Fakeons and quantum gravity

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- How the idea of fake particle ("fakeon") leads to what I claim to be the right theory of quantum gravity
- Physical implications and predictions
- Classical limit
- FLRW solution
- Cosmology

The problem of quantum gravity is to make it renormalizable and unitary at the same time

$$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

Higher derivatives lead to renormalizability, but violate unitarity, unless ...

Unitarity: $$S^\dagger S = 1$$

$S = 1 + iT$

$$2\,\mathrm{Im}T = T^\dagger T$$ Optical theorem

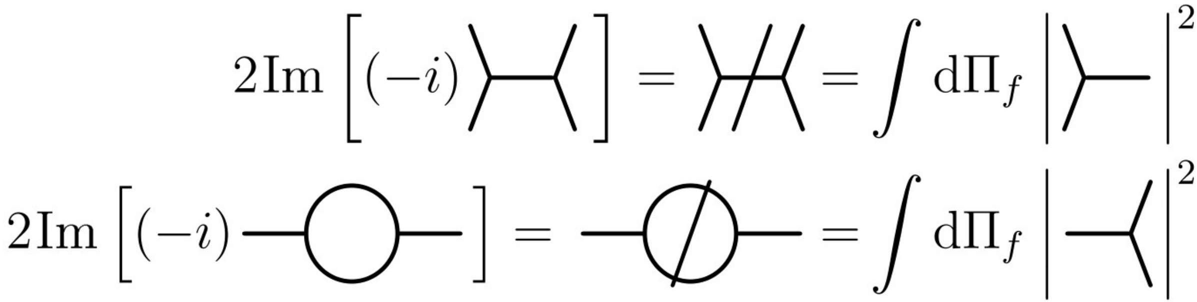
$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \langle b|T^\dagger|n\rangle \overbrace{\,}^{(-1)^{\sigma_n}} \langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in W$$

$\sigma_n = 0, 1$

In general, W may contain unphysical states in higher-derivative theories (those with $\sigma_n = 1$)

Feynman prescription $\longrightarrow$ pseudounitarity equation

Diagrammatic version of the optical theorem:



Cutting equations

Propagator:

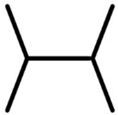
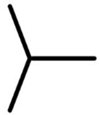
$$G(p,m) = \frac{1}{p^2 - m^2}$$

A prescription is needed

The Feynman prescription gives

$$G_+(p,m,\epsilon) = \frac{1}{p^2 - m^2 + i\epsilon}$$

The optical theorem is ok:

$2\,\mathrm{Im}\Big[(-i)$  $\Big] =$  $= \int d\Pi_f \Big|$  $\Big|^2 \geq 0$

Indeed:

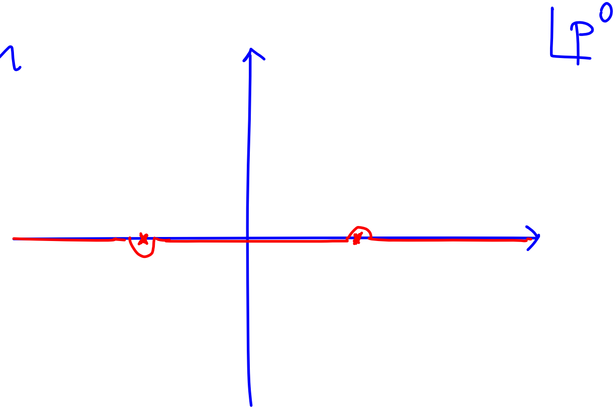
$$\mathrm{Im}\left[-\frac{1}{p^2-m^2+i\epsilon}\right] = \pi\delta(p^2-m^2) \quad \geq 0$$

Ghost: opposite residue $\quad -\dfrac{1}{p^2-m^2+i\epsilon}$

The optical theorem is violated

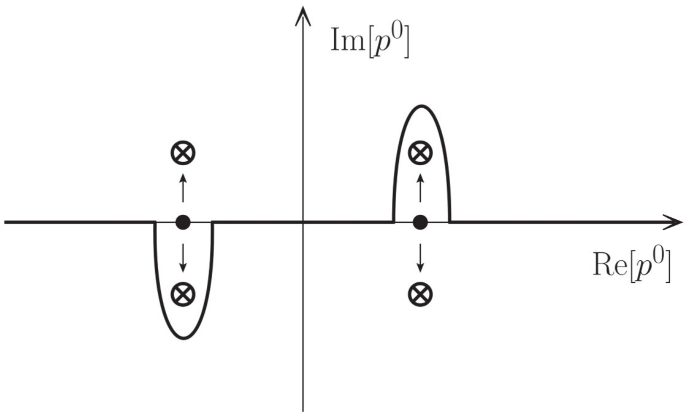
Note that the prescription is crucial

Correct answer: the following



$$G(p,m) = \frac{1}{p^2 - m^2}$$

Write $\pm \dfrac{p^2 - m^2}{(p^2 - m^2)^2}$ and split the poles into pairs



This is achieved by inserting an infinitesimal width as follows :

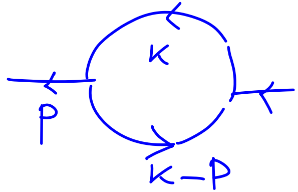
$$\mathbb{G}_{\pm}(p, m, \mathcal{E}^2) = \pm \frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4}$$

Note that the residue is zero on shell :

$\Rightarrow$ NO PARTICLE

However, the story is NOT that simple...

Example : bubble diagram



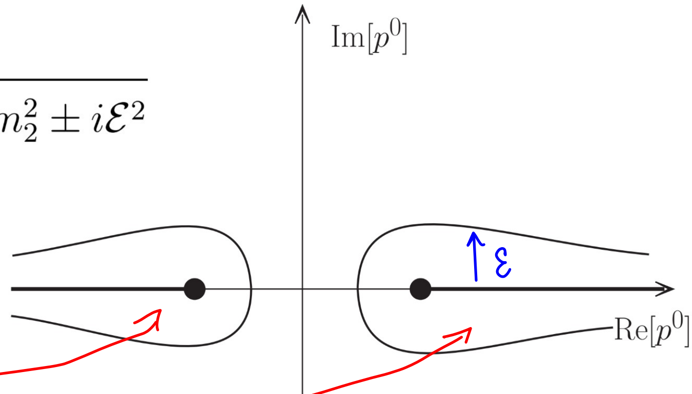
$$i\mathcal{M} \propto \int \frac{dk^0}{2\pi} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \mathbb{G}_\pm(p-k, m_1, \mathcal{E}^2)\mathbb{G}_\pm(k, m_2, \mathcal{E}^2)$$

LW

$$= \int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p, \mathbf{k})$$

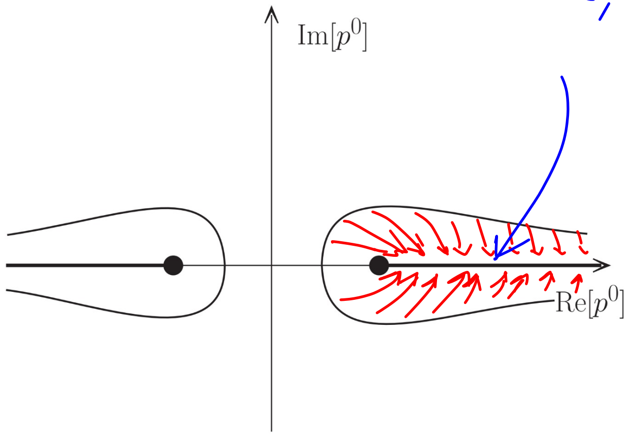
$w(p, \vec{k})$ is singular for

$$|p^0| = \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_1^2 \pm i\mathcal{E}^2} + \sqrt{\mathbf{k}^2 + m_2^2 \pm i\mathcal{E}^2}$$



Here Lorentz invariance & analiticity are violated

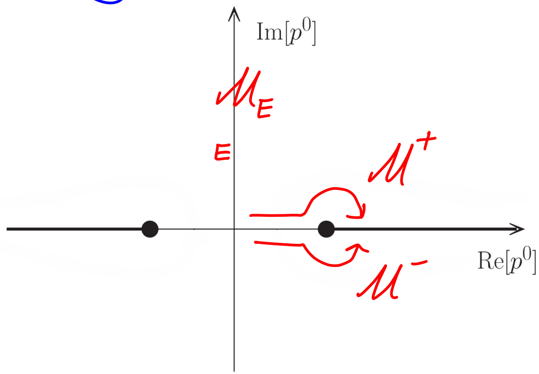
calculate
INSIDE, then take $\varepsilon \to 0$



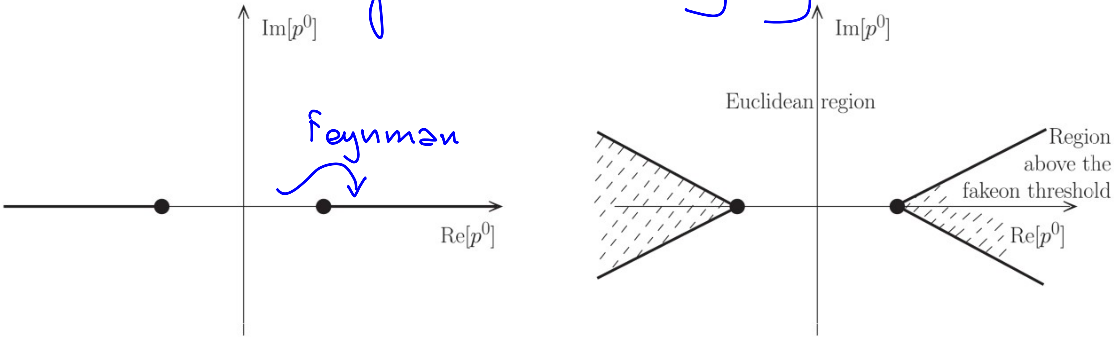
Lorentz invariance and analyticity are recovered in the limit

Result: average continuation

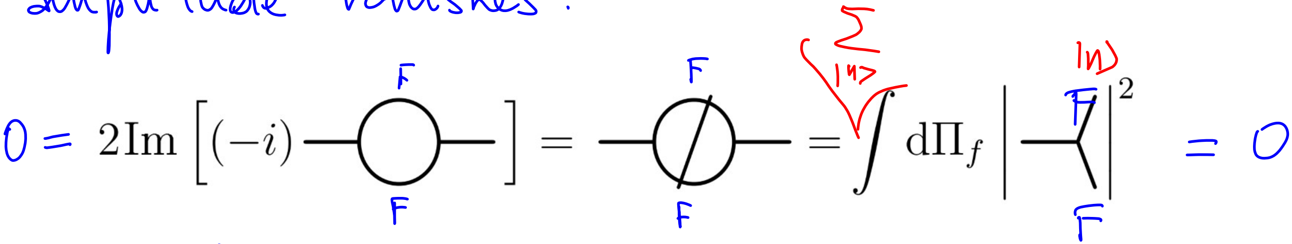
$$\frac{1}{2}\left(\mathcal{M}^+ + \mathcal{M}^-\right)$$



Regionwise analyticity



Since the prescription is symmetric w.r.t. the real axis, the imaginary part of the amplitude vanishes:



$\Rightarrow$ the fakeon F MUST be projected away :

From

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \overset{1,0}{\cancel{(-1)^{\sigma_n}}}\, \langle b|T^\dagger|n\rangle\langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in W$$

to

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in V} \langle b|T^\dagger|n\rangle\langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in V$$

with $V \subset W$, $|F\rangle \in W$, $|F\rangle \notin V$

$$\ln(q^2 - m^2 + i\varepsilon) \rightarrow \frac{1}{2}\ln(q^2 - m^2)^2$$

To all orders:
D. Anselmi (2018), Fakeons & Lee-Wick models, JHEP

Quantum gravity

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

Consider the (renormalizable) higher-derivative action

$\zeta, \alpha, \xi > 0$

$$S_{\rm QG} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right] + S_{\mathfrak{m}}(g,\Phi)$$

Eliminate the higher derivatives by means of extra fields:

D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renormalization.com and arXiv:1806.03605 [hep-th].

$$\mathcal{S}_{\rm QG}(g,\phi,\chi,\Phi) = S_{\rm H}(g) + S_\chi(g,\chi) + S_\phi(\tilde{g},\phi) + S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}$

$$S_{\rm H}(g) = -\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R, \qquad S_\phi(g,\phi) = \frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi - \frac{m_\phi^2}{\kappa^2}\left(1-\mathrm{e}^{\kappa\phi}\right)^2\right]$$

$$S_\chi(g,\chi) = S_{\rm H}(\tilde{g}) - S_{\rm H}(g) - 2\int\chi_{\mu\nu}\frac{\delta S_{\rm H}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2)\Big|_{g\to\tilde{g}}.$$

Now, the $\chi_{\mu\nu}$ action has the wrong overall sign :

$$S_\chi(g,\chi) = -\frac{\zeta}{\kappa^2} S_{\rm PF}(g,\chi,m_\chi^2) - \frac{\zeta}{2\kappa^2}\int \sqrt{-g}R^{\mu\nu}(\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho) + S_\chi^{(>2)}(g,\chi)$$

where S_{PF} is the covariantized Pauli-Fierz action

This means that $\chi_{\mu\nu}$ MUST be quantized as a fakeon. This way, we have <u>both</u> <u>renormalizability</u> and unitarity.

Instead, ϕ can be quantized either as a fakeon or as a physical particle

		Fermions		Bosons	
Quarks	u	c	t	γ	H
	d	s	b	$W^{\pm}$	g
Leptons	e	μ	τ	Z^0	ϕ
	ν_e	ν_μ	ν_τ	g	χ

QG triplet

fakeon ?

fakeon

The fields of the standard model and quantum gravity could explain everything we know

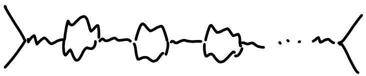
Graviton multiplet : $\{ h_{\mu\nu}, \phi, \chi_{\mu\nu} \}$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$$

fluctuation of the metric

massive scalar

spin-2 fakeon of mass m_χ



Fakeon width :

$$\Gamma_\chi = -\alpha_\chi C m_\chi \qquad C = \frac{N_s + 6N_f + 12N_v}{120}$$

$\Gamma_\chi < 0$: causality is violated by $\chi_{\mu\nu}$

$$\alpha_\chi = \left(\frac{m_\chi}{M_{Pl}}\right)^2$$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

JHEP 11 (2018) 21

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th]

PROJECTION

At the level of generating functionals: $\Gamma(\varphi, \chi)$

φ = physical fields χ = fakeons

Solve $\delta\Gamma(\varphi, \chi)/\delta\chi = 0$ by means of the fakeon prescription

Let $\langle\chi\rangle$ denote the solution

Projected functionals: $\Gamma_{\mathrm{pr}}(\varphi) = \Gamma(\varphi, \langle\chi\rangle)$

$$Z_{\mathrm{pr}}(J) = \int [\mathrm{d}\varphi \mathrm{d}\chi] \exp\left(iS(\varphi, \chi) + i\int J\varphi\right) = \exp\left(iW_{\mathrm{pr}}(J)\right)$$

NO SOURCE J_χ FOR χ

Projection = integrating out the fakeons with the fakeon prescription

Classical limit

The action

$$\mathcal{S}_{\mathrm{QG}}(g,\phi,\chi,\Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g,\chi) + S_{\phi}(\tilde{g},\phi) + S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

is NOT the classical limit, because it is unprojected

Unprojected field equations:

$$g_{\mu\nu}: \quad R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{\kappa^2}{\zeta}\left[\mathrm{e}^{3\kappa\phi} f T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + f T^{\mu\nu}_{\phi}(\tilde{g},\phi) + T^{\mu\nu}_{\chi}(g,\chi)\right]$$

$$\phi: \quad -\frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) - \frac{m_\phi^2}{\kappa}\left(\mathrm{e}^{\kappa\phi}-1\right)\mathrm{e}^{\kappa\phi} = \frac{\kappa\mathrm{e}^{3\kappa\phi}}{3\zeta}T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)\tilde{g}_{\mu\nu},$$

$$\chi_{\mu\nu}: \quad \frac{1}{\sqrt{-g}}\frac{\delta S_\chi(g,\chi)}{\delta\chi_{\mu\nu}} = \mathrm{e}^{3\kappa\phi} f T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + f T^{\mu\nu}_{\phi}(\tilde{g},\phi),$$

At the tree level, the subtleties about integration paths and average continuations are not important, so we can take

$$\frac{p^2-m^2}{(p^2-m^2)^2+\mathcal{E}^4} = \mathcal{P}\frac{1}{p^2-m^2}$$

Use it to solve the fakeon equations

The projected classical action is then

$$S_{\mathrm{QG}}(g,\phi,\Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g,\langle\chi\rangle) + S_{\phi}(\bar{g},\phi) + S_{\mathfrak{m}}(\bar{g}\mathrm{e}^{\kappa\phi},\Phi)$$

where $\langle\chi\rangle$ is the solution

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + 2\langle\chi_{\mu\nu}\rangle$$

Example : $$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}\left(\dot{x}^2 - \tau^2 \ddot{x}^2\right) + x F_{\mathrm{ext}}(t)$$

$$m \frac{d^2}{dt^2} \underbrace{\left(1 + \tau^2 \frac{d^2}{dt^2}\right)}_{\longrightarrow\ \text{invert with } \mathcal{P}\frac{1}{1+\tau^2\frac{d^2}{dt^2}}} x = F_{\mathrm{ext}}$$

The projected equation is

$$m\ddot{x} = \int_{-\infty}^{\infty} \mathrm{d}u \, \frac{\sin(|u|/\tau)}{2\tau} F_{\mathrm{ext}}(t-u)$$

$\longrightarrow$ violation of microcausality

$\cancel{F = ma}$ $\qquad \langle F \rangle = ma$!!

The FLRW metric

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - a^2(t)\mathrm{d}\sigma^2$$

$g_{\mu\nu}$, ϕ, $\cancel{\chi_{\mu\nu}}$

$$\mathrm{d}\sigma^2 = \frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2$$

Unprojected equations $\left(\frac{\mathcal{L}}{\sqrt{-g}} \sim R + R^2 \right)$

$$\Sigma\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \frac{4\pi G}{3}(\rho - 3p), \qquad \Upsilon\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2}\right) = -4\pi G(\rho + p),$$

$$\Sigma(1 - \mathrm{e}^{-\kappa\phi}) = -\frac{8\pi G}{3m_\phi^2}(\rho - 3p)$$

where

$$\Sigma = 1 + \frac{1}{m_\phi^2}\left(3\frac{\dot{a}}{a} + \frac{\mathrm{d}}{\mathrm{d}t}\right)\frac{\mathrm{d}}{\mathrm{d}t}, \qquad \Upsilon = \Sigma + \frac{2}{m_\phi^2}\left[\frac{k}{a^2} + 3\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}}{a}\right)\right].$$

Projection. Define $\langle A \rangle_X \equiv \frac{1}{2}\left[\left.\frac{1}{X}\right|_{\rm rit} + \left.\frac{1}{X}\right|_{\rm adv} \right] A$

and use it to define $\frac{1}{\Sigma}$ and $\frac{1}{\Upsilon}$

(Partially) projected equations:

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{4\pi G}{3} \langle \rho - 3p \rangle_\Sigma$$

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = -4\pi G \langle \rho + p \rangle_\Upsilon$$

$$1 - \mathrm{e}^{-\kappa\phi} = -\frac{8\pi G}{3m_\phi^2} \langle \rho - 3p \rangle_\Sigma$$

(it is NOT over yet ....!!)

OR :

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\tilde{\rho},$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G\tilde{p}.$$

where

$$\tilde{\rho} = \frac{1}{4}\langle\rho - 3p\rangle_\Sigma + \frac{3}{4}\langle\rho + p\rangle_\Upsilon$$

$$\tilde{p} = \frac{1}{4}\langle\rho + p\rangle_\Upsilon - \frac{1}{4}\langle\rho - 3p\rangle_\Sigma$$

The projection can be handled exactly for radiation combined with the vacuum energy:

$$p = \frac{\rho}{3} + p_0$$

p_0 = constant

Exact solution: $\rho_0 = 3\sigma^2/(8\pi G)$ $\quad \tilde{p} = (\tilde{\rho} - 4\rho_0)/3$

σ, σ' = constants

$$\rho(t) = \frac{3}{8\pi G}\left(\sigma^2 + \frac{\sigma''^2}{4a^4}\right), \qquad \sigma''^2 = \sigma'^2\left(1 + \frac{4\sigma^2}{m_\phi^2}\right) \qquad \tilde{\rho}(t) = \frac{3}{8\pi G}\left(\sigma^2 + \frac{\sigma'^2}{4a^4}\right)$$

$$a(t) = \sqrt{\frac{\sinh(\sigma t)}{\sigma}\left(\sigma' \cosh(\sigma t) - \frac{k}{\sigma}\sinh(\sigma t)\right)}$$

But in general the projection is defined perturbatively (since it comes from quantum gravity, which is defined perturbatively)

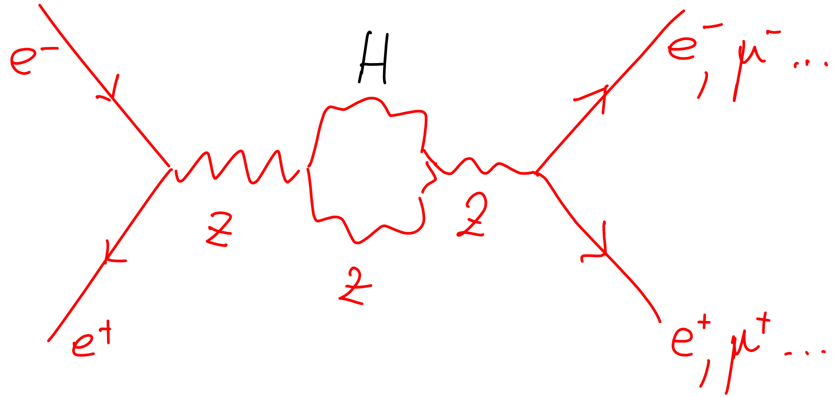
→ The classical equations are defined perturbatively: one may have to face asymptotic series and nonperturbative effects (just to write the equations)

Example: cosmic dust ($p=0$) or $p = w\rho$
$w \neq \frac{1}{3}, -1$

The fakeon prescription can be applied universally

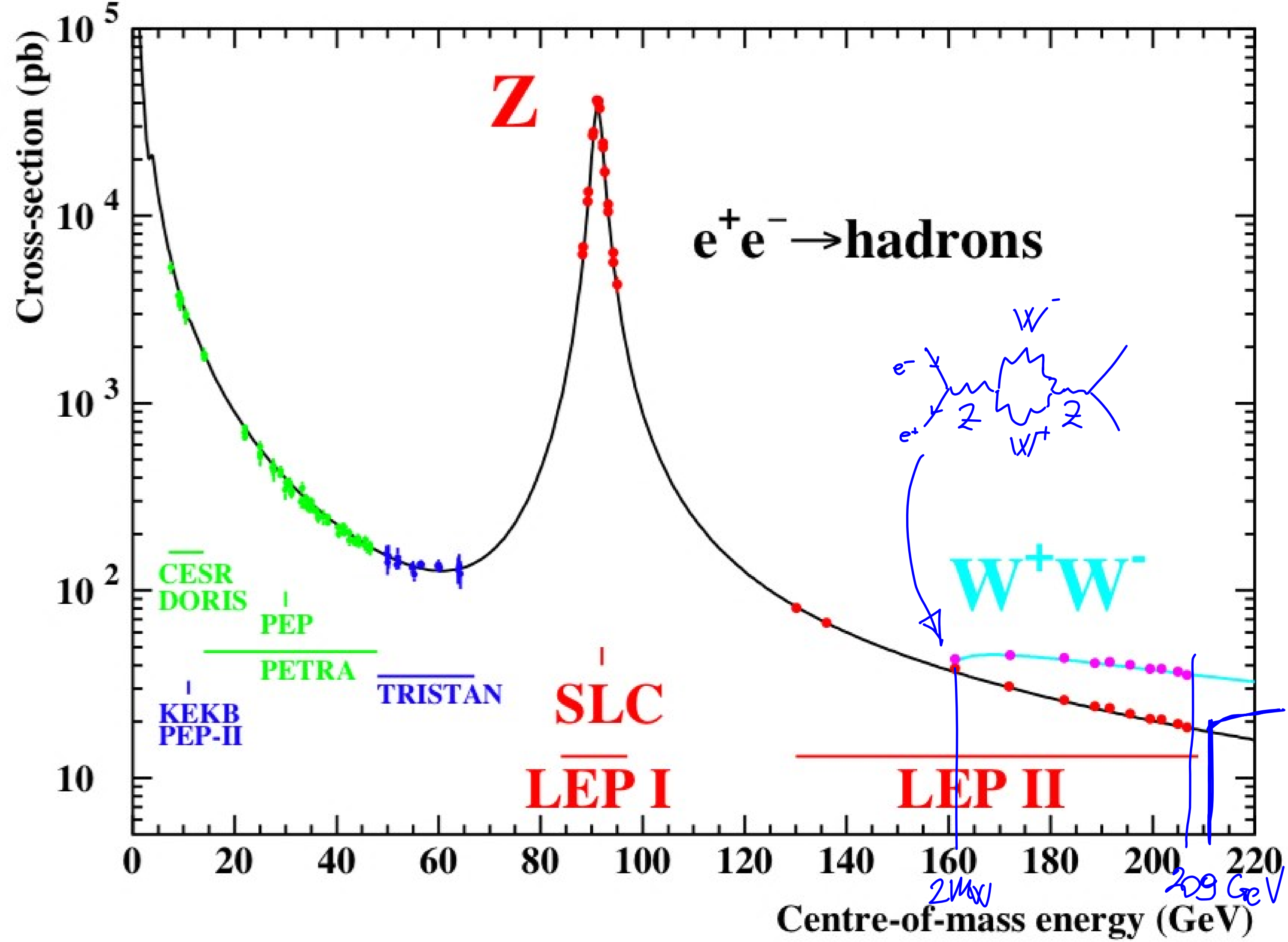
We should talk about "quantum field theory of particles and fakeons".

Is H a fakeon? Maybe...



$$m_H + m_Z = 216\ \mathrm{GeV}$$

D. A., On the nature of the Higgs boson, MPLA 33 (2019) 1950123
arXiv: 1811.02600 [hep-ph]



Cross-section (pb)
Centre-of-mass energy (GeV)
Z
$e^+e^- \rightarrow$ hadrons
W^+W^-
CESR
DORIS
PEP
PETRA
KEKB
PEP-II
TRISTAN
SLC
LEP I
LEP II
e^-
e^+
Z
Z
W^-
W^+
$2M_W$
209 GeV

Comments on alternative approaches to the problem of quantum gravity

--- **string theory** is criticized for being nonpredictive. Moreover, its calculations often require mathematics that is not completely understood

--- **loop quantum gravity** is even more challenging, because it is at an earlier stage of development

--- **holography** (AdS/CFT correspondence) do not admit a weakly coupled expansion

--- **asymptotic safety** in nonperturbative as well.

None is as close to the standard model as the solution based on the fakeon idea, which is a quantum field theory, admits a perturbative expansion in terms of Feynman diagrams and allows us to make calculations with a comparable effort. It can be coupled to th standard model straightforwardly.

Our solution bests its competitors in calculatibity, predictivity and falsifiability. It is also rather rigid, because it contains only two new parameters. It could turn out to be the most predictive theory ever.