Predictions in inflationary cosmology from quantum gravity with purely virtual quanta

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Based on: arXiv:2005.10293

In collaboration with:

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Quantum gravity and purely virtual quanta (fakeons)

- Fakeon:

  a degree of freedom that mediates interactions and circulate inside loops but cannot appear as asymptotic state.
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Quantum gravity with fakeon

- Degrees of freedom of the theory:

massless spin-2 $h_{\mu\nu}$, (graviton)	massive scalar ϕ, (inflaton)	massive spin-2 $\chi_{\mu\nu}$. (fakeon)

- Parameters:

$$M_{\mathrm{Pl}}, \qquad \Lambda_{CC}, \qquad m_\phi, \qquad m_\chi.$$

- Physical content in cosmology:

Scalar perturbations   Tensor perturbations   No vectors

Predictions in inflationary cosmology

- Amplitudes and spectral indices in de Sitter and quasi de Sitter.

$A_{\mathcal{R}}$	A_T	r	$n_{\mathcal{R}} - 1$	n_T
$\frac{m_\phi^2 N^2}{3\pi M_{\rm Pl}^2}$	$\frac{4m_\phi^2}{\pi M_{\rm Pl}^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$\frac{12}{N^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$-\frac{2}{N}$	$-\frac{3}{2N^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$

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- The bound restricts the possible values for the tensor-to-scalar ratio r

$$\frac{1}{9} \lesssim \frac{N^2}{12} r \lesssim 1.$$

$$N = 60 \qquad 0.4 \lesssim 1000r \lesssim 3,$$

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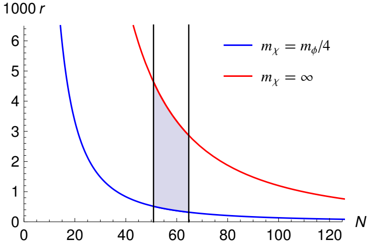

N in the range $n_{\mathcal{R}} = 0.9649 \pm 0.0042$ at 68% CL.

Action and properties

$$S_{\mathrm{QG}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int\sqrt{-g}\left[R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right]$$

Properties:

- Unitarity
  [D. Anselmi and MP, PRD 96 (2017) 045009, D. Anselmi, JHEP 02 (2018) 141].
- Renormalizability (once Λ_{CC} is reinstated).
- Violation of microcausality [D. Anselmi and MP, JHEP 11 (2018) 21].
- No violation of macrocausality
  [D. Anselmi and A. Marino, Class. Quantum Grav. 37 (2020) 095003].

Frameworks

Classical background: FLRW $\quad g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2).$
De Sitter expansion

$$\varepsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}.$$

- Geometric framework

$$S_{\text{geom}}(g) = -\frac{M_{\text{Pl}}^2}{16\pi}\int \sqrt{-g}\left[R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right]$$

m_ϕ and H are unrelated

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  - Expand in ε with H^2/m_χ^2 fixed;
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- Inflaton framework

$$S_{\mathrm{infl}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int \sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + S_\phi(g,\phi),$$

$$S_\phi(g,\phi) = \frac{1}{2}\int \sqrt{-g}\left(\nabla_\mu\phi\nabla^\mu\phi - 2V(\phi)\right), \qquad V(\phi) = \frac{m_\phi^2}{2\hat{\kappa}^2}\left(1 - \mathrm{e}^{\hat{\kappa}\phi}\right)^2$$

Action for cosmological perturbations

$$S(u) = \frac{1}{2}\int \mathrm{d}t\ a(t)^3\left[f(t)\dot{u}^2 - h(t)\ddot{u}^2 - g(t)u^2\right], \qquad u \equiv u_{\mathbf{k}}(t)$$

$$\Downarrow$$

$$S'(U,V) = \frac{1}{2}\int \mathrm{d}t\ Z\left(\dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma U V\right).$$

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Procedure:

i) Solve the EOM for the V by means of the fakeon Green function;

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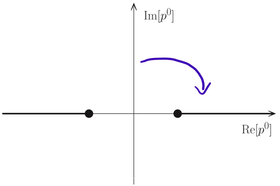
- The physical variable is still $u = F(U,V)$.
- After the procedure, the two-point function is

$$\langle uu\rangle = \langle F(U,V)F(U,V)\rangle, \qquad \text{with} \qquad V = V(U).$$

Fakeon prescription and fakeon Green function

- The fakeon prescription comes from high-energy physics and deals with scattering amplitudes.

$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$



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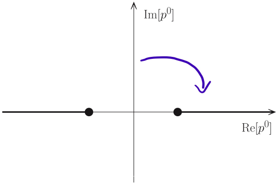
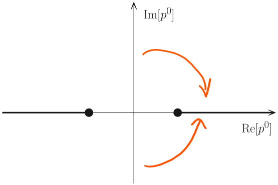


$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{A}_{+}(p) + \mathcal{A}_{-}(p)\right].$$



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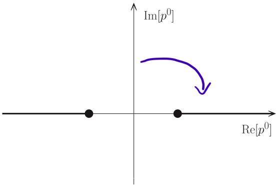

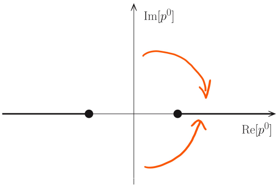

- Classical level: fakeon Green function.

$$G_{\text{f}} = \frac{1}{2}\left(G_{\text{adv}} + G_{\text{ret}}\right).$$

Example:

$$\ddot{V} + \omega^2 V = \left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \omega^2\right) V = F(t) \qquad \Rightarrow \qquad V(t) = (G_{\text{f}} * F)(t).$$

This is enough for tree level correlation functions in cosmology.

Fakeon Green function in FLRW spacetime

- In flat spacetime

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + A^2\right) G_{\mathrm{f}}(t,t') = \delta(t-t'), \qquad A = \text{const.}$$

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$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + H^2 n_\chi^2 + \frac{k^2}{a^2}\right) G_{\mathrm{f}}(t,t') = \delta(t-t'), \qquad n_\chi = \sqrt{\frac{m_\chi^2}{H^2} - \frac{1}{4}}.$$

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- There are two cases where G_{f} is known
  - $k/(aH) \to \infty$ (flat space in conformal time);
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$$G_{\mathrm{f}}(t,t')=\frac{i\pi\,\mathrm{sgn}(t-t')}{4H\sinh(n_\chi\pi)}\left[J_{in_\chi}(\check{k})J_{-in_\chi}(\check{k}')-J_{in_\chi}(\check{k}')J_{-in_\chi}(\check{k})\right],\qquad \check{k}^{(\prime)}=\frac{k}{a(t^{(\prime)})H}.$$

Projected action

$$S'(U,V) = \frac{1}{2}\int \mathrm{d}t\ Z\left(\dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma UV\right).$$

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$$S^{\mathrm{prj}}(U) = S'(U, V(U)), \qquad V(U) = -G_{\mathrm{f}} * (\sigma U).$$

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- In de Sitter expansion $\sigma = \mathcal{O}(\varepsilon)$ and $V = \mathcal{O}(\varepsilon)$.

  $\Rightarrow$ the nonlocal term $\sigma UV = \mathcal{O}(\varepsilon^2)$.

  The action can be written in the Mukhanov form

$$S_w^{\mathrm{prj}} = \frac{1}{2}\int \mathrm{d}\tau \left[w'^2 - \bar{k}^2 w^2 + \frac{w^2}{\tau^2}\left(\nu_{\mathrm{t}}^2 - \frac{1}{4}\right)\right], \qquad \bar{k} = k\Big(1 + \mathcal{O}(\varepsilon)\Big).$$

- The nonlocalities enter in the correlation functions since the physical variable is still

$$u = U + \alpha V(U), \qquad U = U(w).$$

Consistency condition

The fakeon eom's can always be turned into the form

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Imposing a no-tachyon condition on $m(t)^2$ in the flat-space limit

$$m(t)^2\big|_{k/(aH)\to 0} > 0. \qquad \Rightarrow \qquad n_\chi \in \mathbb{R}.$$

It can be seen also from the Green function

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All perturbations give the same bound

$$\boxed{m_\chi > \frac{m_\phi}{4}.}$$

Results: Amplitudes and spectral indices

Tensor: up to order ε^2 in the geometric framework;
up to order ε in the inflaton framework.

Scalar: up to order ε in both frameworks.

Vector: Projected away by the fakeon prescription.

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- Higher order corrections

$$A_{\mathcal{R}} = \frac{GN^2m_\phi^2}{3\pi}\left(1-\frac{\ln N}{6N}+\mathcal{O}\left(\frac{1}{N}\right)\right).$$

$$A_T = \frac{8G}{\pi}\frac{m_\chi^2 m_\phi^2}{m_\phi^2+2m_\chi^2}\left(1-\frac{3m_\chi^2}{N(m_\phi^2+2m_\chi^2)}\left(1+\frac{\ln N}{12N}\right)+\mathcal{O}\left(\frac{1}{N^2}\right)\right).$$

With $N=60$, the first correction to A_T is between 0.3% and 2.5%.

Results: ratio

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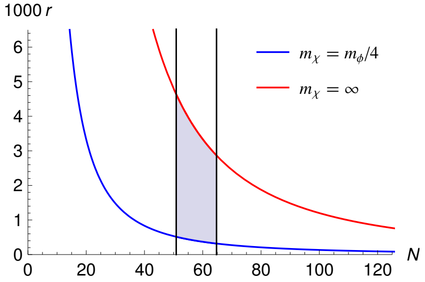

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Perspectives

- Higher orders (loop amplitudes, running of spectral indices).
- Different backgrounds (radiation, matter...).
- Other phases of the universe expansion (Reheating?...).
- ...