Predictions in inflationary cosmology from quantum gravity with purely virtual quanta

Marco Piva


In collaboration with:

D. Anselmi (Università di Pisa)  E. Bianchi (Penn State University)
Quantum gravity and purely virtual quanta (fakeons)

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  a degree of freedom that mediates interactions and circulate inside loops but cannot appear as asymptotic state.

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Quantum gravity with fakeon

• Degrees of freedom of the theory:

  massless spin-2 $h_{\mu\nu}$, (graviton)
  massive scalar $\phi$, (inflaton)
  massive spin-2 $\chi_{\mu\nu}$, (fakeon)

• Parameters:

  $M_{\text{Pl}}$, $\Lambda_{\text{CC}}$, $m_{\phi}$, $m_{\chi}$.

• Physical content in cosmology:

  Scalar perturbations  Tensor perturbations  No vectors
## Predictions in inflationary cosmology

- Amplitudes and spectral indices in de Sitter and quasi de Sitter.

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- A consistency condition gives a bound on the fakeon mass
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- The bound restricts the possible values for the tensor-to-scalar ratio $r$
  \[ \frac{1}{9} \lesssim \frac{N^2}{12} \lesssim 1. \]

  $N = 60 \quad 0.4 \lesssim 1000 r \lesssim 3,$
  $-0.4 \lesssim 1000n_T \lesssim -0.05.$
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\[N = 60 \quad 0.4 \lesssim 1000 r \lesssim 3, \quad -0.4 \lesssim 1000 n_T \lesssim -0.05.\]

\(N\) in the range \(n_R = 0.9649 \pm 0.0042\) at 68% CL.
Action and properties

\[ S_{\text{QG}}(g) = -\frac{M_{\text{Pl}}^2}{16\pi} \int \sqrt{-g} \left[ R + \frac{1}{2m^2_X} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{R^2}{6m^2_\phi} \right] \]

Properties:

- Unitarity

- Renormalizability (once \( \Lambda_{CC} \) is reinstated).

- Violation of microcausality [D. Anselmi and MP, JHEP 11 (2018) 21].

- No violation of macrocausality
  [D. Anselmi and A. Marino, Class. Quantum Grav. 37 (2020) 095003].
Frameworks

Classical background: FLRW 
\[ g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2). \]

De Sitter expansion
\[ \varepsilon = - \frac{\dot{H}}{H^2}, \quad \eta = 2\varepsilon - \frac{\ddot{\varepsilon}}{2H\varepsilon}. \]

- Geometric framework
\[ S_{\text{geom}}(g) = - \frac{M_{\text{Pl}}^2}{16\pi} \int \sqrt{-g} \left[ R + \frac{1}{2m^2_\chi} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} - \frac{R^2}{6m^2_\phi} \right] \]

\( m_\phi \) and \( H \) are unrelated
\[ \varepsilon \sim m^2_\phi (6H^2) \ll 1, \quad m_\chi \sim H \quad \text{or} \quad m_\chi \sim m_\phi. \]
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- Expand in \( \varepsilon \) with \( H^2/m_\chi^2 \) fixed;
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- Expand in \( \varepsilon \) with \( H^2/m^2_\chi \) fixed;
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- Inflaton framework

\[ S_{\text{infl}}(g) = -\frac{M^2_{\text{Pl}}}{16\pi} \int \sqrt{-g} \left[R + \frac{1}{2m^2_\chi} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}\right] + S_\phi(g, \phi), \]

\[ S_\phi(g, \phi) = \frac{1}{2} \int \sqrt{-g} (\nabla_\mu \phi \nabla^\mu \phi - 2V(\phi)), \quad V(\phi) = \frac{m^2_\phi}{2\hat{\kappa}^2} \left(1 - e^{\hat{\kappa}\phi}\right)^2 \]
Action for cosmological perturbations

\[ S(u) = \frac{1}{2} \int dt \, a(t)^3 \left[ f(t)u^2 - h(t)\dot{u}^2 - g(t)u^2 \right], \quad u \equiv u_\mathbf{k}(t) \]

\[ \Downarrow \]

\[ S'(U, V) = \frac{1}{2} \int dt \, Z \left( \dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma UV \right). \]
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Procedure:

i) Solve the EOM for the \( V \) by means of the fakeon Green function;

ii) Insert the solution back in \( S' \);

iii) Quantize the new action with the standard methods.
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ii) Insert the solution back in \( S' \);

iii) Quantize the new action with the standard methods.

- The physical variable is still \( u = F(U, V) \).

- After the procedure, the two-point function is

\[ \langle uu \rangle = \langle F(U, V)F(U, V) \rangle, \quad \text{with} \quad V = V(U). \]
Fakeon prescription and fakeon Green function

- The fakeon prescription comes from high-energy physics and deals with scattering amplitudes.

\[ A_+(p) = A(p + i\epsilon) \]
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\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon) \quad \mathcal{A}_{AV}(p) = \frac{1}{2} \left[ \mathcal{A}_+(p) + \mathcal{A}_-(p) \right].
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\[ \mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon) \quad \text{and} \quad \mathcal{A}_{AV}(p) = \frac{1}{2} [\mathcal{A}_+(p) + \mathcal{A}_-(p)]. \]

- Classical level: fakeon Green function.

\[ G_f = \frac{1}{2} (G_{adv} + G_{ret}) \]

Example:

\[ \ddot{V} + \omega^2 V = \left( \frac{d^2}{dt^2} + \omega^2 \right) V = F(t) \quad \Rightarrow \quad V(t) = (G_f * F)(t). \]

This is enough for tree level correlation functions in cosmology.
Fakeon Green function in FLRW spacetime

- In flat spacetime

\[
\left( \frac{d^2}{dt^2} + A^2 \right) G_f(t, t') = \delta(t - t'), \quad A = \text{const.}
\]

\[
G_f(t, t') = \frac{\sin(A|t - t'|)}{2A}.
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- In FLRW spacetime (tensor perturbations in de Sitter in inflaton framework)

\[ \left( \frac{d^2}{dt^2} + H^2 n^2_\chi + \frac{k^2}{a^2} \right) G_f(t, t') = \delta(t - t'), \quad n_\chi = \sqrt{\frac{m^2_\chi}{H^2} - \frac{1}{4}}. \]
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\[
G_f(t,t') = \frac{i\pi \text{sgn}(t-t')}{4H \sinh(n_\chi \pi)} \left[ J_{i n_\chi}(\tilde{k}) J_{-i n_\chi}(\tilde{k}') - J_{i n_\chi}(\tilde{k}') J_{-i n_\chi}(\tilde{k}) \right], \quad \tilde{k}' = \frac{k}{a(t')H}.
\]
Projected action

\[
S'(U, V) = \frac{1}{2} \int dt \ Z \left( \dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma U V \right).
\]

\[\downarrow\]

\[
S^{\text{proj}}(U) = S'(U, V(U)), \quad V(U) = -G_f * (\sigma U).
\]
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\[ S^\text{prj}(U) = S'(U, V(U)), \quad V(U) = -G_f \ast (\sigma U). \]

- In de Sitter expansion \( \sigma = \mathcal{O}(\varepsilon) \) and \( V = \mathcal{O}(\varepsilon) \).
  \[ \Rightarrow \] the nonlocal term \( \sigma UV = \mathcal{O}(\varepsilon^2) \).

The action can be written in the Mukhanov form

\[ S^\text{prj}_w = \frac{1}{2} \int d\tau \left[ w'^2 - \bar{k}^2 w^2 + \frac{w^2}{\tau^2} \left( \nu_t^2 - \frac{1}{4} \right) \right] , \quad \bar{k} = k \left( 1 + \mathcal{O}(\varepsilon) \right) . \]

- The nonlocalities enter in the correlation functions since the physical variable is still

\[ u = U + \alpha V(U), \quad U = U(w). \]
Consistency condition

The fakeon eom’s can always be turned into the form

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Imposing a no-tachyon condition on $m(t)^2$ in the flat-space limit

$$m(t)^2 \bigg|_{k/(aH) \to 0} > 0. \quad \Rightarrow \quad n_\chi \in \mathbb{R}.$$  

It can be seen also from the Green function

$$G_f(t, t') \xrightarrow[k \to 0]{} \frac{1}{2Hn_\chi} \sin (Hn_\chi |t - t'|)$$
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All perturbations give the same bound

$$m_\chi > \frac{m_\phi}{4}.$$
Results: Amplitudes and spectral indices

**Tensor:** up to order $\varepsilon^2$ in the geometric framework;
up to order $\varepsilon$ in the inflaton framework.

**Scalar:** up to order $\varepsilon$ in both frameworks.

**Vector:** Projected away by the fakeon prescription.
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- Higher order corrections

\[ A_\mathcal{R} = \frac{G N^2 m_\phi^2}{3\pi} \left( 1 - \frac{\ln N}{6N} + \mathcal{O} \left( \frac{1}{N} \right) \right). \]

\[ A_T = \frac{8G}{\pi} \frac{m_\chi^2 m_\phi^2}{m_\phi^2 + 2m_\chi^2} \left( 1 - \frac{3m_\chi^2}{N(m_\phi^2 + 2m_\chi^2)} \left( 1 + \frac{\ln N}{12N} \right) + \mathcal{O} \left( \frac{1}{N^2} \right) \right). \]

With $N = 60$, the first correction to $A_T$ is between 0.3% and 2.5%.
Results: ratio

From the consistency condition $m_\chi > m_\phi / 4$ we have

$$\frac{1}{9} \lesssim \frac{N^2}{12} r \lesssim 1.$$ $\quad N = 60 \quad 0.4 \lesssim 1000r \lesssim 3, \quad -0.4 \lesssim 1000n_T \lesssim -0.05.$
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Conclusions

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**Perspectives**

- Higher orders (loop amplitudes, running of spectral indices).
- Different backgrounds (radiation, matter...).
- Other phases of the universe expansion (Reheating?...).
- ...

