A predictive theory of quantum gravity from fake particles

Damiano Anselmi

- The idea of fake particle ("fakeon") allows us to make sense of higher-derivative theories (under certain assumptions)
- It can also be applied to non-higher-derivative theories
- It amounts to a novel quantization prescription (alternative to the Feynman $i\epsilon$)
- It leads to an essentially unique theory of quantum gravity

Summary of the talk :

- Brief introduction to the new idea
- Physical implications and predictions
- Classical limit
- Non-perturbative aspects of the classical limit
- FLRW solution, Hubble constant, new interpretation of the equations of motion ...

- The fakeon is a purely virtual quantum
- It simulates a physical particle in many situations, but does not belong to the physical spectrum of asymptotic states
- It must be introduced and later projected away (like the Faddeev-Popov ghosts), to have unitarity
- The fakeon prescription/projection does not follow from a gauge principle
- It leads to the violation of microcausality

The problem of quantum gravity is to make it renormalizable and unitary at the same time

$$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

Higher derivatives lead to renormalizability, but violate unitarity $\quad S^+ S = 1$

$$\sum_{|n\rangle} \underset{\text{out}}{\langle a|} S^+ \overset{(-1)}{\underbrace{|n\rangle\langle n|}_{\text{intermediate}}} S \, \underset{\text{in}}{|b\rangle} = \langle \underset{\text{out}}{a} | \underset{\text{in}}{b} \rangle$$

$$\sum_{|n\rangle\in V} |n\rangle\langle n| = \mathbb{1}$$

For example, the Stelle theory of higher-derivative gravity is described by the action

$$S_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right]$$

quantized as usual (which means with the Feynman prescription)

It has propagators of the form

$$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

So its S matrix is not unitary and its Hamiltonian is not bounded from below

We need something radically new

Propagator: $$\pm \frac{1}{p^2 - m^2}$$

Feynman: $$\pm \frac{1}{p^2 - m^2 + i\epsilon}$$ $\pm \delta(p^2 - m^2)$

Fakeon: 1) $$\pm \frac{p^2 - m^2}{(p^2 - m^2)^2}$$

2) $$\pm \frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4}$$ $(= 0$ on shell $!)$

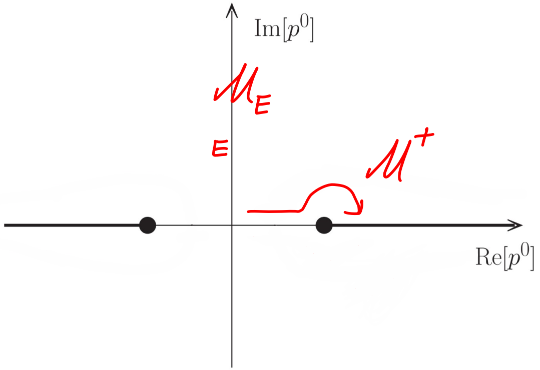
$\longrightarrow$ $0 \cdot \delta(p^2 - m^2)$

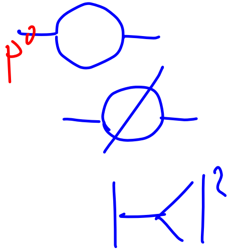
BEWARE !! $\Uparrow$ $\Downarrow$

3) follow a certain procedure in loop diagrams

Beyond tree level :

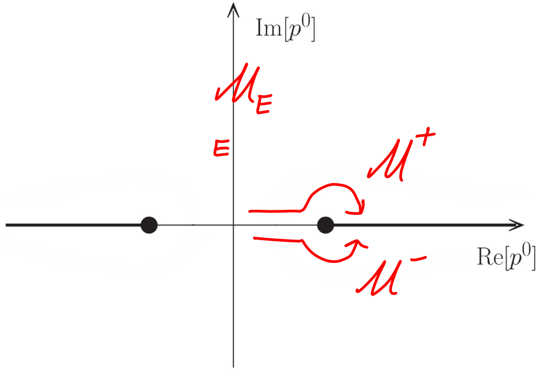
Feynman
analytic continuation





Im. part

fakeon



Result: average continuation $\frac{1}{2}\left(\mathcal{M}^+ + \mathcal{M}^-\right)$

The so-defined theory
IS UNITARY

D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141, 18A1 Renormalization.com and arXiv:1801.00915 [hep-th].

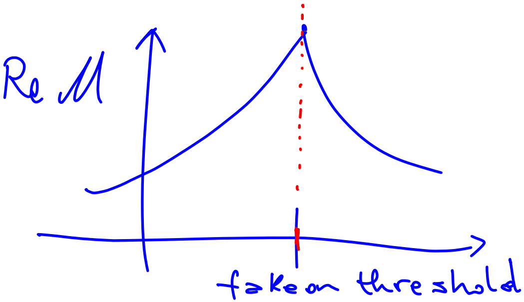
In formulas:

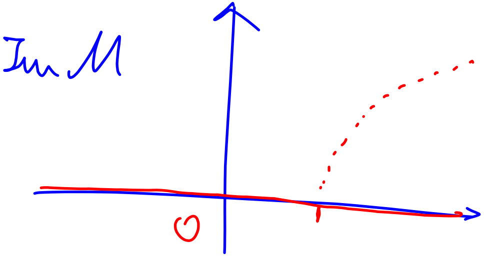
$$\int_0^1 dx \ln\left[-p^2 x(1-x) + m_1^2 x + m_2^2(1-x) - i\epsilon\right]$$

(Feynman)

becomes:

$$\frac{1}{2}\int_0^1 dx \ln\left[\left(-p^2 x(1-x) + m_1^2 x + m_2^2(1-x)\right)^2\right]$$





Quantum gravity

Consider the (renormalizable) higher-derivative action

$$S_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right] + S_{\mathfrak{m}}(g, \Phi)$$

$\zeta, \alpha, \xi > 0$

$R_{\mu\nu}\chi^{\mu\nu} + \chi_{\mu\nu}\chi^{\mu\nu}$

↑ Standard Model

If quantized as usual, this theory has a ghost

Solution: quantize the would-be ghost as a fakeon

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

Eliminate the higher derivatives by means of extra fields.

$$\mathcal{S}_{\mathrm{QG}}(g,\phi,\chi,\Phi)=S_{\mathrm{H}}(g)+S_{\chi}(g,\chi)+S_{\phi}(\tilde{g},\phi)+S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

where $\tilde{g}_{\mu\nu}=g_{\mu\nu}+2\chi_{\mu\nu}$

$$S_{\mathrm{H}}(g)=-\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R,\qquad S_{\phi}(g,\phi)=\frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_{\mu}\phi\nabla^{\mu}\phi-\frac{m_{\phi}^2}{\kappa^2}\left(1-\mathrm{e}^{\kappa\phi}\right)^2\right]$$
$$S_{\chi}(g,\chi)=S_{\mathrm{H}}(\tilde{g})-S_{\mathrm{H}}(g)-2\int\chi_{\mu\nu}\frac{\delta S_{\mathrm{H}}(\tilde{g})}{\delta g_{\mu\nu}}+\frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu}-\chi^2)\big|_{g\to\tilde{g}}.$$

D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renormalization.com and arXiv:1806.03605 [hep-th].

Now, the $\chi_{\mu\nu}$ action has the wrong overall sign :

$$S_\chi(g,\chi) = -\frac{\zeta}{\kappa^2} S_{\mathrm{PF}}(g,\chi,m_\chi^2) - \frac{\zeta}{2\kappa^2}\int \sqrt{-g}\, R^{\mu\nu}(\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho) + S_\chi^{(>2)}(g,\chi)$$

where S_{PF} is the covariantized Pauli-Fierz action

This means that $\chi_{\mu\nu}$ MUST be quantized as a fakeon. This way, we have both renormalizability and unitarity.

Instead, ϕ can be quantized either as a fakeon or as a physical particle

Graviton multiplet : $\{ h_{\mu\nu}, \phi, \chi_{\mu\nu} \}$

$m_\chi = \frac{\zeta}{\alpha}$

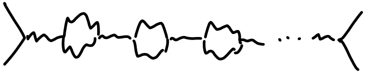
$m_\phi = \frac{\zeta}{\xi}$

$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$

fluctuation of the metric

massive scalar

spin-2 fakeon of mass m_χ



Fakeon width :

$$\Gamma_\chi = - \alpha_\chi C m_\chi$$

$$C = \frac{N_s + 6 N_f + 12 N_v}{120}$$

$$\alpha_\chi = \left(\frac{m_\chi}{M_{Pl}} \right)^2$$

$\Gamma_\chi < 0$: causality is violated by $\chi_{\mu\nu}$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

JHEP 11 (2018) 21

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th]

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \longrightarrow \mathrm{sgn}(t)\,\theta(\Gamma t)\,\exp\left(-imt-\frac{\Gamma t}{2}\right)$$

$$\equiv G_{BW}(t)$$

$$\varphi(x) = \int d^4y \; G_{BW}(x-y)\, J(y)$$

$\Gamma < 0$:

$$\varphi(t) = -\int_t^{\infty} dt' \, e^{-im(t-t') - \frac{\Gamma}{2}(t-t')} J(t')$$

The violation is short-range $\Delta t \simeq \frac{1}{m}, \frac{1}{|\Gamma|}$

but survives the classical limit !!

Projection:

do <u>not</u> use $a_f^\dagger$ of the fake particle f

$(a_f^\dagger)^{n_f}(a_{ph}^\dagger)^{n_{ph}}|0\rangle$: total Fock space W

$(a_{ph}^\dagger)^{n_{ph}}|0\rangle$: projected Fock space V

The free Hamiltonian is bounded from below in V

PROJECTION

$$Z_{\mathrm{pr}}(J) = \int [\mathrm{d}\varphi \mathrm{d}\chi] \exp\left(iS(\varphi, \chi) + i\int J\varphi \right) = \exp\left(iW_{\mathrm{pr}}(J)\right)$$

NO SOURCE J_χ FOR χ

Projection = integrating out the fakeons with the fakeon prescription

At the level of generating functionals: $\Gamma(\varphi, \chi)$

φ = physical fields

χ = fakeons

Solve $\delta\Gamma(\varphi, \chi)/\delta\chi = 0$ by means of the fakeon prescription

Let $\langle\chi\rangle$ denote the solution

Projected functionals: $\Gamma_{\mathrm{pr}}(\varphi) = \Gamma(\varphi, \langle\chi\rangle)$

Classical limit

D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity, Class. and Quantum Grav. 36 (2019) 065010, arXiv:1809.05037 [hep-th]

The action

$$\mathcal{S}_{\mathrm{QG}}(g,\phi,\chi,\Phi)=S_{\mathrm{H}}(g)+S_{\chi}(g,\chi)+S_{\phi}(\tilde{g},\phi)+S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

is NOT the classical limit, because it is unprojected

Unprojected field equations :

$g_{\mu\nu}$:

$$R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R=\frac{\kappa^2}{\zeta}\left[\mathrm{e}^{3\kappa\phi}fT_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)+fT_{\phi}^{\mu\nu}(\tilde{g},\phi)+T_{\chi}^{\mu\nu}(g,\chi)\right.$$

ϕ :

$$-\frac{1}{\sqrt{-\tilde{g}}}\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right)-\frac{m_\phi^2}{\kappa}\left(\mathrm{e}^{\kappa\phi}-1\right)\mathrm{e}^{\kappa\phi}=\frac{\kappa\mathrm{e}^{3\kappa\phi}}{3\zeta}T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)\tilde{g}_{\mu\nu},$$

$\chi_{\mu\nu}$:

$$\frac{1}{\sqrt{-g}}\frac{\delta S_\chi(g,\chi)}{\delta\chi_{\mu\nu}}=\mathrm{e}^{3\kappa\phi}fT_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)+fT_{\phi}^{\mu\nu}(\tilde{g},\phi),$$

At the tree level, the subtleties about integration paths and average continuations are not important, so we can take

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P}\frac{1}{p^2 - m^2}$$

Use it to solve the fakeon equations

The projected classical action is then

$$S_{\mathrm{QG}}\ (g, \phi, \Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g, \langle\chi\rangle) + S_{\phi}(\bar{g}, \phi) + S_{\mathfrak{m}}(\bar{g}\mathrm{e}^{\kappa\phi}, \Phi)$$

where $\langle \chi \rangle$ is the solution

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + 2\langle\chi_{\mu\nu}\rangle$$

Example : $\mathcal{L}_{\text{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) + x F_{\text{ext}}(t)$

$$m \frac{d^2}{dt^2} \underbrace{\left(1 + \tau^2 \frac{d^2}{dt^2}\right)}_{\longrightarrow \text{ invert with } \mathcal{P} \frac{1}{1+\tau^2 \frac{d^2}{dt^2}}} x = F_{ext}$$

The projected equation is

$$m\ddot{x} = \int_{-\infty}^{\infty} \mathrm{d}u \frac{\sin(|u|/\tau)}{2\tau} F_{\text{ext}}(t-u)$$

$\longrightarrow$ violation of microcausality

$\cancel{F = m a}$ $\qquad \langle F \rangle = m a$!!

But in general the projection is defined perturbatively (since it comes from quantum gravity, which is defined perturbatively)

→ The classical equations are defined perturbatively: one may have to face asymptotic series and nonperturbative effects (just to write the equations)

Example :

$$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) - V(x,t) \qquad V = \frac{\lambda}{4!} x^4$$

Unprojected equations of motion :

$$m \frac{d^2}{dt^2} \underbrace{\left(1 + \tau^2 \frac{d^2}{dt^2}\right)} x = - \frac{\lambda}{3!} x^3$$

Projected equations of motion :

$$m \ddot{x} = - \frac{\lambda}{3!} \langle x^3 \rangle \qquad \langle \dots \rangle = \text{same average as before}$$

By iterating the projection, we get

$\omega = 0 \qquad \tilde{\lambda} \equiv \lambda / m$

$$\ddot{x} = -\frac{\tilde{\lambda} x}{6}\left(x^2 - 6\tau^2\dot{x}^2\right) - \frac{\tilde{\lambda}^2\tau^2 x}{12}\left(x^4 - 48\tau^2 x^2\dot{x}^2 + 372\tau^4\dot{x}^4\right)$$
$$-\frac{\tilde{\lambda}^3\tau^4 x}{6}\left(x^6 - 156\tau^2 x^4\dot{x}^2 + 4572\tau^4 x^2\dot{x}^4 - 31152\tau^6\dot{x}^6\right) + \mathcal{O}(\tilde{\lambda}^4)$$

$$\frac{\mathcal{L}}{m} = \frac{\dot{x}^2}{2} - \frac{\tilde{\lambda} x^2}{4!}\left(x^2 + 12\tau^2\dot{x}^2\right) + \frac{\tau^2\tilde{\lambda}^2 x^2}{72}\left(x^4 - 54\tau^2 x^2\dot{x}^2 + 372\tau^4\dot{x}^4\right) + \mathcal{O}(\tilde{\lambda}^3)$$

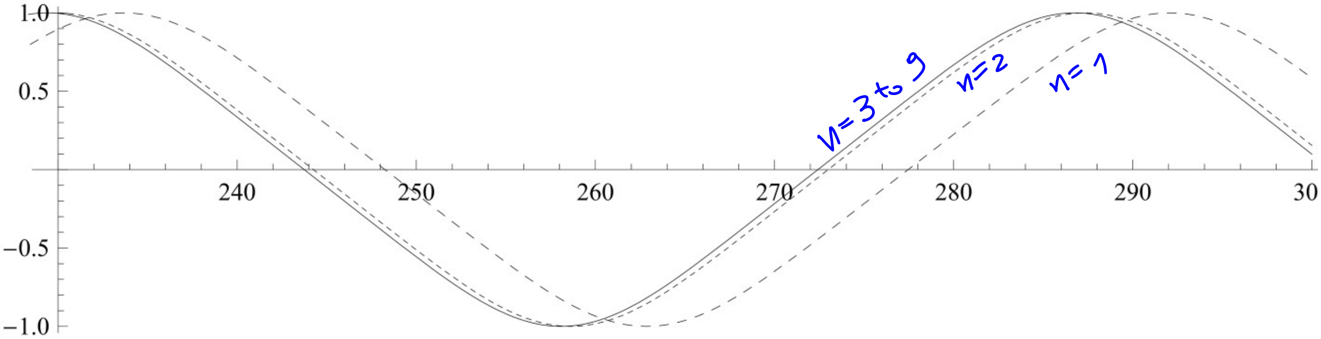
Growth of the coefficients:

$c_{nm}\, x^n \dot{x}^m$

n	5	10	15	20	25
$c_{n,0}$	10^0	10^6	10^{13}	10^{22}	10^{32}
$c_{n,n}$	10^9	10^{28}	10^{52}	10^{78}	$\boxed{10^{107}}$

!!

Solution with $x(0) = 1$, $\dot{x}(0) = 0$, $m = \tau = 1$, $\lambda = \frac{1}{10}$



n=1 : fairly good

n = 2 : good

n=3 to n=9 : excellent

n>9 : meaningless

$$n_{max} \simeq \left[\frac{1}{|\lambda|}\right]$$

The FLRW metric

D. Anselmi, Fakeons and the classicization of quantum gravity: the FLRW metric, J. High Energy Phys. 04 (2019) 61, arXiv:1901.09273 [gr-qc]

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \qquad dt^2 - a^2(t)d\sigma^2$$

$g_{\mu\nu}$, ϕ, $\cancel{\chi_{\mu\nu}}$

fakeon

$$d\sigma^2 = \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Unprojected equations $\left(\frac{\mathcal{L}}{\sqrt{-g}} \sim R + \frac{1}{m_\phi^2} R^2 \right)$

$$\Sigma\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \frac{4\pi G}{3}(\rho - 3p), \qquad \Upsilon\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2}\right) = -4\pi G(\rho + p),$$

where

$$\Sigma = 1 + \frac{1}{m_\phi^2}\left(3\frac{\dot{a}}{a} + \frac{d}{dt}\right)\frac{d}{dt}, \qquad \Upsilon = \Sigma + \frac{2}{m_\phi^2}\left[\frac{k}{a^2} + 3\frac{d}{dt}\left(\frac{\dot{a}}{a}\right)\right].$$

Projection. Define $\langle A\rangle_X \equiv \frac{1}{2}\left[\left.\frac{1}{X}\right|_{\mathrm{rit}} + \left.\frac{1}{X}\right|_{\mathrm{adv}}\right] A$

and use it to define $\frac{1}{\Sigma}$ and $\frac{1}{\Upsilon}$

$\underline{\text{(Partially)}}$ projected equations :

$$\downarrow$$

$$\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{4\pi G}{3}\langle\rho - 3p\rangle_\Sigma$$
$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} - \frac{k}{a^2} = -4\pi G\langle\rho + p\rangle_\Upsilon$$

(it is NOT over yet .....!!)

OR :

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\tilde{\rho},$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -8\pi G\tilde{p}.$$

where

$$\tilde{\rho} = \frac{1}{4}\langle\rho - 3p\rangle_\Sigma + \frac{3}{4}\langle\rho + p\rangle_\Upsilon$$

$$\tilde{p} = \frac{1}{4}\langle\rho + p\rangle_\Upsilon - \frac{1}{4}\langle\rho - 3p\rangle_\Sigma$$

The projection can be handled exactly for radiation combined with the vacuum energy:

$$p = \frac{\rho}{3} + p_0$$

p_0 = constant

Exact solution:

$$\rho_0 = 3\sigma^2/(8\pi G) \qquad \tilde{p} = (\tilde{\rho} - 4\rho_0)/3$$

σ, σ' = constants

$$\rho(t) = \frac{3}{8\pi G}\left(\sigma^2 + \frac{\sigma''^2}{4a^4}\right), \qquad \sigma''^2 = \sigma'^2\left(1 + \frac{4\sigma^2}{m_\phi^2}\right) \qquad \tilde{\rho}(t) = \frac{3}{8\pi G}\left(\sigma^2 + \frac{\sigma'^2}{4a^4}\right)$$

$$a(t) = \sqrt{\frac{\sinh(\sigma t)}{\sigma}\left(\sigma' \cosh(\sigma t) - \frac{k}{\sigma}\sinh(\sigma t)\right)}$$

predictive

$$\varphi(t) = \int_{t_1}^{t} \mathrm{d}t' G_H^{\mathrm{f}}(t-t') J(t') \qquad \text{for } t_1 + \Delta t \lesssim t \leqslant t_2.$$

error :

$$\delta J \equiv \int_{-\infty}^{t_1} \mathrm{d}t' G_H^{\mathrm{f}}(t-t') J(t')$$

small outside due to damping

Fuzziness relation :

$$\Delta x^2 \sim \frac{1}{m^2}$$

Shortest interval of time ever measured directly (laser pulses) : 10^{-17} s ⟸ "BAD"

Shortest distance ever measured (LIGO) : 10^{-17} cm

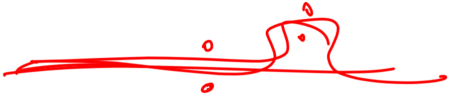
Special relativity does not help : mixing space and time it can only spread the error on time into space

Sometimes the universe "conspires" to make the violation disappear : Hubble constant > 0

FLRW background

(Anselmi and Marino, arXiv:1909.12873)

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi\right)+m^2\varphi=J.$$



$$\Sigma = 1+\frac{3H}{m^2}\frac{\mathrm{d}}{\mathrm{d}t}+\frac{1}{m^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2},$$

in $t_1 \leq t \leq t_2$

H = constant

$$G^{\mathrm{f}}_H(t) = \left.\frac{1}{\Sigma}\right|_f = :m^2\,\mathrm{sgn}(H)\underbrace{\theta(Ht)}_{\text{arrow of time restored}}\mathrm{e}^{-\frac{3}{2}Ht}\frac{\sin(t\sigma)}{\sigma}$$

CAUSAL FOR $H>0$

$$\sigma = \sqrt{m^2-\left(\frac{3H}{4}\right)^2}$$

$$\varphi(t)=\int_{-\infty}^{t}\mathrm{d}t'\,G^{\mathrm{f}}_H(t-t')J(t')$$

← ??

CAN THE VIOLATION BE AMPLIFIED ?

The violation of microcausality predicted by quantum gravity is short range
(Anselmi and Marino, arXiv:1909.12873)

$$G_{\mathrm{f}}(x)=\frac{m^4}{8\pi^2}\left[\frac{K_1\left(im\sqrt{x^2-i\epsilon}\right)}{m\sqrt{x^2-i\epsilon}}+\frac{K_1\left(-im\sqrt{x^2+i\epsilon}\right)}{m\sqrt{x^2+i\epsilon}}\right]$$

Basically, it is a Yukawa plus an anti-Yukawa. It vanishes outside the light cones

Relatively low frequencies are dumped

$$\langle J\rangle_{\mathrm{f}}(x)=\boxed{\int_{-m}^{m}}\frac{\mathrm{d}\omega}{2\pi}\mathrm{e}^{-i\omega x^0}\int\mathrm{d}^3\mathbf{y}\frac{m^2\mathrm{e}^{-\sqrt{m^2-\omega^2}|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|}\tilde{J}(\omega,\mathbf{y})$$
$$+\int_{|\omega|\geqslant m}\frac{\mathrm{d}\omega}{2\pi}\mathrm{e}^{-i\omega x^0}\int\mathrm{d}^3\mathbf{y}\frac{m^2\cos(\sqrt{\omega^2-m^2}|\mathbf{x}-\mathbf{y}|)}{4\pi|\mathbf{x}-\mathbf{y}|}\tilde{J}(\omega,\mathbf{y})$$

only extremely high ($\sim 10^{36}$ Hz) frequencies are not

Conclusions

$$\alpha_\chi = \frac{m_\chi^2}{M_{Pl}^2} \qquad \alpha_\phi = \frac{m_\phi^2}{M_{Pl}^2} \qquad \left(\frac{E}{M_{Pl}}\right)^4$$

No other approach to quantum gravity is as close to the standard model as the solution based on the fakeon idea

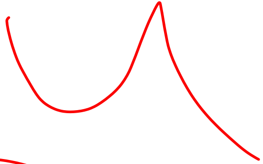
$$|T_\chi| \sim m_\chi \alpha_\chi$$

- **It is a quantum field theory**
- **It admits a perturbative expansion in terms of Feynman diagrams**
- **It allows us to make calculations with an effort comparable to the one of the SM**
- **It can be coupled to the standard model straightforwardly**

- **It is rather rigid (only two new parameters, m_χ and m_ϕ)**
- **It is as fundamental as the standard model (NO "living with ghosts", thanks!)**
- **It could be the most predictive theory ever: it is expected to be perturbative from zero energy to an energy equal to** $E \simeq M_{Pl} \cdot e^{10^{1\cancel{4}0}}$!!

Hawking, Hertog, "Living with ghosts", PRD 65 (2002) 103515

(for $m_\chi \sim 10^{12}$ GeV)



$$\alpha_\chi \simeq \frac{m_\chi^2}{M_{Pl}^2} \simeq 10^{-1\cancel{4}6} : \qquad 10^{-14} \ln \frac{E^2}{\mu^2} \simeq 1 \qquad \text{for } E^2 \simeq M_{Pl}^2 e^{10^{14}}$$

The quest for purely virtual quanta

- The relation between higher-derivatives and the violation of microcausality has been known since the Abraham-Lorentz force of classical electrodynamics

~~runaway solution~~ $\longrightarrow$ violation of microcausality

$$F = ma - \tau m \dot{a} \quad \Longrightarrow$$

$$= m\left(1 - \tau \frac{d}{dt}\right) a$$

$$ma = \langle F \rangle = \frac{1}{\tau} \int_t^\infty dt' e^{(t-t')/\tau} F(t')$$

$$\tau \sim \frac{e^2}{m}$$

[J.D. Jackson, Classical electrodynamics, chap. 17]

P.A.M. Dirac, Classical theory of radiating electrons, Proc. Roy. Soc. London A 167 (1938) 148.

- In the Lee - Wick models (particular HD QFTs) it was noted right away by Lee and Wick

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

- By the way, the fakeons allow us to understand the LW models better and actually complete their formulation, which was plagued with several issues

D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, arXiv:1703.04584

D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141, 18A1 arXiv:1801.00915 [hep-th].

D. Anselmi and M. Piva, Perturbative unitarity in Lee-Wick quantum field theory, Phys. Rev. D 96 (2017) 045009 arXiv:1703.05563

The Feynman-Wheeler electrodynamics could contain the classical massless fakeon with propagator

$$\mathcal{P}\,\frac{1}{p^2}$$

However, this option violates causality at-large, so Feynman and Wheeler developed an "emitter-absorber theory" to eliminate the massless fakeon and recover causality

J.A. Wheeler and R.P. Feynman, Interaction with the absorber as the mechanism of radiation, Rev. Mod. Phys. 17 (1945) 175;

J.A. Wheeler and R.P. Feynman, Classical electrodynamics in terms of direct interparticle action, Rev. Mod. Phys. 21 (1949) 425.

Bollini, Rocca, The Wheeler propagator

Int. J. Theor. Phys. 37 (1998) 2877, arXiv:hep-th/9807010

studied $\mathcal{P}\frac{1}{p^2-m^2}$ inside Feynman diagrams

and concluded they could be viable

Not correct: it violates

- renormalizability
- unitarity
- stability

$$\frac{\ln \Lambda_{UV}}{p^2}$$

U.G. Aglietti and D.A., Inconsistency of Minkowski higher-derivative theories, EPJC 77 (2017) 84 and arXiv: 1612.06510 't Hooft - Veltman, Diagrammar, CERN 73-09, § 6.1

Every diagram, when multiplied by the appropriate source functions and integrated over all x contributes to the S-matrix. The contribution to the T-matrix, defined by

$$S = 1 + iT \tag{6.7}$$

is obtained by multiplying by a factor -i. Unitarity of the S-matrix implies an equation for the imaginary part of the so defined T matrix

$$T - T^{\dagger} = iT^{\dagger}T \, . \tag{6.8}$$

The T-matrix, or rather the diagrams, are also constrained by the requirement of causality. As yet nobody has found a definition of causality that corresponds directly to the intuitive notions; instead formulations have been proposed involving the off-mass-shell Green's functions. We will employ the causality requirement in the form proposed by Bogoliubov that has at least some intuitive appeal and is most suitable in connection with a diagrammatic analysis. Roughly speaking Bogoliubov's condition can be put as follows: if a space-time point x_1 is in the future with respect to some other space-time point x_2, then the diagrams involving x_1 and x_2 can be rewritten in terms of functions that involve positive energy flow from x_2 to x_1 only.

The trouble with this definition is that space-time points cannot be accurately pinpointed with relativistic wave packets corresponding to particles on mass-shell. Therefore this definition cannot be formulated as an S-matrix constraint. It can only be used for the Green's functions.

Other definitions refer to the properties of the fields. In particular there is the proposal of Lehmann, Symanzik and Zimmermann that the fields commute outside the light cone. Defining fields in terms of diagrams, this definition can be shown to reduce to Bogoliubov's definition. The formulation of Bogoliubov causality in terms of cutting rules for diagrams will be given in Section 6.4.

Personal position:

- it is hard to elevate micro causality
  to a fundamental physical principle
- quantum gravity does not allow it at small distances
- micro causality is probably a blunder,
  due to our limited insight and poor
  experimental accuracy

Downgraded to: unusual form of the equations,
fuzziness of the input / output relation ...

	Fermions	Bosons
Quarks	u c t d s b	γ H $W^{\pm}$ g Z^0 ϕ g χ
Leptons	e μ τ ν_e ν_μ ν_τ	

QG triplet

fakeon ?

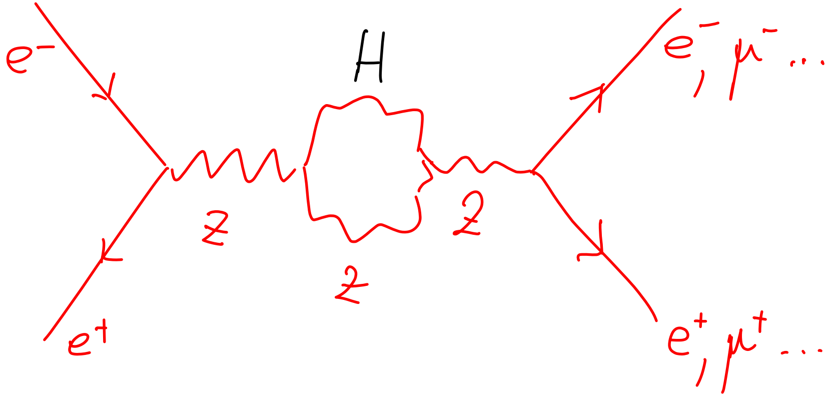
fakeon

The fields of the standard model and quantum gravity could explain everything we know

The fakeon prescription can be applied universally

We should talk about "quantum field theory of particles and fakeons".

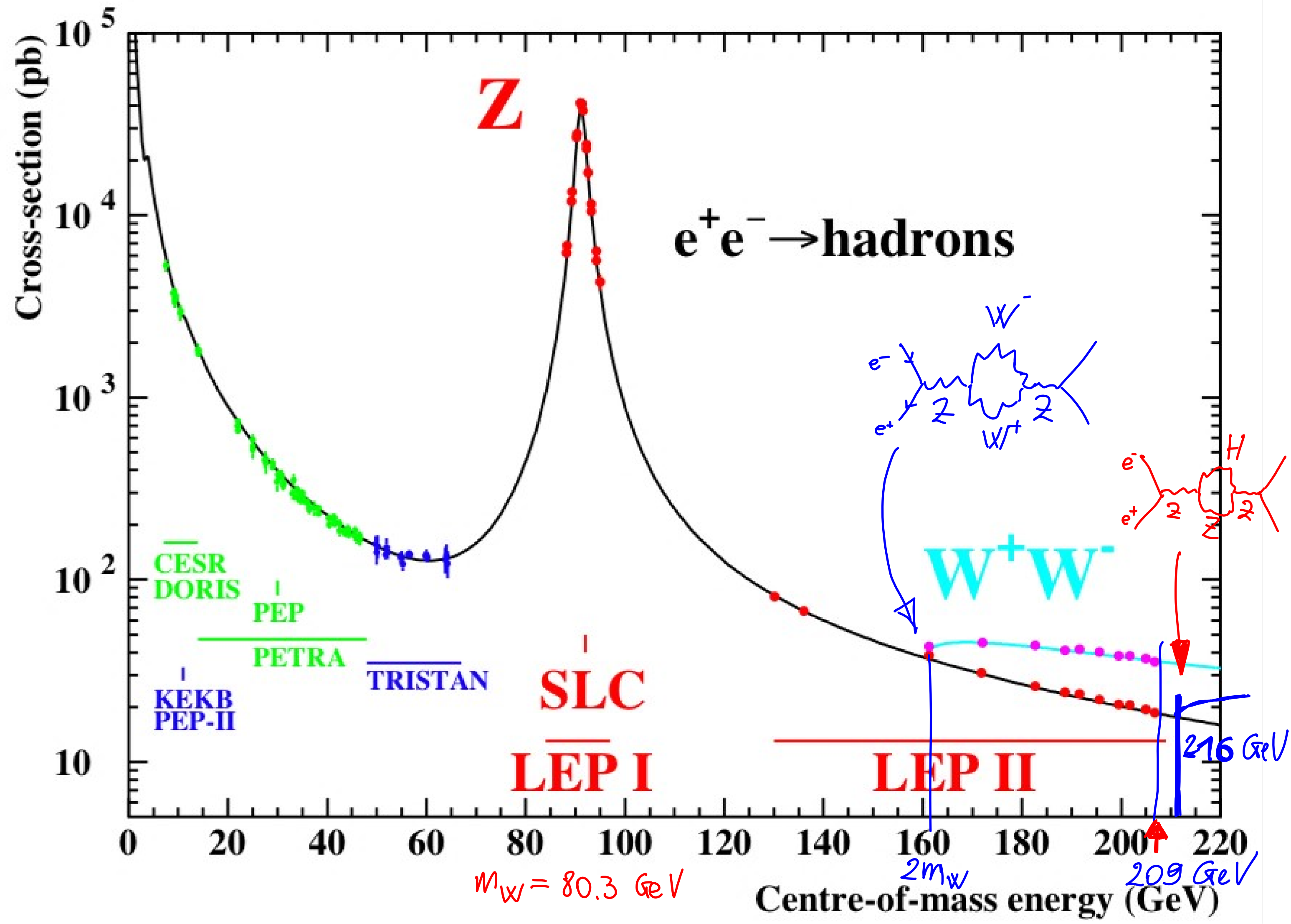
Is H a fakeon? Maybe...



$m_H + m_Z = 216$ GeV

D. A., On the nature of the Higgs boson, MPLA 33 (2019) 1950123

arXiv: 1811.02600 [hep-ph]


Cross-section (pb)
10^5
10^4
10^3
10^2
10
Z
e+e−→hadrons
W+W-
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)
e-
e+
Z
W-
W+
Z
H
e-
e+
Z
Z
Z
2m_W
209 GeV
216 GeV
m_W = 80.3 GeV