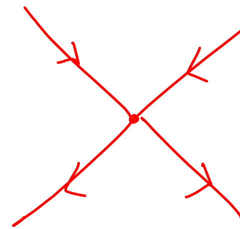
# Primordial cosmology from quantum gravity with purely virtual particles

*Damiano Anselmi*

The Fermi theory of the weak interactions is nonrenormalizable, because it contains a vertex with four fermions, muliplied by the Fermi constant, which has a negative dimension in units of mass

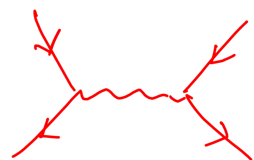
$$G_F(\bar{\psi}\psi)^2$$



$$[G_F] = -2$$

The problem can be solved by introducing massive intermediate bosons, Z and W

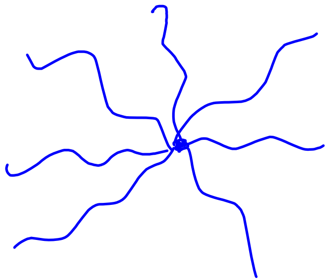
$$g\bar{\psi}A_\mu\psi$$



$$[g] = 0$$

Yang-Mills theory was formulated in 1954, but it was not known, at that time, how to give mass to the intermediate bosons.
Much later, the spontaneous symmetry breaking mechanism and asymptotic freedom turned it into a cornerstone of particle physics

The Einstein-Hilbert action and the Starobinsky R+R^2 theory inflation are nonrenormalizable, like the Fermi theory. They contain vertices with infintely many external legs, and a coupling of negative dimension



$$[G_N] = -2$$

Again, the problem can be solved by introducing intermediate massive particles

Again, the particles are of a type **not known before**, but this time we need to **change the very meaning of the word "particle"** to gain renormalizability without losing unitarity


$$[g] = 0$$

## Summary of the talk

- **Purely virtual particles** (fakeons): quantization prescription and comparison with alternative possibilities and approaches

- The quantum field theory of gravity

- Predictions in primordial cosmology:

  - fakeon projection on curved backgrounds

  - **cosmic RG flow:** fine structure constant, beta function, running coupling, resummation of leading and subleading logs...

  - perturbation spectra

  - r, r + 8 nT, tilts, running coefficients, etc.

The fakeon is a purely virtual particle

- It simulates a physical particle in many situations, but does not belong to the physical spectrum of asymptotic states

- It must be first introduced and projected away at the very end, to have unitarity, but "keep everything local"

- Think of the Faddeev-Popov ghosts: they must be introduced to fix the gauge (while keeping the action local), but must be projected away at the end to recoved unitarity

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \longrightarrow -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2\lambda}(\partial_\mu A^\mu)^2-\bar{C}\partial D C$$

(projected) classical action: ill-defined propagators $\longleftarrow$ (unprojected) gauge-fixed classical action, useful to define propagators and vertices

- The key idea is a new quantization prescription for the poles of the free propagators, alternative to the Feynman i epsilon one

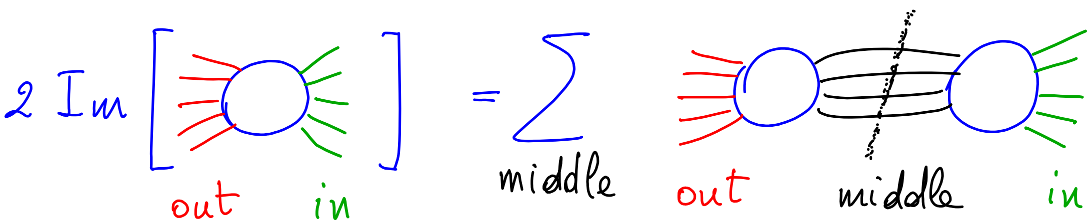
$$\frac{1}{p^2 - m^2 + i\epsilon} \rightarrow ?$$

- Differently from the projection involved in the gauge-fixing procedure, the fakeon prescription/projection **does not follow from a symmetry principle** and it is not protected by that to all orders

- The fakeon can be proved to be **consistent to all orders with unitarity** by means of a completely different approach, which involves surgically changing how we calculate Feynman diagrams

Unitarity: $S^\dagger S = 1$ $\quad$ $S$ = scattering matrix

$2\,\mathrm{Im}\,T = T T^\dagger$ $\quad$ $S = 1 + iT$



- The problem of quantum gravity is to make it renormalizable and unitary at the same time

- A quick way to gain renormalizability is by means of higher-derivatives, but the Feynman prescription then leads to the violation of unitarity (Stelle, 1977)

$$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

a ghost (?)

Let us compare quantization prescriptions for the unprescribed propagator

$$\pm\frac{1}{p^2-m^2}$$

1) Feynman $\pm\dfrac{1}{p^2-m^2+i\epsilon}$ Imaginary part: $\pm\,\delta(p^2-m^2)$

2) Principal value

$$\pm\,\mathcal{P}\frac{1}{p^2-m^2}=\pm\frac{1}{2}\left(\text{Feynman}+\text{Feynman}^*\right)=$$

$$=\pm\frac{1}{2}\left(\text{Ret}+\text{Adv}\right)$$

Imaginary part: $0\cdot\delta(p^2-m^2)$

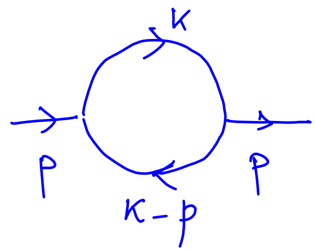
3) fakeon ---> ?

# Bubble diagram

D. Anselmi, The quest for purely virtual quanta: fakeons versus Feynman-Wheeler particles, J. High Energy Phys. 03 (2020) 142, arXiv:2001.01942 [hep-th]

| Feynman | Principal value | Fakeon |
|---|---|---|
| $\dfrac{1}{p^2-m^2+i\epsilon}$ | $\mathcal{P}\dfrac{1}{p^2-m^2}$ | $\left.\dfrac{1}{p^2-m^2}\right\vert_f$ |

bubble diagram



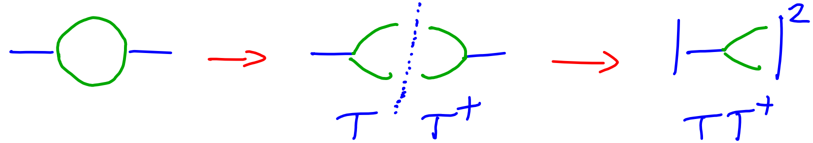

$$\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{(p-k)^2-m_1^2+i\epsilon}\,\frac{1}{k^2-m_2^2\pm i\epsilon}$$

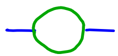
!!!

# Counterterms :

| Feynman | Principal value | Fakeon |
| --- | --- | --- |
| $\dfrac{i \ln \Lambda^2}{4\pi^2}$ | $\dfrac{i \ln \Lambda^2}{4\pi^2} \, \dfrac{m_1^2 - m_2^2}{p^2}$ | $\dfrac{i \ln \Lambda^2}{4\pi^2}$ |
| local | non local | local |
| ok ! | crazy ! | ok ! |

Unitarity:




$2\,\mathrm{Im}\big[(-i)$  $\big] = \dfrac{\sqrt{p^2-(m_1+m_2)^2}\sqrt{p^2-(m_1-m_2)^2}}{8\pi p^2}$ times:

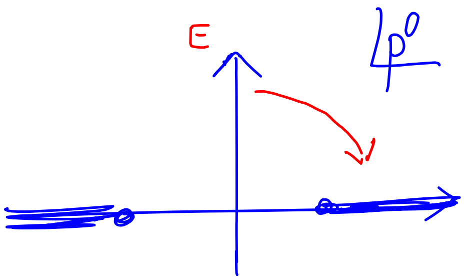
| | | |
|---|---|---|
| $\theta\left(p^2-(m_1+m_2)^2\right)$ | $-\theta\left((m_1-m_2)^2-p^2\right)$ | $0$ |
| Feynman | Principal value | Fakeon |
| $p^2 \geq (m_1+m_2)^2$ | $p^2 \leq (m_1-m_2)^2$ | nothing |
| ok! | crazy! | ok! |

# Feynman prescription vs fakeon prescription

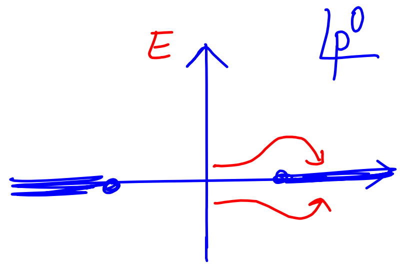
1. Start from the Euclidean version of your loop integral

2. Initiate the Wick rotation

3. Complete the Wick rotation nonanalytically by means of the average continuation



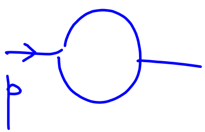

Feynman




fakeon

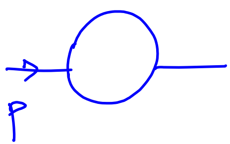
massless case

In formulas the Feynman prescription gives



$$= \text{const.} \times \ln(p^2 - i\epsilon)$$

while the fakeon prescription gives



$$= \text{same const.} \times \frac{1}{2} \ln(p^2)^2$$

Basically,

$$\ln(p^2 - i\epsilon) \longrightarrow \frac{1}{2} \ln(p^2)^2$$

same real part
(in particular, same divergent part)

but DIFFERENT IMAGINARY (ABSORPTIVE) part

unique unitary

Warning: the approach is new, different from previous ones, and leads to different quantitative and qualitative predictions,

**BUT**

the difference is subtle

What do you gain from it?

- A truly **fundamental** theory (other approaches aim at effective theories)

- **Uniqueness**, if we remain on the path laid out by the standard model of particle physics

- **Predictivity**, calculabity, falsifiability, testability within our lifetime (thanks to primordial cosmology)

What entities **CAN** be quantized as purely virtual particles?

- those that with the Feynman prescription would be **GHOSTS**

- those that with the Feynman prescription would be legit **PHYSICAL PARTICLES**

What entities **CAN NOT** be turned into purely virtual particles? **TACHYONS**

$$\pm \frac{1}{p^2 - m^2} \qquad \mathrm{Re}\left[m^2\right] \geqslant 0$$

**Time ordering (past, present, future) loses physical meaning at energies greater than the mass m of the fakeon**

In the late 1960s Lee and Wick claimed they could make sense of higher-derivative theories

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

The existing formulations of the Lee-Wick models have ambiguities and other serious issues, related to using dressed propagators, requiring the presence pairs complex conjugate poles and a problematic classical limit $\frac{1}{p^2 - m^2 + i\Gamma}$

Ultimately, they are unable to really eradicate the ghosts from the theory: they can at most expect that they decay and can be ignored for practical purposes

**Most of all, they lead to essentially different physics and cannot be considered satisfactory at the fundamental level**

Those approaches are something like "living with ghosts" (NO THANKS!)

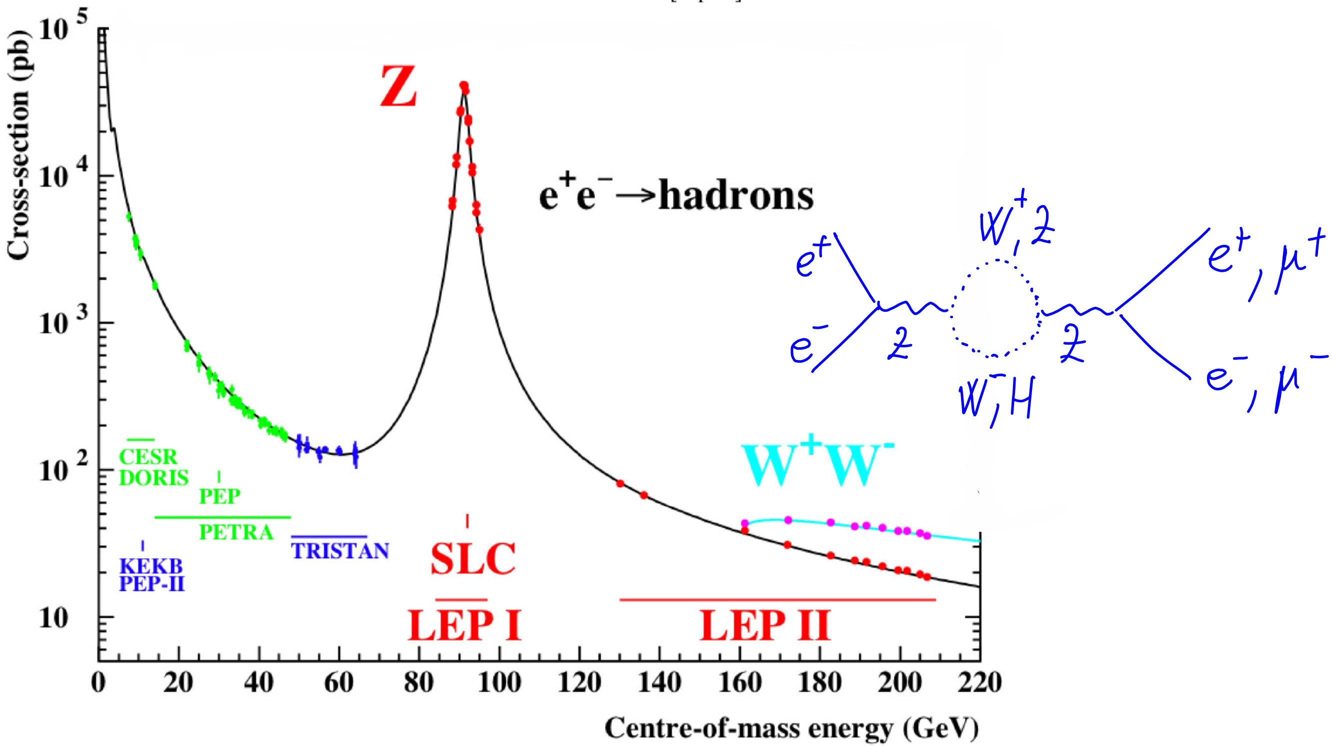
Hawking, Hertog, "Living with ghosts", PRD 65 (2002) 103515 arXiv:hep-th/0107088

The muons are a good example why this procedure does NOT provide definitive answers: it is not enough to have UNSTABLE entities (e.g. ghosts) to claim that they do not belong to the physical spectrum of asymptotic states

Example: are there fakeons among the standard model?

Is the Higgs boson a fakeon?

D. Anselmi, **On the nature of the Higgs boson**, Mod. Phys. Lett. A 34 (2019) 1950123, arXiv: 1811.02600 [hep-th].



Yet, the problems of the Lee-Wick models can be cured by means of the fakeon idea

Average continuation

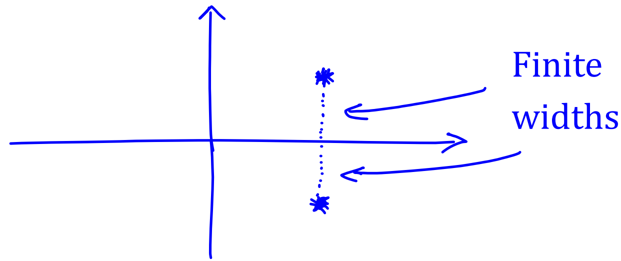
D. Anselmi and M. Piva, **A new formulation of Lee-Wick quantum field theory**, J. High Energy Phys. 06 (2017) 066 and arXiv: 1703.04584 [hep-th]

D. Anselmi and M. Piva, **Perturbative unitarity of Lee-Wick quantum field theory**, Phys. Rev. D 96 (2017) 045009 and arXiv: 1703.05563 [hep-th]

I stress again that, once reformulated, the models become something physically very different from what Lee and Wick originally intended, since the LW models do not have the goal of projecting states away consistently, which is crucial to obtain a fundamental theory instead of an effective one

But there was another problem:

LW models apparently need pairs of complex conjugate poles




This requires MORE higher-derivatives

$$\int (R + R^2 + R\Box R)\sqrt{-g}$$

but then gravity is superrenormalizable (unlike the gauge interactions of the standard model) and NOT UNIQUE

The new models do not have this problem and with a few tricks they can handle poles on the real axis (zero widths at the tree level)

D. Anselmi, **On the quantum field theory of the gravitational interactions**, J. High Energy Phys. 06 (2017) 086 and arXiv: 1704.07728 [hep-th].

QG becomes unique

D. Anselmi, **Fakeons and Lee-Wick models**, J. High Energy Phys. 02 (2018) 141 and arXiv: 1801.00915 [hep-th].

All-order theorems here

# Quantum gravity

Consider the (renormalizable) higher-derivative Lagrangian

$$S_{\mathrm{geom}}(g,\Phi) = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right)$$

If quantized as usual, this theory has a ghost

K.S. Stelle, Renormalization of higher derivative quantum gravity, Phys. Rev. D 16 (1977) 953.

Solution: quantize the would-be ghost as a purely virtual particle

D. Anselmi, **On the quantum field theory of the gravitational interactions**, J. High Energy Phys. 06 (2017) 086 and arXiv: 1704.07728 [hep-th].

IMPORTANT: the action

$$S_{\text{geom}}(g,\Phi) = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right)$$

is NOT the classical limit of the quantum field theory of gravity, because it is unprojected

It should be regarded more or less like a gauge-fixed action: it is useful to derive simple Feynman rules, but it contains things (e.g. the Faddeev-Popov ghosts) that should be dropped to do physics

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\lambda}(\partial_\mu A^\mu)^2 - \bar{C}\partial D C$$

The classical limit is obtained by CLASSICIZING the quantum field theory of gravity, since the fakeon is purely quantum

D. Anselmi, **Fakeons, microcausality and the classical limit of quantum gravity**, Class. and Quantum Grav. 36 (2019) 065010 and arXiv: 1809.05037 [hep-th]

D. Anselmi, **Fakeons and the classicization of quantum gravity: the FLRW metric**, J. High Energy Phys. 04 (2019) 61 and arXiv: 1901.09273 [gr-qc]

## The triplet of quantum gravity

| | | | inflationary perturbation spectrum |
| --- | --- | --- | --- |
| $h_{\mu\nu}$ | $=$ | graviton, fluctuation of the metric | ✓ |
| $\phi$ | $=$ | inflaton, mass $m_\phi$ | ✓ |
| $\chi_{\mu\nu}$ | $=$ | fakeon, spin 2, mass $m_\chi$ | ✗ |

Renormalization constants, beta functions, widths, absorptive parts, dressed propagators, checks of the optical theorem...

D. Anselmi and M. Piva, **Quantum gravity, fakeons and microcausality**, J. High Energy Phys. 11 (2018) 21 and arXiv: 1806.03605 [hep-th].

D. Anselmi and M. Piva, **The ultraviolet behavior of quantum gravity**, J. High Energy Phys. 05 (2018) 27 and arXiv: 1803.07777 [hep-th].

# Primordial cosmology from quantum gravity

We use

$$S_{\rm QG} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(D_\mu\phi D^\mu\phi - 2V(\phi)\right),$$

$$V(\phi) = \frac{3m_\phi^2}{32\pi G}\left(1 - \mathrm{e}^{\phi\sqrt{16\pi G/3}}\right)^2$$

Starobinsky potential

which is the previous action up to the introduction of an auxiliary field $\phi$ and a Weyl transformation to eliminate $R^2$

$$S_{\rm geom}(g,\Phi) = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \cancel{\frac{R^2}{6m_\phi^2}}\right)$$

D. Anselmi, E. Bianchi and M. Piva, **Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term**, J. High Energy Phys. 07 (2020) 211 and arXiv:2005.10293 [hep-th].

D. Anselmi, **Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity**, arXiv: 2007.15023 [hep-th]

D. Anselmi, **High-order corrections to inflationary perturbation spectra in quantum gravity**, arXiv: 2010.04739 [hep-th]

The Friedmann equations do not change

$$H = \dot{a}/a$$

$$\dot{H} = -4\pi G\dot{\phi}^2, \qquad H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + 2V(\phi)\right), \qquad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} = -V'(\phi),$$

Define the coupling
("fine structure constant" of inflation)

$$\alpha = \sqrt{\frac{4\pi G}{3}}\frac{\dot{\phi}}{H} = \sqrt{-\frac{\dot{H}}{3H^2}}$$

The Friedmann equations become

$$\boxed{\begin{aligned}\dot{\alpha} &= m_\phi\sqrt{1-\alpha^2} - H(2+3\alpha)\left(1-\alpha^2\right)\\ \dot{H} &= -3\alpha^2H^2\end{aligned}}$$

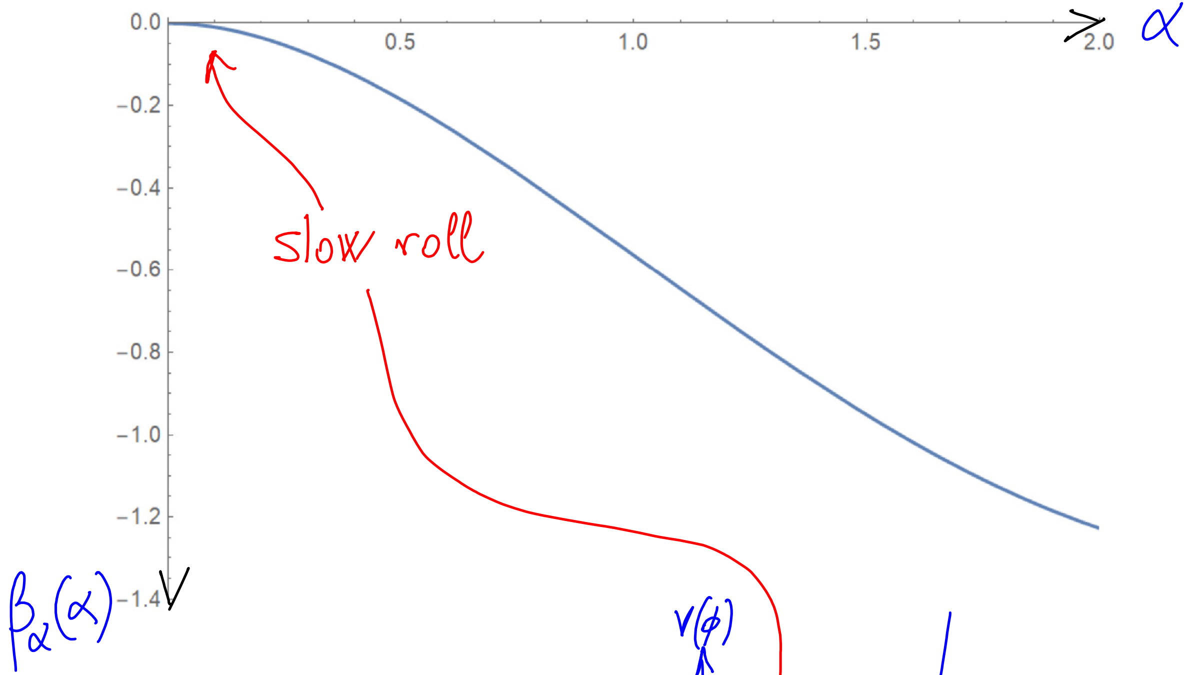
We can introduce the beta function
of the "cosmic RG flow"

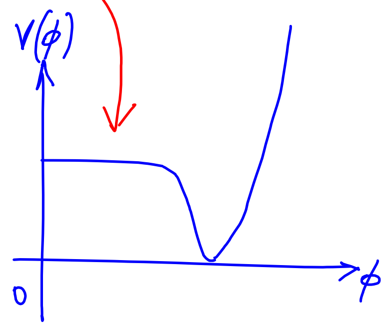
$$\beta_\alpha = \frac{d\alpha}{d\ln|\tau|}$$

$$\left(\tau = -\int_t^{+\infty}\frac{\mathrm{d}t'}{a(t')} \quad \text{= conformal time}\right)$$

$$\beta_\alpha = -2\alpha^2\left[1 + \frac{5}{6}\alpha + \frac{25}{9}\alpha^2 + \frac{383}{27}\alpha^3 + \frac{8155}{81}\alpha^4 + \frac{72206}{81}\alpha^5 + \frac{2367907}{243}\alpha^6 + \mathcal{O}(\alpha^7)\right]$$

D. Anselmi, **Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity**, arXiv: 2007.15023 [hep-th]

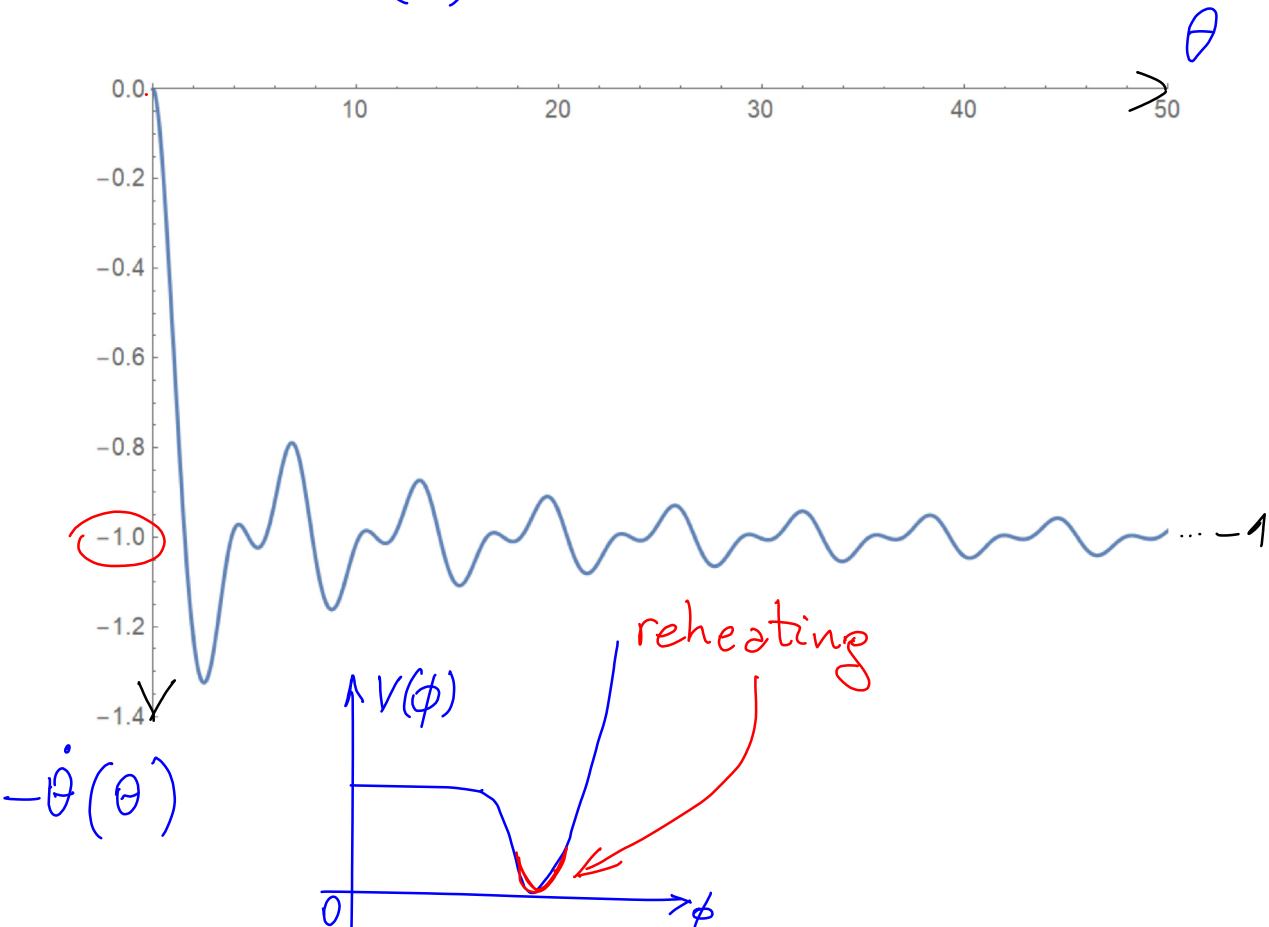



$$\beta_\alpha(\alpha) = -2\alpha^2 + O(\alpha^3)$$

[like pure QCD]

$\Theta = \arcsin(\alpha)$



Leading log running coupling $$\alpha = \frac{\alpha_k}{1 + 2\alpha_k \ln(-k\tau)}$$

$$\left[\beta_\alpha(\alpha) = -2\alpha^2\right]$$

Running coupling to the NNLL order

$$\alpha = \frac{\alpha_k}{\lambda}\left(1 - \frac{5\alpha_k}{6\lambda}\ln\lambda\right)\left[1 + \frac{25\alpha_k^2}{12\lambda^2}\left(1 - \lambda - \frac{\ln\lambda}{3}(1 - \ln\lambda)\right)\right]$$

$$\lambda \equiv 1 + 2\alpha_k \ln\eta, \; \underline{\eta = -k\tau}$$

RG equations:

Spectra: $\mathcal{P}(k) = \tilde{\mathcal{P}}(\alpha_k)$ in the superhorizon limit

$\alpha_k = \alpha(1/k) =$ running coupling at momentum $k$

## "Dynamical" tensor-to-scalar ratio

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_\mathcal{R}(k)}$$

## Tilts

$$n_T = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_T}{\partial \alpha_k}, \qquad n_\mathcal{R} - 1 = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_\mathcal{R}}{\partial \alpha_k}$$

## Running coefficients

$$\frac{\mathrm{d}^n n_T}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_T, \qquad \frac{\mathrm{d}^n n_\mathcal{R}}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_\mathcal{R}$$

Results

Effects of the fakeon $\chi_{\mu\nu}$ everywhere

$$\mathcal{P}_T(k) = \frac{4m_\phi^2\zeta G}{\pi}\left[1 - 3\zeta\alpha_k\left(1 + 2\alpha_k\gamma_M + 4\gamma_M^2\alpha_k^2 - \frac{\pi^2\alpha_k^2}{3}\right) + \frac{\zeta^2\alpha_k^2}{8}(94 + 11\xi)\right.$$
$$\left. + 3\gamma_M\zeta^2\alpha_k^3(14+\xi) - \frac{\zeta^3\alpha_k^3}{12}(614 + 191\xi + 23\xi^2) + \mathcal{O}(\alpha_k^4)\right]$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2}\left[1 + (5 - 4\gamma_M)\alpha_k + \left(4\gamma_M^2 - \frac{40}{3}\gamma_M + \frac{7}{3}\pi^2 - \frac{67}{12} - \underbrace{\frac{\xi}{2}F_s(\xi)}\right)\alpha_k^2 + \mathcal{O}(\alpha_k^3)\right]$$

First effect of the fakeon

where

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad \tilde{\gamma}_M = \gamma_M - \frac{i\pi}{2}, \qquad \gamma_M = \gamma_E + \ln 2,$$

$$F_s(\xi) = 1 + \frac{\xi}{4} + \frac{\xi^2}{8} + \frac{\xi^3}{8} + \frac{7\xi^4}{32} + \frac{19}{32}\xi^5 + \frac{295}{128}\xi^6 + \frac{1549}{128}\xi^7 + \frac{42271}{512}\xi^8 + \mathcal{O}(\xi^9)$$

$$n_T = -6\left[1 + 4\gamma_M\alpha_k + (12\gamma_M^2 - \pi^2)\alpha_k^2\right]\zeta\alpha_k^2 + \left[24 + 3\xi + 4(31 + 2\xi)\gamma_M\alpha_k\right]\zeta^2\alpha_k^3$$
$$-\frac{1}{8}(1136 + 566\xi + 107\xi^2)\zeta^3\alpha_k^4 + \mathcal{O}(\alpha_k^5),$$

$\xi = \frac{m_\phi^2}{m_\chi^2}$, $\zeta = \frac{2m_\chi^2}{m_\phi^2 + 2m_\chi^2}$

$$n_{\mathcal{R}} - 1 = -4\alpha_k + \frac{4\alpha_k^2}{3}(5 - 6\gamma_M) - \frac{2\alpha_k^3}{9}(338 - 90\gamma_M + 72\gamma_M^2 - 42\pi^2 + \underbrace{9\xi F_{\rm s}}) + \mathcal{O}(\alpha_k^4)$$

$$r + 8n_T = -192\zeta\alpha_k^3 + 8(202\zeta + 65\xi\zeta - 144\gamma_M - 8\pi^2 + 3\xi F_{\rm s})\zeta\alpha_k^4 + \mathcal{O}(\alpha_k^5)$$

$0 \cdot \alpha_k^2$

A coincidence makes the lowest order of this quantity vanish,
but it is definitely not zero

# Expansion in leading and subleading logs

$$\alpha(1/\kappa) = \alpha\left(\alpha_*, \ln \kappa/\kappa_*\right)$$

$\kappa_* =$ pivot scale $= 0.05\ \mathrm{Mpc}^{-1}$

$\alpha_* =$ pivot coupling $= 0.0087 \pm 0.0090 \sim \frac{1}{115}$

from $\mathcal{P}_{\mathcal{R}}$ and $n_{\mathcal{R}}$ Planck collaboration, Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211 [astro-ph.CO].

Expansion in powers of $\alpha_*$ , but

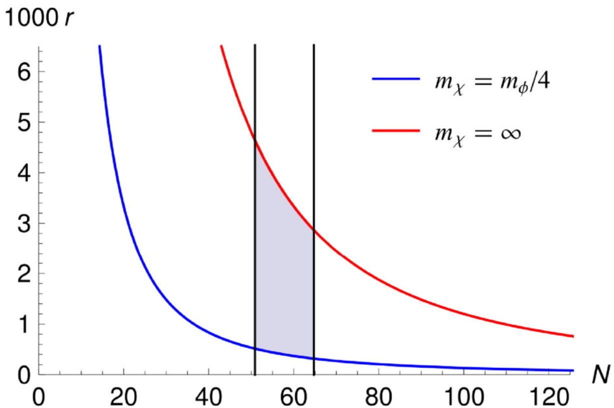
the product $\alpha_* \ln \frac{\kappa}{\kappa_*}$ is considered of order zero

and treated exactly

D. Anselmi, E. Bianchi and M. Piva, **Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term**, J. High Energy Phys. 07 (2020) 211 and arXiv:2005.10293 [hep-th].

The consistency of the fakeon projection on a curved background puts a lower bound on $m_\chi$ :

$$\frac{m_\phi}{4} < m_\chi < \infty$$

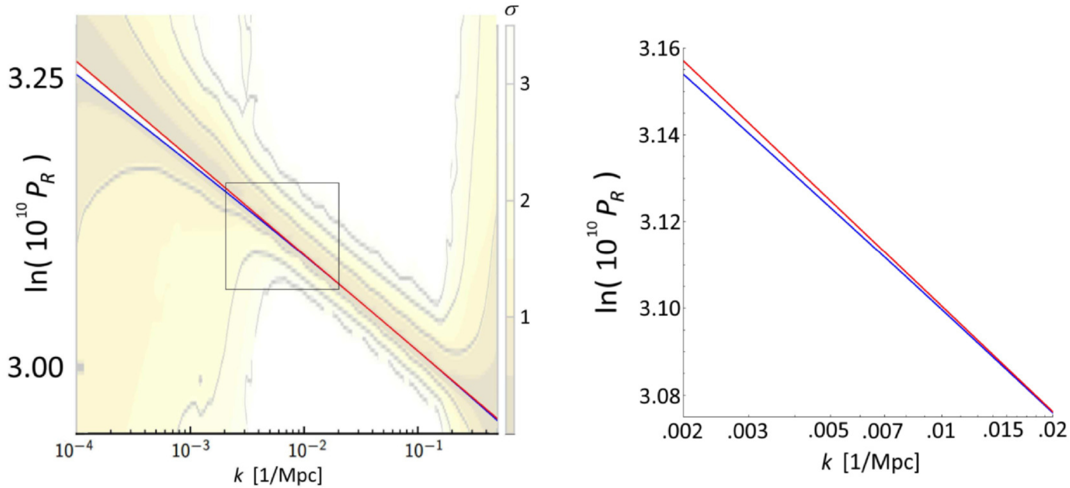



Allowed values of the tensor-to-scalar ratio $r$

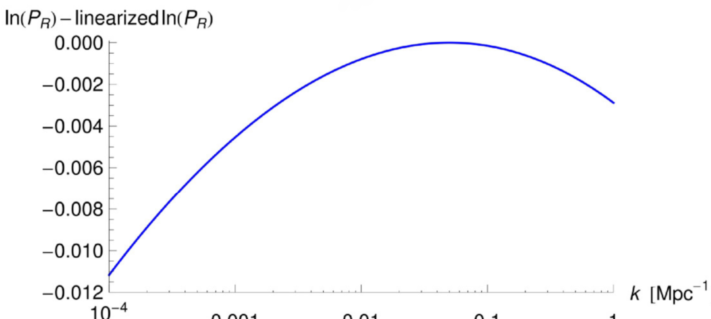
$$0.4 \lesssim 1000r \lesssim 3,$$

$$-0.4 \lesssim 1000n_T \lesssim -0.05$$

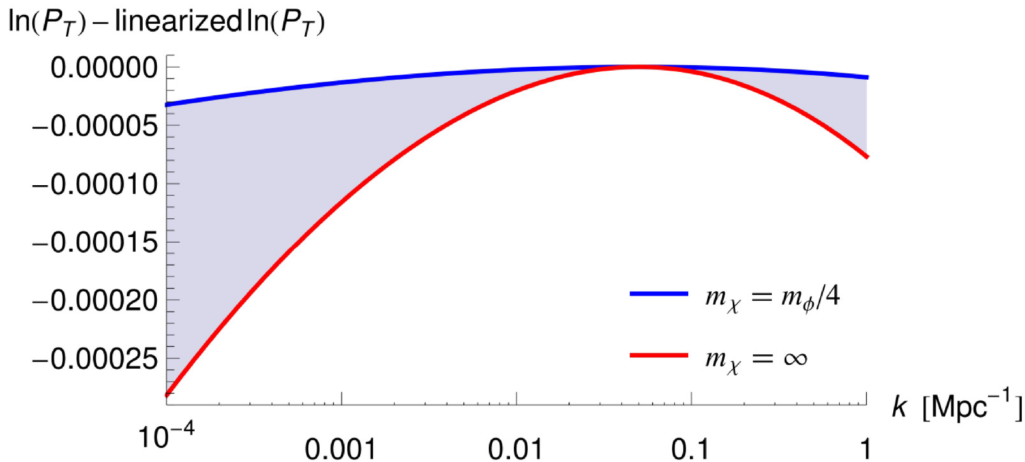
for $N = 60$




D. Anselmi, Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, arXiv:2007.15023




Running of the scalar spectrum




Running of the tensor spectrum

# Derivation (tensor spectrum)

$$S_{\rm QG} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(D_\mu\phi D^\mu\phi - 2V(\phi)\right)$$

Expand with

$$g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2) - 2a^2\left(u\delta_\mu^1\delta_\nu^1 - u\delta_\mu^2\delta_\nu^2 + v\delta_\mu^1\delta_\nu^2 + v\delta_\mu^2\delta_\nu^1\right),$$

*u=u(t,z)* $\longrightarrow$ $u_{\mathbf{k}}(t)$

*v=v(t,z)*

$k = |\mathbf{k}|$

The quadratic Lagrangian is

$$(8\pi G)\frac{\mathcal{L}_{\rm t}}{a^3} = \underbrace{\dot{u}^2 - \frac{k^2}{a^2}u^2}_{\text{Starobinsky limit}} - \frac{1}{m_\chi^2}\underbrace{\left[\ddot{u}^2 - 2\left(H^2 - \frac{3}{2}\alpha^2 H^2 + \frac{k^2}{a^2}\right)\dot{u}^2 + \frac{k^4}{a^4}u^2\right]}_{\text{HD fakeon correction}}$$

$m_\chi \to \infty$ $\searrow$ Mukhanov-Sasaki equation

$\longrightarrow$ spectrum

# Limit $m_\chi \to \infty$ of infinitely heavy fakeon

$$(8\pi G)\frac{\mathcal{L}_{\mathrm{t}}}{a^3} = \dot{u}_{\mathbf{k}}\dot{u}_{-\mathbf{k}} - \frac{k^2}{a^2}u_{\mathbf{k}}u_{-\mathbf{k}}$$

$u_{\vec{k}} \longrightarrow w_{\vec{k}}$ :

$$w = au\sqrt{\frac{k}{4\pi G}} \qquad \eta = -k\tau$$

Mukhanov-Sasaki action

$$S_{\mathrm{t}} = \frac{1}{2}\int \mathrm{d}\eta \left[w_{\mathbf{k}}'^2 - w_{\mathbf{k}}^2 + (2+\sigma_{\mathrm{t}})\frac{w_{\mathbf{k}}^2}{\eta^2}\right]$$

$$2 + \sigma_{\mathrm{t}} = 2\tau^2 a^2 H^2\left(1 - \frac{3}{2}\alpha^2\right)$$

$$\sigma_{\mathrm{t}} = 9\alpha^2 + 48\alpha^3 + 364\alpha^4 + \mathcal{O}(\alpha^5)$$

no $\alpha_0 w_1(\eta)$ $\qquad$ $\alpha_0 = \alpha(1/k)$

**Expansion** $$w(\eta) = w_0(\eta) + \alpha_0^2 w_2(\eta) + \alpha_0^3 w_3(\eta) + \cdots$$

$$w_n'' + w_n - \frac{2w_n}{\eta^2} = \frac{g_n(\eta)}{\eta^2}$$

Bunch-Davies vacuum condition

$$w(\eta) \sim \frac{\mathrm{e}^{i\eta}}{\sqrt{2}} \qquad \text{for large } \eta$$

$$g_0(\eta) = g_1(\eta) = 0$$

$$g_2 = 9w_0, \qquad g_3 = 12w_0(4 - 3\ln\eta), \qquad g_4 = 9w_2 + 2w_0(182 - 159\ln\eta + 54\ln^2\eta)$$

**Solutions** $$w_0 = \frac{(\eta + i)}{\eta\sqrt{2}}\mathrm{e}^{i\eta}$$

$$w_2(\eta) = \frac{3}{\eta\sqrt{2}}\left[2i\mathrm{e}^{i\eta} + (\eta - i)\mathrm{e}^{-i\eta}\left(\mathrm{Ei}(2i\eta) - i\pi\right)\right],$$

$$w_3(\eta) = -\frac{12i\sqrt{2}\mathrm{e}^{i\eta}\ln\eta}{\eta} + \frac{3\sqrt{2}(\eta - i)\mathrm{e}^{-i\eta}}{\eta}\Bigg(2i\pi - \frac{\pi^2}{12} - i\pi\gamma_M - 2(\ln\eta + 1)\mathrm{Ei}(2i\eta)$$
$$+ i\pi\ln\eta + (\ln\eta + \gamma_M)^2 + 4i\eta\, F_{2,2,2}^{1,1,1}(2i\eta)\Bigg),$$

# THEN,

- Quantize *w* as usual by means of creation and annihilation operators

- Compute the two-point function < *w* w >

- Go back to *u* through the *w* definition $$w = au\sqrt{\frac{k}{4\pi G}}$$

- Compute the two-point function < *u* *u* >

- Finally, work out the spectrum $\mathcal{P}_T(k)$ from

$$\langle \hat{u}_{\mathbf{k}}(\tau)\hat{u}_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{\pi^2}{8k^3}\mathcal{P}_T(k)$$

# Quantum gravity case

$$(8\pi G)\frac{\mathcal{L}_t}{a^3} = \dot{u}^2 - \frac{k^2}{a^2}u^2 - \frac{1}{m_\chi^2}\left[\ddot{u}^2 - 2\left(H^2 - \frac{3}{2}\alpha^2 H^2 + \frac{k^2}{a^2}\right)\dot{u}^2 + \frac{k^4}{a^4}u^2\right]$$

$$(8\pi G)\frac{\mathcal{L}_t}{a^3} = \dot{u}^2 - \frac{k^2}{a^2}u^2 - \frac{1}{m_\chi^2}\left[\ddot{u}^2 + X\right]$$

Add: $$+\frac{1}{m_\chi^2}\left[\,U - \ddot{u} + Y\,\right]^2$$

U = new field

Result: one more field
(now we have ***u*** and ***U***)
but no higher derivatives

Next: Change variables from ***u, U*** to ***U, V*** to diagonalize the action in the de Sitter limit

***V*** = other new field = c(t) ***u*** + d(t) ***U***,

where c(t) and d(t) are suitable functions of the background

It is a nontrivial fact that this turns out TO BE POSSIBLE!

$$(8\pi G)\frac{\mathcal{L}_{\mathrm{t}}'}{a^3} = \dot{U}^2 - \frac{k^2}{a^2}U^2 - \dot{V}^2 + \left[m_\chi^2 + \frac{m_\phi^2}{2} + \frac{k^2}{a^2}\right]V^2$$

plus corrections of order $\alpha$

wrong sign!!!!

This is the fakeon that we must project away: *V* -----> $V(U) = \mathcal{O}(\alpha)$

## The fakeon projection

The classical limit of the fakeon prescription is just (in flat space):

$$\frac{1}{2}\left[\frac{1}{p^2-m^2+i\epsilon}+\frac{1}{p^2-m^2-i\epsilon}\right]$$

$$=\frac{1}{2}\left(\text{Feynman} + \text{Feynman}^*\right)$$

which, after Fourier transform reads

$$\frac{1}{2\,m}\sin\left(m\,|t-t'|\right)$$

No-tachyon condition: m real

After normalizing the coefficient to one, we get

$$\hat{L} = -\frac{1}{2}\left(\frac{\mathrm{d}\hat{V}}{\mathrm{d}t}\right)^2 + \frac{m(t)^2}{2}\hat{V}^2, \qquad m(t)^2 = m_\chi^2 - \frac{m_\phi^2}{16} + \frac{k^2}{a^2}$$

time-dependent mass

What happens on a nontrivial background (which is like a time-dependent mass)?

Luckily, the de Sitter limit is like flat space (in conformal time) for *k/(aH)* large and like flat space (in cosmological time) for *k/(aH)* small

In either case we know the solution, so we know it everywhere (and the other case is a consistency check)

BUT (!!!!!!) the NO-TACHYON condition demands $m_\chi > \frac{m_\phi}{4}$

Elimination of the higher derivatives by means of an extra field $U$:

$$\mathcal{L}'_{\rm t} = \mathcal{L}_{\rm t} + \frac{a^3}{8\pi G m_\chi^2}\left[m_\chi^2\sqrt{\gamma}U - \ddot{u} - 3H\left(1 - \frac{4\alpha^2 H^2}{m_\chi^2\gamma}\right)\dot{u} - fu\right]^2$$

$$f = m_\chi^2\gamma + \frac{k^2}{a^2} + \frac{\alpha^2 H^2}{m_\chi^2\gamma}\left(3m_\chi^2 - 12H^2 + 24\alpha H^2 - \frac{2\alpha^2 H^2(17m_\chi^2 - 38H^2)}{m_\chi^2\gamma}\right) \qquad \gamma = 1 + 2\frac{H^2}{m_\chi^2}.$$

Diagonalization of the de Sitter limit:

$$u = \frac{U+V}{\sqrt{\gamma}}$$

$$\begin{aligned}(8\pi G)\frac{\mathcal{L}'_{\rm t}}{a^3} = {} & \dot{U}^2 - \frac{hk^2}{a^2}U^2 - \frac{9}{8}m_\phi^2\xi\zeta\alpha^2\left[1 - \frac{\zeta\alpha}{6}(40-7\xi) + \frac{\zeta^2\alpha^2}{144}(2800 - 3806\xi - 497\xi^2)\right]U^2 \\ & -\dot{V}^2 + \left[m_\chi^2 + \frac{m_\phi^2}{2}(1-3\alpha) + \frac{k^2}{a^2}\right]V^2 + \frac{3m_\phi^2\zeta\alpha^2}{2}\left(1 - \xi + \frac{4\xi\zeta k^2}{m_\phi^2 a^2}\right)UV \\ & -\frac{3m_\phi^2\zeta^2\alpha^3}{4}\left[6 - 22\xi + \xi^2 + (6 - 7\xi - 2\xi^2)\frac{4\xi\zeta k^2}{m_\phi^2 a^2}\right]UV\end{aligned}$$

*$V(U)=O(\alpha^2)$*

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad h = 1 - 3\xi\zeta^2\alpha^2 + \frac{3\xi\zeta^3\alpha^3}{2}(6 - 7\xi - 2\xi^2) + \mathcal{O}(\alpha^4).$$

Fakeon projection:

insert the ansatz $$V = \alpha^2 (v_1 + v_2\alpha) U + v_3\alpha^3 \dot{U} + \mathcal{O}(\alpha^4)$$ $v_1, v_2, v_3$ unknown constants

into the *V* equation of motion. Using the *U* equation of motion, we find

$$V = -\frac{3\xi\zeta^2\alpha^2}{4}\left[1 - \xi - (6 - 19\xi - 2\xi^2)\frac{\zeta\alpha}{2}\right] U + \frac{3(1-\xi)\xi^2\zeta^3\alpha^3}{m_\phi}\dot{U} + \mathcal{O}(\alpha^4)$$

projected Mukhanov-Sasaki action :

$$S_{\mathrm{t}}^{\mathrm{prj}} = \frac{1}{2}\int \mathrm{d}\eta \left( w'^2 - hw^2 + 2\frac{w^2}{\eta^2} + \sigma_{\mathrm{t}}\frac{w^2}{\eta^2} \right)$$

mass "renormalization"

where

$$w = \frac{aU\sqrt{k}}{\sqrt{4\pi G}}, \qquad \sigma_{\mathrm{t}} = 9\zeta\alpha^2 + \frac{3\zeta^2\alpha^3}{2}(32 + 43\xi) + \zeta^3\alpha^4 \underbrace{F_{\mathrm{t}}(\xi)} + \mathcal{O}(\alpha^5)$$

$$F_{\mathrm{t}}(\xi) = 364 + \frac{4037}{8}\xi + \frac{6145}{16}\xi^2 + \frac{81}{2}\xi^3$$

$$h = 1 - 3\xi\zeta^2\alpha^2 + \frac{3\xi\zeta^3\alpha^3}{2}(6 - 7\xi - 2\xi^2) + \mathcal{O}(\alpha^4)$$

Making the change of variables $\eta = -\kappa\tau \longrightarrow \tilde{\eta}(\eta)$

with $\tilde{\eta}'(\eta) = \sqrt{h(\eta)},\ \tilde{\eta}(0) = 0$

we obtain the action

$$\tilde{S}_{\mathrm{t}}^{\mathrm{prj}} = \frac{1}{2}\int \mathrm{d}\tilde{\eta}\left(\tilde{w}'^2 - \tilde{w}^2 + \frac{2\tilde{w}^2}{\tilde{\eta}^2} + \tilde{\sigma}_{\mathrm{t}}\frac{\tilde{w}^2}{\tilde{\eta}^2}\right)$$

with

$$\tilde{w}(\tilde{\eta}(\eta)) = h(\eta)^{1/4} w(\eta), \qquad \tilde{\sigma}_{\mathrm{t}} = \frac{\tilde{\eta}^2(\sigma_{\mathrm{t}} + 2)}{\eta^2 h} + \frac{\tilde{\eta}^2}{16h^3}\left(4hh'' - 5h'^2\right) - 2$$

The Bunch-Davies vacuum condition in these variables is the standard one:

$$\tilde{w}(\tilde{\eta}) \simeq \frac{\mathrm{e}^{i\tilde{\eta}}}{\sqrt{2}} \qquad \text{for } \tilde{\eta} \to \infty,$$

# THEN,

- Go back to the original variables, $\eta$ , $w(\eta)$ ...
- Work out the Bunch-Davies vacuum conditions for those
- Solve the Muk.-Sas. equation for $w(\eta) = w_0 + \alpha_k w_1 + \alpha_k^2 w_2 + \cdots$
- Go back to $U$ through the $w$ definition $$w = \frac{aU\sqrt{k}}{\sqrt{4\pi G}}$$
- Find $V(U)$ through the fakeon projection $$V = \alpha^2 (v_1 + v_2\alpha) U + v_3\alpha^3\dot{U} + \mathcal{O}(\alpha^4)$$
- Find $u$ through the de Sitter diagonalization $$u = \frac{U+V}{\sqrt{\gamma}} \qquad \gamma = 1 + 2\frac{H^2}{m_\chi^2}$$
- Finally, work out the spectrum $\mathcal{P}_T$ from $\langle u \; u \rangle$

# Conclusions

## From particle physics to inflationary cosmology and back

The constraints originated by high-energy physics (locality, renormalizability and unitarity) allow us to overcome the arbitrariness of classical theories and formulate a basically unique quantum field theory of gravity, which contains just two parameters more than the Einstein theory, which are the mass of the inflaton and the mass of the spin-2 fakeon.

Other constraints coming from cosmology allow us to derive a condition that binds the two masses. Altogether, the constraints from cosmology and those from high-energy physics produce a very predictive theory, which could be tested in the forthcoming years thanks to inflation.

We have worked out the spectra, the tensor-to-scalar ratio, the tilts, the running coefficients and the combination $r+8n_T$ to the next-to-next-to-leading log order. Thanks to the "cosmic" renormalization-group flow, we have resummed the leading and subleading logs and expressed all quantities as power series in the running inflationary fine structure constant $\alpha \sim 1/115$.

Hopefully, primordial cosmology will turn into an arena for precision tests of quantum gravity, which will experience the same success achieved by the standard model of particle physics in the past decades!