^3}{8} + \frac{7\xi^4}{32} + \frac{19}{32}\xi^5 + \frac{295}{128}\xi^6 + \frac{1549}{128}\xi^7 + \frac{42271}{512}\xi^8 + \mathcal{O}(\xi^9)$$

γ_E being the Euler-Mascheroni constant. While $\mathcal{P}_T$ is exact in ξ, so far the NNLL contribution to $\mathcal{P}_\mathcal{R}$ has been determined only as an asymptotic expansion in powers of ξ.

5.3 Predictions

A number of other quantities can be calculated from the spectra, such as the "dynamical" tensor-to-scalar ratio

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_\mathcal{R}(k)} \tag{5.13}$$

the tilts

$$n_T = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_T}{\partial \alpha_k}, \qquad n_\mathcal{R} - 1 = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_\mathcal{R}}{\partial \alpha_k},$$

and the running coefficients

$$\frac{\mathrm{d}^n n_T}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_T, \qquad \frac{\mathrm{d}^n n_\mathcal{R}}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_\mathcal{R}.$$

Using (5.12), we find, for example,

$$\begin{aligned} n_T = & -6\left[1 + 4\gamma_M\alpha_k + (12\gamma_M^2 - \pi^2)\alpha_k^2\right]\zeta\alpha_k^2 + \left[24 + 3\xi + 4(31 + 2\xi)\gamma_M\alpha_k\right]\zeta^2\alpha_k^3 \\ & -\frac{1}{8}(1136 + 566\xi + 107\xi^2)\zeta^3\alpha_k^4 + \mathcal{O}(\alpha_k^5), \\ n_\mathcal{R} - 1 = & -4\alpha_k + \frac{4\alpha_k^2}{3}(5 - 6\gamma_M) - \frac{2\alpha_k^3}{9}(338 - 90\gamma_M + 72\gamma_M^2 - 42\pi^2 + 9\xi F_\mathrm{s}) + \mathcal{O}(\alpha_k^4). \end{aligned} \tag{5.14}$$

The first two corrections to the usual relation $r + 8n_T \simeq 0$ are

$$r + 8n_T = -192\zeta\alpha_k^3 + 8(202\zeta + 65\xi\zeta - 144\gamma_M - 8\pi^2 + 3\xi F_\mathrm{s})\zeta\alpha_k^4 + \mathcal{O}(\alpha_k^5). \tag{5.15}$$

We discuss the validity of the predictions by expressing the results in terms of a pivot scale k_* and evolving $\alpha(1/k)$ from k_* to k by means of the RG evolution equations. The spectra become functions of $\ln(k_*/k)$ and the pivot coupling $\alpha_* \equiv \alpha(1/k_*)$. With $k_* = 0.05$ Mpc^{-1} and (for definiteness) $\xi \sim F_\mathrm{s} \sim 1$, the data reported in [28] give $\ln(10^{10}\mathcal{P}_\mathcal{R}^*) = 3.044 \pm 0.014$ and $n_\mathcal{R}^* = 0.9649 \pm 0.0042$, where the star superscript means that the quantity is evaluated at the pivot scale. The second formula of (5.12) and formula (5.14) give the values

$$\alpha_* = 0.0087 \pm 0.0010, \qquad m_\phi = (2.99 \pm 0.37) \cdot 10^{13}\mathrm{GeV}$$

for the "fine structure constant" α_* and the inflaton mass, respectively. The value of m_χ will be known as soon as the tensor-to-scalar ratio r will be measured. The bound $m_\chi > m_\phi/4$ gives $4 \cdot 10^{-4} \lesssim r \lesssim 3.5 \cdot 10^{-3}$ at the pivot scale.

The first formula of (5.12) predicts the tensor spectrum $\mathcal{P}_T$ with a relative theoretical error $\sim \alpha_*^4 \sim 10^{-8}$. The relative error on the tensor tilt n_T is $\sim \alpha_*^3 \sim 10^{-6}$. As far as the quantities involving the scalar fluctuations are concerned, we have to take into account that the function $F_{\mathrm{s}}(\xi)$ is only partially known. It can be shown that the relative theoretical errors of the scalar spectrum $\mathcal{P}_{\mathcal{R}}$ and the scalar tilt $n_{\mathcal{R}} - 1$ are around $\alpha_*^3 \sim 10^{-6}$ for $\xi < 1/2$, 10^{-5} for $1/2 < \xi < 1$ and 10^{-4} for $1 < \xi < 16$.

If primordial cosmology turns into an arena for precision tests of quantum gravity, the predictions might have a chance to be tested in the incoming years [29].

6 Phenomenology of fake particles

Fakeons can be used to propose models of new physics beyond the standard model. For example, the popular inert doublet model [30] has rather different phenomenological properties if the second doublet is taken to be a fakeon [4]. Since the fake doublet avoids the Z-pole constraints regardless of the chosen mass scale, there is room for new effects below the electroweak scale. In addition, the absence of on-shell propagation prevents fakeons from inducing missing energy signatures in collider experiments.

Other types of standard model extensions by means of fakeons predict measurable interactions at energy scales that are usually precluded. For example, the interactions between a fake scalar doublet and the muon can explain discrepancies concerning the measurement of the muon anomalous magnetic moment [5]. The experimental results can be matched for fakeon masses below the electroweak scale without contradicting precision data and collider bounds on new light degrees of freedom.

An important topic for the phenomenology of particle physics is the treatment of dressed propagators. Since a fakeon appears to have a sort of "mass" and a sort of "width", but it is not a particle, we should provide physical meanings for such two quantities. In the next section we explain that the mass is the scale of the violation of microcausality. The width, instead, has a thoroughly new interpretation.

The resummation of self-energy diagrams into dressed propagators in the case of purely virtual particles reveals some unexpected facts, which, in turn, highlight nontrivial properties of long-lived unstable particles. We summarize here the main points, the details being available in ref. [31].

We factor out the normalization factor Z of the propagator. We also include the corrections Δm to the mass m into m itself by default. This way, we can focus our attention on the width Γ, since Z and Δm do not play crucial roles. The formally resummed dressed propagators of physical particles φ, fake particles χ and ghosts ϕ then read, around the peaks,

$$\hat{P}_\varphi \simeq \frac{i}{p^2 - m^2 + i(\epsilon + m\Gamma)}, \qquad \hat{P}_\chi \simeq \frac{i(p^2 - m^2)}{(p^2 - m^2)(p^2 - m^2 + im\Gamma) + \epsilon^2},$$
$$\hat{P}_\phi \simeq -\frac{i}{p^2 - m^2 + i(\epsilon - m\Gamma)}, \tag{6.1}$$

respectively. It is easy to show that they differ by infinitely many contact terms, which do not admit well-defined sums, such as

$$\Delta_{\hat{\Gamma}}(x) \equiv \sum_{n=0}^{\infty} \frac{(-\hat{\Gamma}^2)^n}{(2n)!} \delta^{(2n)}(x), \tag{6.2}$$

where $x \equiv (p^2 - m^2)/m^2$ and $\hat{\Gamma} = \Gamma/m$ ($\Gamma \geqslant 0$). Specifically,

$$\mathrm{Im}[im^2(\hat{P}_\varphi - \hat{P}_\phi)]\Big|_{\epsilon\rightarrow 0} = 2\pi\Delta_{\hat{\Gamma}}(x), \qquad \mathrm{Im}[im^2(\hat{P}_\varphi - \hat{P}_\chi)]\Big|_{\epsilon\rightarrow 0} = \pi\Delta_{\hat{\Gamma}}(x).$$

It turns out that $\Delta_{\hat{\Gamma}}(x)$ is not a well-defined mathematical distribution. What does that mean? The problem is that the peak region is outside the convergence domain of the geometric series and can only be reached in the case of physical particles, from the convergence region, by means of analyticity. In the other cases, nonperturbative effects become important.

Not only. Ill-defined quantities also appear in the case of unstable, long-lived physical particles, when we separate their observation from the observation of their decay products. By the optical theorem, the imaginary part $2\mathrm{Re}[\hat{P}_\varphi]$ is equal to the sum of the cross sections $\Omega_{\varphi\text{particle}}$ and $\Omega_{\varphi\text{decay}}$ of the processes $e^+e^- \rightarrow \varphi$ and $e^+e^- \rightarrow$ decay products of φ, which can be read by cutting the diagrams contributing to the dressed propagators. The former is the process where the particle is physically observed before it decays (as in the case of the muon). The latter is the process where its decay products are observed, instead (as in the case of the Z boson).

We find

$$\Omega_{\varphi\text{particle}} \simeq \frac{\epsilon}{(p^2 - m^2)^2 + (\epsilon + m\Gamma)^2}, \qquad \Omega_{\varphi\text{decay}} \simeq \frac{m\Gamma}{(p^2 - m^2)^2 + (\epsilon + m\Gamma)^2}, \tag{6.3}$$

so the limit $\epsilon \rightarrow 0$ tells us that the muon is unobservable:

$$\Omega_{\varphi\text{particle}} \rightarrow 0, \qquad \Omega_{\varphi\text{decay}} \rightarrow \frac{m\Gamma}{(p^2 - m^2)^2 + m^2\Gamma^2}. \tag{6.4}$$

This is not a surprising result, if we recall that the scattering processes are supposed to occur between incoming states at $t = -\infty$ and outgoing states at $t = +\infty$, which makes it impossible to observe an unstable particle. However, the observation of the muon is a fact and we should be able to account for it.

In practical situations the scattering processes take some finite time interval Δt, much larger than the duration $\bar{\Delta}t$ of the interactions involved in the process, but not equal to infinity. The prediction $\Omega_{\varphi\text{particle}} = 0$ remains correct when Δt is much larger than, say, the muon lifetime τ_μ, but fails for $\bar{\Delta}t \ll \Delta t \lesssim \tau_\mu$.

To solve the impasse, we introduce the energy resolution $\Delta E \sim 1/\bar{\Delta}t$. In principle, we should undertake the task of rederiving all the basic formulas of quantum field theory for scattering processes where incoming and outgoing states are separated by a finite Δt. The results will depend on ΔE, since $\Delta E = 0$ is only compatible with $\bar{\Delta}t = \infty$, hence $\Delta t = \infty$. A clever shortcut is to guess how ΔE may affect the results.

Generically, we can expect that ΔE will affect the formulas more or less everywhere. However, in most places we can neglect it, especially when it redefines quantities that are already present (like the mass m). The ΔE dependence cannot be ignored if it affects a "zero", such as the imaginary part of the denominator of the propagator around the peak.

Thus, we assume that when ΔE is different from zero the predictions coincide with the ones we have written above, provided we make the replacement

$$\epsilon \rightarrow \epsilon + 2m\Delta E, \tag{6.5}$$

after which we can legitimately take ϵ to zero. The form of the ΔE dependence appearing here is not crucial, as long as the correction vanishes when ΔE tends to zero. Making the replacement in formulas (6.3) and letting ϵ tend to zero, we obtain

$$\Omega_{\varphi\text{particle}} \simeq \frac{2m\Delta E}{(p^2 - m^2)^2 + m^2(2\Delta E + \Gamma)^2}, \tag{6.6}$$

$$\Omega_{\varphi\text{decay}} \simeq \frac{m\Gamma}{(p^2 - m^2)^2 + m^2(2\Delta E + \Gamma)^2}, \tag{6.7}$$

The results show that $\Omega_{\varphi\text{particle}}$ is no longer zero. Phenomenologically we may distinguish two opposite cases:

— The case of the Z boson, which is $\Delta E \ll \Gamma/2$. There,

$$\Omega_{\varphi\text{particle}} \simeq 0, \qquad \Omega_{\varphi\text{decay}} \simeq \frac{m\Gamma}{(p^2 - m^2)^2 + m^2\Gamma^2},$$

so we do not see the particle: we see its decay products. The results do not depend on ΔE to the first degree of approximation.

— The case of the muon, which is $m \gg \Delta E \gg \Gamma/2$. There,

$$\Omega_{\varphi\text{particle}} \simeq \frac{2\Delta E}{(p^2 - m^2)^2 + 4m^2\Delta E^2} \simeq \pi\delta(p^2 - m^2), \qquad \Omega_{\varphi\text{decay}} \simeq 0, \tag{6.8}$$

so we see the particle and not its decay products. Again, the results do not depend on ΔE to the first degree of approximation.

In the intermediate situations, where ΔE and Γ are comparable, we see both the particle and its decay products and the results depend on ΔE.

Ultimately, this has to do with the energy-time uncertainty relation $\Delta E \sim 1/\bar{\Delta}t$. Indeed, $\Delta E = 0$ implies an infinite time uncertainty, during which every unstable particle has enough time to decay before being observed. An infinite amount of time is required to determine an energy with absolute precision, and such an amount of time is available only for stable particles. It is impossible to observe an unstable particle with infinite resolving power on its energy.

However, quantum field theory is not quantum mechanics, where wave functions allow us to keep time, coordinates, energy and momenta, and their uncertainty relations, under a satisfactory control. In quantum field theory, as it is usually formulated, we renounce any determination of time and coordinates and tacitly assume infinite resolving powers on energy and momenta. This means that we have a worse control on the built-in uncertainty relations. It may occur that we unawaredly try and calculate something that is impossible to calculate, because it violates such relations, as in the case of $\Omega_{\varphi\text{particle}}$ with no ΔE. The theory cannot return a meaningful result there, otherwise it would be in contradiction with the premises it is built on. Not unexpectedly, we find mathematical problems in the forms of ill-defined distributions, which may appear term by term or in the resummations.

In the case of fakeons something similar happens, but more invasively, since analyticity is less powerful there. Making the replacement $\epsilon \to m\Delta E$ (with a different factor with respect to (6.5), for convenience), the convergence region of $\hat{P}_\chi$ is delimited by the condition

$$\frac{m\Gamma|p^2 - m^2|}{(p^2 - m^2)^2 + m^2\Delta E^2} < 1,$$

which holds for every p if and only if

$$\Delta E > \frac{\Gamma}{2}. \tag{6.9}$$

With the conventions just chosen, this bound coincides with the one of physical particles. The difference is that in the case of physical particles we can cross the obstacle by means of analyticity (unless we separate the observation of the particle from the observation of its decay products, as said). Instead, we cannot cross it in the case of purely virtual particles, because the fakeon prescription is not analytic.

Ghosts exhibit somewhat similar features, in this respect, but we do not discuss them here.

It is conceivable that (6.9) encodes a new type of uncertainty relation, a "peak uncertainty", which expresses the impossibility of approaching the fakeon too closely, given its nature of particle that cannot be brought to reality. It also gives a meaning to the fakeon width, while the fakeon mass codifies the violation of microcausality/microlocality.

These properties suggest that certain processes may involve nonperturbative aspects. A way to avoid them is by restricting the invariant masses $M = \sqrt{p^2}$ of the sets of external states mediated by fakeons by means of the conditions

$$|M^2 - m^2| > m\Gamma. \tag{6.10}$$

So doing, we keep the processes far enough from the regions of the fakeon peaks, which allows us to take ΔE to zero. Under these assumptions, we can make predictions about scattering processes at arbitrarily high energies.

However, conditions like (6.10) do not allow us to sum over the whole phase spaces of the final states, because such a sum includes contributions from the regions of the fakeon peaks. For that purpose, we may propose effective formulas for the complete dressed propagators, argued from the general properties of fakeons. An example is

$$\hat{P}_\chi = \frac{i(p^2 - m^2)}{(p^2 - m^2)(p^2 - m^2 + im\Gamma) + \gamma^2 m^{2+2\delta}\Gamma^{2-2\delta}}, \tag{6.11}$$

where γ and δ are constants, satisfying $\gamma > 0$, $0 < \delta < 1$. This formula can be obtained by choosing

$$\Delta E = \gamma\Gamma \left(\frac{m}{\Gamma}\right)^\delta, \tag{6.12}$$

which fulfills (6.9) in the classical limit $\Gamma \to 0$, where (6.11) correctly tends to the principal value of $i/(p^2 - m^2)$. An expression like (6.12) could be originated by nonperturbative effects or describe the impact of the experimental setup.

If some relatively light fakeon exists in nature, it should be possible to detect the peak uncertainty experimentally. Instead of seeing a resonance, as we expect for a normal particle, we should see a bump, or a smeared peak, with a shape that might even depend on the experimental setup in a way that could be difficult, or impossible, to predict.

7 Peak uncertainty and micro acausality

A violation of microcausality, with typical scale equal to the fakeon mass, is associated with the intrinsic nonlocal nature of the fakeon projection. Consider the toy model described by the Lagrangian

$$\mathcal{L}(x, Q, t) = \frac{m}{2}\dot{x}^2 - m\dot{x}\dot{Q} + \frac{mM^2}{2}Q^2 + xF_{\text{ext}}(t),$$

where x is the coordinate of a physical particle of mass m, Q is the one of a purely virtual particle of mass M and $F_{\text{ext}}(t)$ is a time-dependent external force. The equations of motion give

$$\ddot{x} = -M^2 Q, \qquad \ddot{Q} + M^2 Q = -\frac{1}{m} F_{\text{ext}}(t).$$

The solution of the Q equation, which reads

$$mQ = -\mathcal{P}\frac{1}{\frac{\mathrm{d}^2}{\mathrm{d}t^2} + M^2} F_{\text{ext}}(t) = -\frac{1}{2M}\int_{-\infty}^{\infty} \mathrm{d}u F_{\text{ext}}(t-u) \sin\left(M|u|\right),$$

is given by the fakeon prescription. The equation of motion for x then reads

$$m\ddot{x} = \frac{M}{2}\int_{-\infty}^{\infty} \mathrm{d}u F_{\text{ext}}(t-u) \sin\left(M|u|\right). \tag{7.1}$$

We see that the integral appearing on the right-hand side receives contributions from the external force in the past and in the future. Due to the oscillating behavior of $(M/2)\sin(M|u|)$, the amount of future effectively contributing is

$$|\Delta u| \simeq \frac{1}{M} \tag{7.2}$$

and disappears for $M \to \infty$, since $\lim_{M\to\infty}(M/2)\sin(M|u|) = \delta(u)$. Thus, (7.2) implies that we cannot make predictions for time intervals shorter than τ. In principle, we could check (7.1) a posteriori, if we manage to measure $x(t)$ and $F_{\text{ext}}(t)$ independently.

This example shows that the violation of microcausality, being encoded in the fakeon mass, does not need a nonvanishing width and survives the classical limit. The peak uncertainty, instead, is encoded in the radiative corrections that give Γ, so it disappears in the classical limit. This does not prevent us, though, from making predictions about processes occurring at higher energies. Finally, the violation of microcausality is always present, while it is possible to have no peak uncertainty (6.9), as in the models of ref. [4], where fakeons have identically vanishing widths due to a $\mathbb{Z}_2$ symmetry.

8 Conclusions

Purely virtual particles have a variety of applications, which range from collider physics, to quantum gravity and primordial cosmology. Fakeons mediate interactions without appearing among the incoming and outgoing states. Their consistency with unitarity can be proved by means of algebraic spectral optical identities. The renormalization of a theory with fakeons coincides with the one of the parent Euclidean theory. Its classical limit is described by an ordinary Lagrangian plus Hermitian, microscopically acausal and nonlocal self-interactions among the physical particles.

Quantum gravity with fakeons propagates the graviton, the inflaton and a massive spin-2 fakeon. It can be coupled straightforwardly to the standard model. Its classicization leads to a constrained primordial cosmology, which predicts the tensor-to-scalar ratio r in the window $0.4 \lesssim 1000r \lesssim 3.5$. The interpretation of inflation as a cosmic RG flow allows us to calculate the perturbation spectra up to high orders.

Fakeons evade various phenomenological constraints that apply to physical particles. It is impossible to get too close to the fakeon peak, because of a peak uncertainty, equal to the fakeon width divided by 2, which is expected to be observable. Instead, the fakeon mass is associated to the scale of the violation of microcausality.

In conclusion, the fakeon diagrammatics gives quantum field theory a chance to surpass its own limitations and offer solutions to long-standing problems, without leaving the realm of perturbation theory and without advocating leaps of faith or uncertain approaches, such as string theory [32], loop quantum gravity [33], holography and the AdS/CFT correspondence [34]. The way paved by purely virtual particles tops the competitors in calculability, predictivity and falsifiability. For example, the sharp predictions about inflationary cosmology leave little room for artificial adjustments, in the case of discrepancies with data. Instead, the main weakness of string theory is its lack of predictivity, because of the landscape of 10^{500} or so false vacua [35]. Loop quantum gravity is extremely challenging from the mathematical point of view, when, in contrast, the fakeon diagrammatics is a relatively simple extension of the usual diagrammatics of physical particles. The AdS/CFT correspondence does have a quantum field theoretical side, but it is a strongly coupled one, which leads to use nonperturbative methods, mostly based on conjectures. A separate discussion applies to the idea of asymptotic safety [36], which is purely quantum field theoretical. Nevertheless, it also requires nonperturbative methods, to deal with the interacting ultraviolet fixed points.

Acknowledgments

This work was supported in part by the European Regional Development Fund through the CoE program grant TK133 and the Estonian Research Council grant PRG803.

References

[1] D. Anselmi, Diagrammar of physical and fake particles and spectral optical theorem, J. High Energy Phys. 11 (2021) 030, 21A5 Renormalization.com and arXiv: 2109.06889 [hep-th].

[2] D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

[3] D. Anselmi, E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211, 20A2 Renormalization.com and arXiv:2005.10293 [hep-th].

[4] D. Anselmi, K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Müürsepp, M. Piva and M. Raidal, Phenomenology of a fake inert doublet model, J. High Energy Phys. 10 (2021) 132, 21A3 Renormalization.com and arXiv:2104.02071 [hep-ph].

[5] D. Anselmi, K. Kannike, C. Marzo, L. Marzola, A. Melis, K. Müürsepp, M. Piva and M. Raidal, A fake doublet solution to the muon anomalous magnetic moment, Phys. Rev. D 104 (2021) 035009, 21A4 Renormalization.com and arXiv:2104.03249 [hep-ph].

[6] G. J. van Oldenborgh and J. A. M. Vermaseren, New Algorithms for One Loop Integrals, Z. Phys. C 46 (1990) 425.

J. Kublbeck, M. Bohm, and A. Denner, Feyn Arts: Computer Algebraic Generation of Feynman Graphs and Amplitudes, Comput. Phys. Commun. 60 (1990) 165;

A. Denner, Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200, Fortsch. Phys. 41 (1993) 307 and arXiv:0709.1075 [hep-ph];

T. Hahn, Loop calculations with FeynArts, FormCalc, and LoopTools, Acta Phys. Polon. B30 (1999) 3469 and arXiv:hep-ph/9910227;

T. Hahn, Generating Feynman diagrams and amplitudes with FeynArts 3, Comput. Phys. Commun. 140 (2001) 418 and arXiv:hep-ph/0012260;

A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, FeynRules 2.0 - A complete toolbox for tree-level phenomenology, Comput. Phys. Commun. 185 (2014) 2250 and arXiv:1310.1921;

H.H. Patel, Package-X: A Mathematica package for the analytic calculation of one-loop integrals, Comput. Phys. Commun. 197 (2015) 276 and arXiv:1503.01469 [hep-ph].

[7] D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141, 18A1 Renormalization.com and arXiv:1801.00915 [hep-th].

[8] D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, 17A1 Renormalization.com and arXiv:1703.04584 [hep-th].

[9] D. Anselmi, Fakeons versus Lee-Wick ghosts: physical Pauli-Villars fields, finite QED and quantum gravity, 22A2 Renormalization.com and arXiv:2202.10483 [hep-th].

[10] T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209.

[11] T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

[12] T.D. Lee, A relativistic complex pole model with indefinite metric, in *Quanta: Essays in Theoretical Physics Dedicated to Gregor Wentzel* (Chicago University Press, Chicago, 1970), p. 260.

[13] N. Nakanishi, Lorentz noninvariance of the complex-ghost relativistic field theory, Phys. Rev. D 3 (1971) 811.

[14] R.E. Cutkosky, P.V Landshoff, D.I. Olive, J.C. Polkinghorne, A non-analytic S matrix, Nucl. Phys. B12 (1969) 281.

[15] B. Grinstein, D. O'Connell and M.B. Wise, Causality as an emergent macroscopic phenomenon: The Lee-Wick O(N) model, Phys. Rev. D 79 (2009) 105019 and arXiv:0805.2156 [hep-th].

[16] D. Anselmi, Quantum field theories of arbitrary-spin massive multiplets and Palatini quantum gravity, J. High Energy Phys. 07 (2020) 176 and arXiv:2006.01163 [hep-th].

[17] M. Piva, Massive higher-spin multiplets and asymptotic freedom in quantum gravity, Phys. Rev. D 105 (2022) 045006 and arXiv:2110.09649 [hep-th].

[18] R.E. Cutkosky, Singularities and discontinuities of Feynman amplitudes, J. Math. Phys. 1 (1960) 429;

M. Veltman, Unitarity and causality in a renormalizable field theory with unstable particles, Physica 29 (1963) 186;

G. 't Hooft, Renormalization of massless Yang-Mills fields, Nucl. Phys. B 33 (1971) 173;

G. 't Hooft, Renormalizable Lagrangians for massive Yang-Mills fields, Nucl. Phys. B 35 (1971) 167;

G. 't Hooft and M. Veltman, *Diagrammar*, CERN report CERN-73-09;

M. Veltman, *Diagrammatica. The path to Feynman rules* (Cambridge University Press, New York, 1994).

[19] D. Anselmi, The quest for purely virtual quanta: fakeons versus Feynman-Wheeler particles, J. High Energy Phys. 03 (2020) 142, 20A1 Renormalization.com and arXiv:2001.01942 [hep-th].

[20] J.C. Ward, An identity in quantum electrodynamics, Phys. Rev. 78, (1950) 182;

Y. Takahashi, On the generalized Ward identity, Nuovo Cimento, 6 (1957) 371;

A.A. Slavnov, Ward identities in gauge theories, Theor. Math. Phys. 10 (1972) 99;

J.C. Taylor, Ward identities and charge renormalization of Yang-Mills field, Nucl. Phys. B33 (1971) 436.

[21] K.S. Stelle, Renormalization of higher derivative quantum gravity, Phys. Rev. D 16 (1977) 953.

[22] D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renormalization.com and arXiv:1806.03605 [hep-th].

[23] D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity, Class. and Quantum Grav. 36 (2019) 065010, 18A4 Renormalization.com and arXiv:1809.05037 [hep-th].

[24] D. Anselmi, High-order corrections to inflationary perturbation spectra in quantum gravity, J. Cosmol. Astropart. Phys. 02 (2021) 029, 20A5 Renormalization.com and arXiv:2010.04739 [hep-th].

[25] N.A. Chernikov and E.A. Tagirov, Quantum theory of scalar field in de Sitter space-time, Ann. Inst. H. Poincaré A IX, 2 (1968) 109;

C. Schomblond and P. Spindel, Conditions d'unicité pour le propagateur $\Delta^1(x;y)$ du champ scalaire dans l'univers de de Sitter, Ann. Inst. H. Poincaré A XXV 1 (1976) 67;

T.S. Bunch and P. Davies, Quantum field theory in de Sitter space: renormalization by point splitting, Proc. Royal Soc. London A 360 (1978) 117.

[26] T. Clunan and M. Sasaki, Tensor ghosts in the inflationary cosmology, Class. Quant. Grav. 27 (2010) 165014 and arXiv:0907.3868 [hep-th];

N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, Inflation with a Weyl term, or ghosts at work, J. Cosmol. Astropart. Phys. 1103 (2011) 040 and arXiv:1012.5202 [gr-qc];

N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, Lorentz-violating vs ghost gravitons: the example of Weyl gravity, J. High Energ. Phys. 2012 (2012) 9 and arXiv:1202.3131 [gr-qc];

C. Fang and QG. Huang, The trouble with asymptotically safe inflation, Eur. Phys. J. C 73, 2401 (2013) and arXiv:1210.7596 [hep-th];

Y.S. Myung and T. Moon, Primordial massive gravitational waves from Einstein-Chern-Simons-Weyl gravity, J. Cosmol. Astropart. Phys. 08 (2014) 061 and arXiv:1406.4367 [gr-qc];

K. Kannike, G. Hütsi, L. Pizza, A. Racioppi, M. Raidal, A. Salvio and A. Strumia, Dynamically induced Planck scale and inflation, J. High Energy Phys. 05 (2015) 065 and arXiv:1502.01334 [astro-ph.CO];

M.M. Ivanov and A.A. Tokareva, Cosmology with a light ghost, J. Cosmol. Astropart. Phys. 12 (2016) 018 and arXiv:1610.05330 [hep-th];

A. Salvio, Inflationary perturbations in no-scale theories, Eur. Phys. J. C77 (2017) 267 and arXiv:1703.08012 [astro-ph.CO].

[27] See also V.F. Mukhanov, H.A. Feldman and R.H. Brandenberger, Theory of cosmological perturbations, Phys. Rept. 215 (1992) 203;

D. Baumann, TASI lectures on inflation, arXiv:0907.5424 [hep-th];

S. Weinberg, *Cosmology*, Oxford University Press, 2008.

[28] Planck collaboration, Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211 [astro-ph.CO].

[29] K.N. Abazajian *et al.*, CMB-S4 Science Book, First Edition, arXiv:1610.02743 [astro-ph.CO].

[30] N.G. Deshpande and E. Ma, Pattern of symmetry breaking with two Higgs doublets, Phys. Rev. D 18 (1978) 2574;

E. Ma, Verifiable radiative seesaw mechanism of neutrino mass and dark matter, Phys. Rev. D 73 (2006) 077301 and hep-ph/0601225;

R. Barbieri, L.J. Hall, and V.S. Rychkov, Improved naturalness with a heavy Higgs: an alternative road to LHC physics, Phys. Rev. D 74 (2006) 015007 and hep-ph/0603188;

L. Lopez Honorez, E. Nezri, J.F. Oliver, and M.H.G. Tytgat, The Inert Doublet Model: An archetype for dark matter, J. Cosmol. Astropart. Phys. 02 (2007) 028 and hep-ph/0612275;

A. Belyaev, G. Cacciapaglia, I.P. Ivanov, F. Rojas-Abatte and M. Thomas, Anatomy of the Inert Two Higgs Doublet Model in the light of the LHC and non-LHC Dark Matter Searches, Phys. Rev. D 97 (2018) 035011 and arXiv:1612.00511 [hep-ph].

[31] D. Anselmi, Dressed propagators, fakeon self-energy and peak uncertainty, 22A1 Renormalization.com and arXiv:2201.00832 [hep-ph].

[32] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory I & II*, Cambridge University Press, 1987;

J. Polchinski, *String Theory I & II*, Cambridge University Press, 1998;

K. Becker, M. Becker, J. Schwarz, *String theory and M-theory: A modern introduction*, Cambridge University Press, 2007;

R. Blumenhagen, D. Lust and S. Theisen, Basic Concepts of String Theory, Springer Verlag, 2012.

[33] A. Ashtekar (ed.): *100 years of relativity. Space-time structure: Einstein and beyond*, World Scientific, 2005;

C. Rovelli, *Quantum Gravity*, Cambridge University Press, 2004;

T. Thiemann, *Modern canonical quantum general relativity*, Cambridge University Press, 2007.

[34] J. Maldacena, The Large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1997) 231 and arXiv:hep-th/9711200;

S. Gubser, I. Klebanov, A. Polyakov, Gauge theory correlators from non-critical string theory, Phys. Lett. B 428 (1998) 105 and arXiv:hep-th/9802109;

E. Witten, Anti-de Sitter space and holography, Adv. Theor. Math. Phys. 2 (1998) 253 and arXiv:hep-th/9802150;

For a review, see V.E. Hubeny, The AdS/CFT correspondence, Class. and Quantum Grav. 32 (2015) 124010 and arXiv:1501.00007 [gr-qc].

[35] M. Douglas, The statistics of string/M theory vacua, J. High Energy Phys. 05 (2003) 46 and arXiv:hep-th/0303194;

S. Ashok and M. Douglas, Counting flux vacua, J. High Energy Phys. 01 (2004) 060 and hep-th/0307049.

[36] S. Weinberg, Ultraviolet divergences in quantum theories of gravitation, in *An Einstein centenary survey*, Edited by S. Hawking and W. Israel, Cambridge University Press, Cambridge 1979, p. 790;

O. Lauscher and M. Reuter, Ultraviolet fixed point and generalized flow equation of quantum gravity, Phys. Rev. D65 (2002) 025013 and arXiv:hep-th/0108040;

O. Lauscher and M. Reuter, Flow equation of quantum Einstein gravity in a higher-derivative truncation, Phys. Rev. D66 (2002) 025026 and arXiv:hep-th/0205062;

for a recent update, see K.G. Falls, C.R. King, D.F. Litim, K. Nikolakopoulos, C. Rahmede, Asymptotic safety of quantum gravity beyond Ricci scalars, Phys. Rev. D 97 (2018) 086006 and arXiv:1801.00162 [hep-th].