One-loop Effective Action Of Quantum Gravity With Fakeons And Its Phenomenology

Marco Piva

Modena, September 9$^{\mathrm{th}}$, 2019

Avenues of Quantum Field Theory in Curved Spacetime

The problem of renormalizability in QG

The problem of renormalizability in QG

Hilbert–Einstein action: unitary but nonrenormalizable theory

$$S_{\rm HE} = -\frac{1}{2\kappa^2}\int \sqrt{-g}R, \quad \kappa^2 = 8\pi G.$$

$$\Gamma^{(\rm ct)}_{\rm HE} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

The problem of renormalizability in QG

Hilbert–Einstein action: unitary but nonrenormalizable theory

$$S_{\mathrm{HE}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}R, \quad \kappa^2 = 8\pi G.$$

$$\Gamma^{(\mathrm{ct})}_{\mathrm{HE}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\big[c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

Stelle action: renormalizable but not unitary theory

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\big[\gamma R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + 2\Lambda_{\mathrm{C}}\big].$$

$$\Gamma^{(\mathrm{ct})}_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\big[a_\gamma R + a_\alpha R_{\mu\nu}R^{\mu\nu} + a_\beta R^2 + 2a_{\mathrm{C}}\big].$$

Unitarity

$$SS^\dagger = 1, \qquad S = 1 + iT.$$

Unitarity equation (optical theorem)

$$2\mathrm{Im}T = TT^\dagger.$$

Cutting equations

$$\Downarrow$$

Pseudo-unitarity equation

$$2\mathrm{Im}T = THT^\dagger.$$

$$H = \mathrm{diag}(\ldots, 1, \ldots, 1, -1, \ldots, -1, \ldots).$$

Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

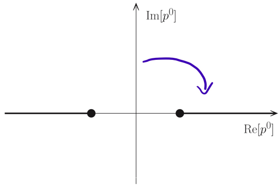
Prescription for the amplitude $\mathcal{A}(p)$

Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

$$\text{Standard particle :} \quad \frac{1}{k^2 - m^2 + i\epsilon}.$$

$$\text{Ghost :} \quad \frac{-1}{k^2 - m^2 + i\epsilon}.$$

Prescription for the amplitude $\mathcal{A}(p)$



$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$

Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

Standard particle : $$\frac{1}{k^2 - m^2 + i\epsilon}.$$

Ghost : $$\frac{-1}{k^2 - m^2 + i\epsilon}.$$

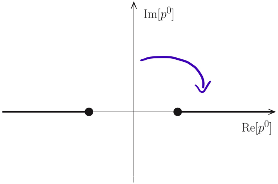
Fake particle (fakeon):

$$\pm \frac{k^2 - m^2}{(k^2 - m^2)^2 + \mathcal{E}^4}.$$

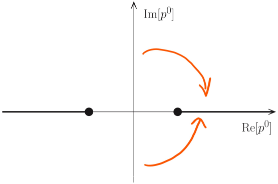
$+$ integration domain deformations

(see D. Anselmi and MP, JHEP 1706 (2017) 066.)

Prescription for the amplitude $\mathcal{A}(p)$



$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$



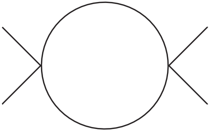
$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{A}_+(p) + \mathcal{A}_-(p)\right].$$

A theory of particles and fakeons is unitary.

D. Anselmi and MP, PRD 96 (2017) 045009. D. Anselmi, JHEP 02 (2018) 141.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4.$$

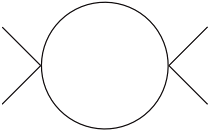
One-loop bubble diagram



Compute the Euclidean amplitude $\mathcal{A}(p)$ and then use the prescription.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4.$$

One-loop bubble diagram



Compute the Euclidean amplitude $\mathcal{A}(p)$ and then use the prescription.

After renormalizing the UV divergence

Feynman prescription

$$\mathcal{A}_+(p) = \frac{1}{2(4\pi)^2}\ln\frac{-p^2 - i\epsilon}{\mu^2}.$$

New presciption

$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{4(4\pi)^2}\ln\frac{(p^2)^2}{\mu^4}.$$

The theory of quantum gravity and fakeons

D. Anselmi, JHEP 1706 (2017) 086.

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right].$$

The theory of quantum gravity and fakeons

D. Anselmi, JHEP 1706 (2017) 086.

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right].$$

The propagator in De Donder gauge ($\Lambda_{\mathrm{C}} = 0,\ \alpha = \xi,\ g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$) is

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \frac{i}{2p^2\left(\zeta - \alpha p^2\right)}\mathcal{I}_{\mu\nu\rho\sigma} = \left\{\frac{1}{p^2} - \frac{1}{p^2 - \zeta/\alpha}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma},$$

$$\mathcal{I}_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}).$$

The theory of quantum gravity and fakeons

D. Anselmi, JHEP 1706 (2017) 086.

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right].$$

The propagator in De Donder gauge ($\Lambda_{\mathrm{C}} = 0,\ \alpha = \xi,\ g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$) is

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \frac{i}{2p^2\left(\zeta - \alpha p^2\right)}\mathcal{I}_{\mu\nu\rho\sigma} = \left\{\frac{1}{p^2} - \frac{1}{p^2 - \zeta/\alpha}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma},$$

$$\mathcal{I}_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}).$$

With the new prescription it turns into

$$\left\{\frac{1}{p^2 + i\epsilon} - \frac{p^2 - \zeta/\alpha}{\left[(p^2 - \zeta/\alpha + i\epsilon)^2 + \mathcal{E}^4\right]_{\mathrm{AV}}}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma}.$$

The theory of quantum gravity and fakeons

D. Anselmi, JHEP 1706 (2017) 086.

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right].$$

The propagator in De Donder gauge ($\Lambda_{\mathrm{C}} = 0,\ \alpha = \xi,\ g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$) is

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \frac{i}{2p^2\left(\zeta - \alpha p^2\right)}\mathcal{I}_{\mu\nu\rho\sigma} = \left\{\frac{1}{p^2} - \frac{1}{p^2 - \zeta/\alpha}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma},$$

$$\mathcal{I}_{\mu\nu\rho\sigma} \equiv (\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}).$$

With the new prescription it turns into

$$\left\{\frac{1}{p^2 + i\epsilon} - \boxed{\frac{p^2 - \zeta/\alpha}{\left[(p^2 - \zeta/\alpha + i\epsilon)^2 + \mathcal{E}^4\right]_{\mathrm{AV}}}}\right\}\frac{i}{2\zeta}\mathcal{I}_{\mu\nu\rho\sigma}.$$

Renormalization

The fakeon prescription does not spoil renormalizability.

Using the Batalin-Vilkovisky formalism we computed the beta functions in a new method by renormalizing the sources of the BRST tf's.

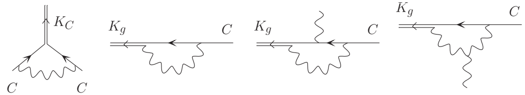
D. Anselmi and MP, JHEP 05 (2018) 027.

$$S_{\text{count}} = \frac{\mu^{-\varepsilon}}{(4\pi)^2\varepsilon}\int\sqrt{-g}\left[2\Delta\Lambda_C + \Delta\zeta R + \Delta\alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\Delta\xi}{6}R^2\right] + (S,\mathcal{F}),$$

$$\begin{aligned}\Delta\alpha &= -\frac{133}{10}, \qquad \Delta\xi = \frac{5}{6} + \frac{5\xi}{\alpha} + \frac{5\xi^2}{3\alpha^2}, \qquad \Delta\zeta = \zeta\left(\frac{5}{6\xi} + \frac{5\xi}{3\alpha^2} + A\right),\\ \Delta\Lambda_C &= \Lambda_C\left(-\frac{5}{\alpha} + \frac{2}{\xi} - 2A\right) - \frac{5\zeta^2}{4\alpha^2} - \frac{\zeta^2}{4\xi^2}.\end{aligned}$$

The beta functions are

$$\beta_\alpha = -\frac{2\kappa^2}{(4\pi)^2}\Delta\alpha, \qquad \beta_\xi = -\frac{2\kappa^2}{(4\pi)^2}\Delta\xi, \qquad \beta_\zeta = -\frac{2\kappa^2}{(4\pi)^2}\Delta\zeta, \qquad \beta_{\Lambda_C} = -\frac{2\kappa^2}{(4\pi)^2}\Delta\Lambda_C.$$



Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_0 = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_{0\text{grav}} + \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_{0\text{fake}} .$$

Possible cases in the diagram (pure gravity case)

i) **Fake-Fake:** AV continuation $\longrightarrow$ purely imaginary;

Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_{0\text{grav}} + \langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_{0\text{fake}} .$$

Possible cases in the diagram (pure gravity case)

i) **Fake-Fake:** AV continuation $\longrightarrow$ purely imaginary;

ii) **Grav-Fake:** AV continuation $\longrightarrow$ purely imaginary;

Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_0 = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_{0\text{grav}} + \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_{0\text{fake}} .$$

Possible cases in the diagram (pure gravity case)

i) **Fake-Fake:** AV continuation $\longrightarrow$ purely imaginary;

ii) **Grav-Fake:** AV continuation $\longrightarrow$ purely imaginary;

iii) **Grav-Grav**: Standard Wick rotation $\longrightarrow$ nontrivial real part.

Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

$$\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_0 = \langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_{0\text{grav}} + \langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle_{0\text{fake}}.$$

Possible cases in the diagram (pure gravity case)

i) **Fake-Fake:** AV continuation $\longrightarrow$ purely imaginary;

ii) **Grav-Fake:** AV continuation $\longrightarrow$ purely imaginary;

iii) **Grav-Grav**: Standard Wick rotation $\longrightarrow$ nontrivial real part.

Absorptive part of graviton self energy (part I)

$\Lambda_C = 0$, massless matter

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_0 = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_{0\text{grav}} + \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_{0\text{fake}} .$$

Possible cases in the diagram (pure gravity case)

i) **Fake-Fake:** AV continuation $\longrightarrow$ purely imaginary;

ii) **Grav-Fake:** AV continuation $\longrightarrow$ purely imaginary;

iii) **Grav-Grav**: Standard Wick rotation $\longrightarrow$ nontrivial real part.

The contributions of type iii) can be evaluated by using

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_0 = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p) \rangle_{0\text{grav}} .$$

In the case of pure gravity the absorptive part can be written as

$$\Gamma_{\text{abs}} = -\int \frac{\delta S_{\text{HD}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu}, \quad \Delta g_{\mu\nu} \text{ piecewise local function of } h_{\mu\nu}.$$

In the case of pure gravity the absorptive part can be written as

$$\Gamma_{\text{abs}} = -\int \frac{\delta S_{\text{HD}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu}, \quad \Delta g_{\mu\nu} \text{ piecewise local function of } h_{\mu\nu}.$$

Adding (massless) matter fields of all types the absorptive part is

$$\begin{aligned}\Gamma_{\text{abs}} \quad = \quad & \frac{i\mu^{-\varepsilon}}{16\pi} \int \sqrt{-g} \left[C \left(R_{\mu\nu} \theta(-\Box_c) R^{\mu\nu} - \frac{1}{3} R \theta(-\Box_c) R \right) + \frac{N_s \eta^2}{36} R \theta(-\Box_c) R \right] \\ & - \int \frac{\delta S_{\text{HD}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu},\end{aligned}$$

$$C = \frac{1}{120} (N_s + 6N_f + 12N_v),$$

$$\eta = \text{nonminimal coupling for scalar fields.}$$

Absorptive part of graviton self energy (part II)

$\Lambda_C = 0$, massive matter

D. Anselmi and MP, JHEP 11 (2018) 021.

Equivalent action:
auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

Absorptive part of graviton self energy (part II)

$\Lambda_C = 0$, massive matter

D. Anselmi and MP, JHEP 11 (2018) 021.

Equivalent action:
auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

$$S_{\mathrm{QG}}(g,\phi,\chi,\Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g,\chi) + S_{\phi}(\tilde{g},\phi) + S_m(\tilde{g}e^{\kappa\phi},\Phi),$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}.$$

$$S_{\mathrm{H}} = -\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R, \quad S_{\phi}(g,\phi) = \frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi - \frac{m_\phi^2}{\kappa^2}(1-e^{\kappa\phi})^2\right].$$

$$S_{\chi}(g,\chi) = S_{\mathrm{H}}(\tilde{g}) - S_{\mathrm{H}}(g) - 2\int\chi_{\mu\nu}\frac{\delta S_{\mathrm{H}}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}\,(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2)\big|_{g\to\tilde{g}}.$$

$$S_m = \text{Standard Model},$$

$$\text{Graviton multiplet:} \qquad G_A = \{h_{\mu\nu}, \phi, \chi_{\rho\sigma}\}.$$

$$\{G, F, F\}, \qquad \{G, S, F\}, \qquad \{G, S, Gh\}.$$

Computation of the absorptive parts $M_{AB} = \langle G_A G_B \rangle_{\text{abs}}^{\text{1-loop}}$

Graviton multiplet: $\qquad G_A = \{h_{\mu\nu}, \phi, \chi_{\rho\sigma}\}.$

$$\{G, F, F\}, \qquad \{G, S, F\}, \qquad \{G, S, Gh\}.$$

Computation of the absorptive parts $M_{AB} = \langle G_A G_B \rangle_{\text{abs}}^{\text{1-loop}}$

$$\Gamma_{\text{abs}}^{hh} = -\int \frac{\delta S_{\text{QG}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu},$$

$$\Gamma_{\text{abs}}^{\Phi} = \frac{i}{16\pi} \int \sqrt{-g} R_{\mu\nu}\, \theta(r_\Phi)\theta(1-r_\Phi)\sqrt{1-r_\Phi}\left[P_\Phi(r_\Phi)\left(R^{\mu\nu} - \frac{1}{3}g^{\mu\nu}R\right) + Q_\Phi(r_\Phi) g^{\mu\nu} R\right],$$

$$r_\Phi = -4m_\Phi^2/\Box, \quad P_\Phi,\ Q_\Phi = \text{polynomials}.$$

Graviton multiplet: $\qquad G_A = \{h_{\mu\nu}, \phi, \chi_{\rho\sigma}\}.$

$$\{G, F, F\}, \qquad \{G, S, F\}, \qquad \{G, S, Gh\}.$$

Computation of the absorptive parts $M_{AB} = \langle G_A G_B \rangle_{\text{abs}}^{\text{1-loop}}$

$$\Gamma_{\text{abs}}^{hh} = -\int \frac{\delta S_{\text{QG}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu},$$

$$\Gamma_{\text{abs}}^{\Phi} = \frac{i}{16\pi} \int \sqrt{-g} R_{\mu\nu}\, \theta(r_\Phi)\theta(1-r_\Phi)\sqrt{1-r_\Phi}\left[P_\Phi(r_\Phi)\left(R^{\mu\nu} - \frac{1}{3}g^{\mu\nu}R\right) + Q_\Phi(r_\Phi)g^{\mu\nu}R\right],$$

$$r_\Phi = -4m_\Phi^2/\Box, \quad P_\Phi,\ Q_\Phi = \text{polynomials}.$$

Contributions to the absorptive part

$$\Gamma_{\text{abs}}^{\text{GFF}} = \Gamma_{\text{abs}}^{hh} + \Gamma_{\text{abs}}^{m},$$

$$\Gamma_{\text{abs}}^{\text{GSF}} = \Gamma_{\text{abs}}^{hh} + \Gamma_{\text{abs}}^{\phi h} + \Gamma_{\text{abs}}^{\phi\phi} + \Gamma_{\text{abs}}^{m},$$

$$\Gamma_{\text{abs}}^{\text{GSGh}} = \Gamma_{\text{abs}}^{\text{GSF}} + \Gamma_{\text{abs}}^{\chi h} + \Gamma_{\text{abs}}^{\chi\phi} + \Gamma_{\text{abs}}^{\chi\chi}.$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\text{Pl}}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\rm Pl}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

Breit-Wigner distribution

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \qquad \longrightarrow \qquad \mathrm{sgn}(t)\theta(\Gamma t)\exp\left(-imt-\frac{\Gamma t}{2}\right).$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\mathrm{Pl}}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

Breit-Wigner distribution

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \qquad \longrightarrow \qquad \mathrm{sgn}(t)\theta(\Gamma t)\exp\left(-imt-\frac{\Gamma t}{2}\right).$$

$$\text{Duration} \sim 1/|\Gamma_\chi|$$

$$\text{If } m_\chi \sim 10^{11}\text{GeV then } 1/|\Gamma_\chi| \sim 7\cdot 10^{-17}\text{s}$$

$$\text{If } m_\chi \sim 10^{12}\text{GeV then } 1/|\Gamma_\chi| \sim 4\cdot 10^{-20}\text{s}$$

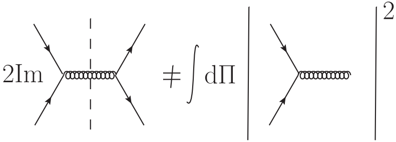
For time intervals of the order $1/|\Gamma_\chi|$ past, present and future, as well as cause and effect lose meaning.

$$2\mathrm{Im}\left[i\left\langle\chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\right\rangle_{s\sim\bar{m}_\chi^2}\right]\xrightarrow[\Gamma_\chi\to0^-]{}\frac{2\pi\kappa^2}{\zeta}Z_\chi\delta(s-\bar{m}_\chi^2)\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s)\geq0.$$

$$2\mathrm{Im}\left[i\left\langle\chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\right\rangle_{s\sim\bar{m}_\chi^2}\right] \xrightarrow[\Gamma_\chi\to 0^-]{} \frac{2\pi\kappa^2}{\zeta} Z_\chi \delta(s-\bar{m}_\chi^2)\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s) \geq 0.$$

In the case of the Stelle theory (GSGh) at tree level

$$\begin{aligned} 2\mathrm{Im}\left[i\left\langle\chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\right\rangle_0\right] &= \frac{2\kappa^2}{\zeta}\mathrm{Im}\left[\frac{1}{p^2-m_\chi^2+i\epsilon}\right]\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s) \\ &= -\frac{2\pi\kappa^2}{\zeta}\delta(s-m_\chi^2)\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s) \leq 0. \end{aligned}$$



The classical limit

At classical level the fakeon prescription is just

$$G_{\mathrm{F}}=\frac{1}{2}\left(G_{\mathrm{adv}}+G_{\mathrm{ret}}\right).$$

The classical limit

At classical level the fakeon prescription is just

$$G_{\mathrm{F}} = \frac{1}{2}\left(G_{\mathrm{adv}} + G_{\mathrm{ret}}\right).$$

Example: HD particle in an external force

$$m\ddot{x} + m\tau^2 \ddddot{x} = \left(1 + \tau^2 \frac{\mathrm{d}^2}{\mathrm{d}t^2}\right) m\ddot{x} = F_{\mathrm{ext}}(t).$$

$$\Downarrow$$

$$m\ddot{x} = \int_{-\infty}^{\infty} G_{\mathrm{F}}(t-t')F_{\mathrm{ext}}(t')\mathrm{d}t' \equiv \langle F_{\mathrm{ext}}\rangle .$$

The classical limit

At classical level the fakeon prescription is just

$$G_{\mathrm{F}} = \frac{1}{2}\left(G_{\mathrm{adv}} + G_{\mathrm{ret}}\right).$$

Example: HD particle in an external force

$$m\ddot{x} + m\tau^2 \ddddot{x} = \left(1 + \tau^2 \frac{\mathrm{d}^2}{\mathrm{d}t^2}\right) m\ddot{x} = F_{\mathrm{ext}}(t).$$

$$\Downarrow$$

$$m\ddot{x} = \int_{-\infty}^{\infty} G_{\mathrm{F}}(t-t')F_{\mathrm{ext}}(t')\mathrm{d}t' \equiv \langle F_{\mathrm{ext}} \rangle .$$

Analogously the classical limit of QG leads to

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left\langle \mathrm{T}_{\mu\nu} \right\rangle .$$

For the explicit case of FLRW metric see D. Anselmi, JHEP 1904 (2019) 061.

Consistent projection

$$\mathcal{F} = \text{metric space of states } |n\rangle \text{ made by means of physical } a^\dagger_{\text{phys}}$$

Consistent projection

$\mathcal{F} =$ metric space of states $|n\rangle$ made by means of physical $a^{\dagger}_{\text{phys}}$

$V = \bar{\mathcal{F}}$ Hilbert space obtained by means of the Cauchy procedure.
V is the Fock space of physical states.

Consistent projection

$\mathcal{F} =$ metric space of states $|n\rangle$ made by means of physical $a^\dagger_{\text{phys}}$

$V = \bar{\mathcal{F}}$ Hilbert space obtained by means of the Cauchy procedure.
V is the Fock space of physical states.

$W =$ Total Fock space of states made by means of $a^\dagger_h$, $a^\dagger_\phi$, $a^\dagger_\chi$ ...

Consistent projection

$\mathcal{F} =$ metric space of states $|n\rangle$ made by means of physical $a^{\dagger}_{\text{phys}}$

$V = \bar{\mathcal{F}}$ Hilbert space obtained by means of the Cauchy procedure.
V is the Fock space of physical states.

$W =$ Total Fock space of states made by means of $a_h^{\dagger}$, $a_{\phi}^{\dagger}$, $a_{\chi}^{\dagger}$ ...

V is a proper subspace of W

Consistent projection

$\mathcal{F}$ = metric space of states $|n\rangle$ made by means of physical $a^\dagger_{\mathrm{phys}}$

$V = \bar{\mathcal{F}}$ Hilbert space obtained by means of the Cauchy procedure.
V is the Fock space of physical states.

W = Total Fock space of states made by means of $a^\dagger_h$, $a^\dagger_\phi$, $a^\dagger_\chi$ ...

V is a proper subspace of W

Perturbative unitarity: the projection $W \longrightarrow V$ is consistent

HD Harmonic oscillator

$$S(x,\dot{x},\ddot{x}) = \int \mathrm{d}t \left[\frac{1}{2} m \left(\dot{x}^2 - \tau^2 \ddot{x}^2 - \omega^2 x^2 \right) \right].$$

$$\Downarrow$$

$$S'(q,\dot{q},Q,\dot{Q}) = \frac{m}{2} \int \mathrm{d}t \left[\dot{q}^2 - \dot{Q}^2 + \frac{Q^2}{\tau^2} - \omega^2 (q+Q)^2 \right].$$

HD Harmonic oscillator

$$S(x,\dot{x},\ddot{x}) = \int \mathrm{d}t \left[\frac{1}{2}m\left(\dot{x}^2 - \tau^2\ddot{x}^2 - \omega^2 x^2\right)\right].$$

$$\Downarrow$$

$$S'(q,\dot{q},Q,\dot{Q}) = \frac{m}{2}\int \mathrm{d}t \left[\dot{q}^2 - \dot{Q}^2 + \frac{Q^2}{\tau^2} - \omega^2(q+Q)^2\right].$$

$$H'(p,q,P,Q) \to H_W(u,p_u,v,p_v) = \Omega\left(a_u^\dagger a_u - \frac{1}{2}\right) - \tilde{\Omega}\left(a_v^\dagger a_v + \frac{1}{2}\right),$$

$$\Omega = \frac{1}{\tau\sqrt{2}}\sqrt{1-\sqrt{1-4\tau^2\omega^2}}, \qquad \tilde{\Omega} = \frac{1}{\tau\sqrt{2}}\sqrt{1+\sqrt{1-4\tau^2\omega^2}}.$$

Projected Hamiltonian

$$H_W(u,p_u,v,p_v) \longrightarrow H_V(u,p_u) = \Omega a_u^\dagger a_u + \frac{1}{2}\left(\Omega - \tilde{\Omega}\right).$$

Space fourier transform of quantum cosmological perturbations have same quadratic actions but with the substitutions

$$m \to m(t), \qquad \omega \to \omega(t), \qquad \mathrm{d}t \to a(t)^3\mathrm{d}t.$$

Summary

- A new quantization prescription gives a unitary and renorm. QFT of gravity.
  - Predictive and possibly testable in the near future.
  - Quantization of cosmological perturbations (E. Bianchi and MP, in preparation).
- New type of degrees of freedom (fakeons).
- New phenomenology due to the presence of fakeons.