From fakeons to quantum gravity

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Basic principles of quantum field theory

unitarity
locality
renormalizability

Fundamental symmetry requirements

- Global Lorentz invariance
- General covariance
- Local Lorentz invariance
- Gauge invariance

Analyticity

Regionwise analyticity

Causality

- Macrocausality
- Microcausality

D. Anselmi, The correspondence principle in quantum field theory and quantum gravity, 18A5 Renorm
http://renormalization.com/18a5

Yang-Mills theory

$$S_{\mathrm{YM}} = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F^a_{\mu\nu}F^{a\mu\nu}$$

D=4

unitarity

locality

(strict) **renormalizability**

Fundamental symmetry requirements

- Global Lorentz invariance
- General covariance
- Local Lorentz invariance
- Gauge invariance

Analyticity

Regionwise analyticity

Causality

- Macrocausality
- Microcausality

The theory is <u>unique</u> in form
(and $D=4$ is uniquely predicted)
if the checked principles are assumed

BUT

THE GAUGE GROUP IS NOT

PREDICTED

why $SU(2) \times SU(3) \times U(1)$? Why no $SU(37)$, $SU(19)$?

Matter sector of the standard model

We basocally have no clue about why it is what it is

Only Known restrictions :

- low energy limit
- cancellation of gauge anomalies

Correspondence fading away already ?

Quantum Gravity

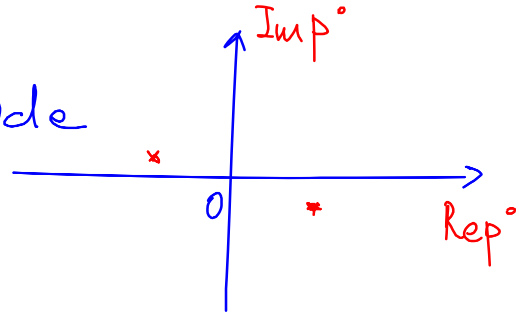
For a review:

D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity, arXiv:1809.05037 [hep-th].

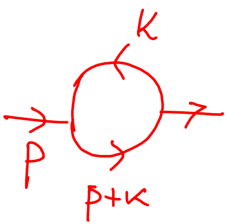
Feynman prescription:

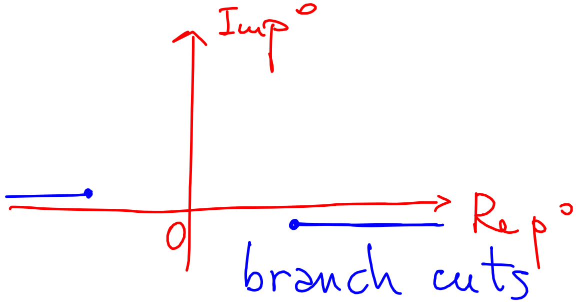
$$\frac{1}{p^2-m^2+i\epsilon} \quad : \quad \text{physical particle}$$

$$-\frac{1}{p^2-m^2+i\epsilon} \quad : \quad \text{ghost}$$



Bubble diagram:





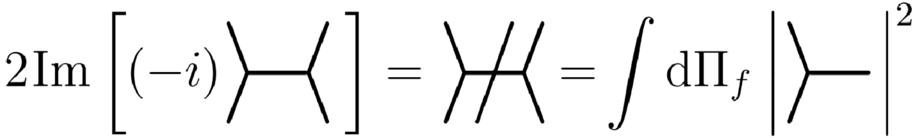
Unitarity

$S = 1 + iT$

$$S^\dagger S = 1 \qquad\qquad 2\,\mathrm{Im}\,T = T^\dagger T$$

optical theorem

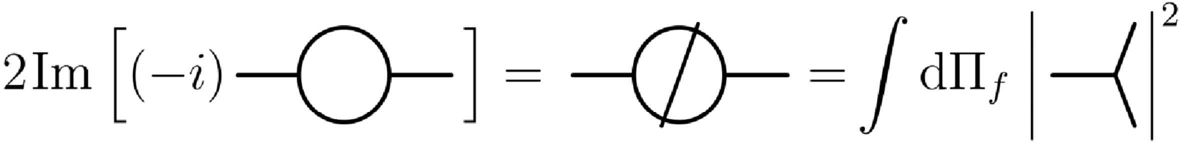
Propagator :



≥ 0

the sign of the propagator cannot change!

Bubble diagram :

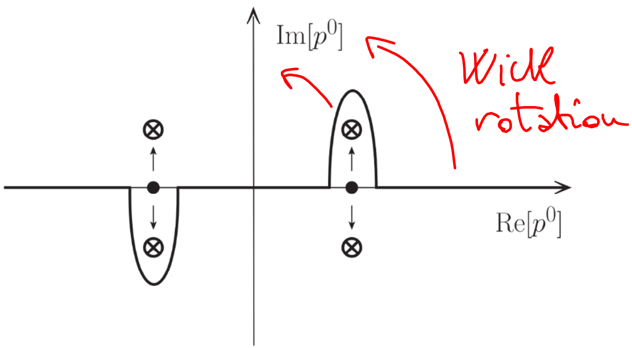


A new prescription can quantize the pole of the propagator as a fakeon, i.e. a fake particle

$$\frac{1}{p^2-m^2} \longrightarrow \frac{p^2-m^2}{(p^2-m^2)^2} \longrightarrow \frac{p^2-m^2}{(p^2-m^2)^2+\varepsilon^4} \xrightarrow[\varepsilon \to 0]{}$$

in a new sense!

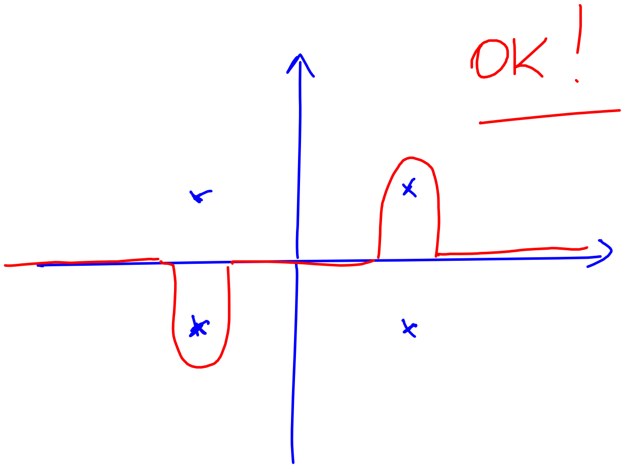
(New type of mathematical distributions?)



The poles of $\frac{1}{p^2-m^2}$ are split into pairs

The integration path is the one following from the Wick rotation

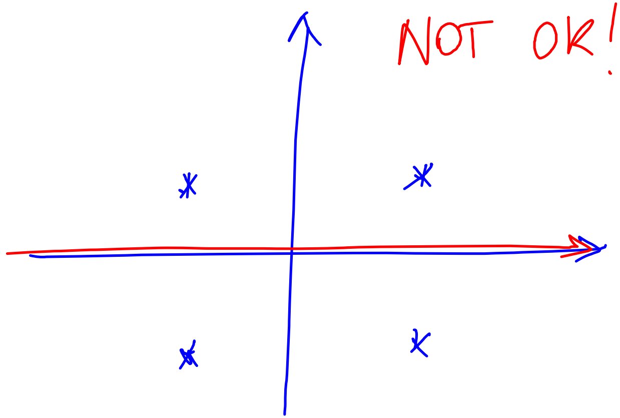
FROM EUCLIDEAN :



Here $\dfrac{p^2-m^2}{(p^2-m^2)^2+\varepsilon^4} =$

= new type of distribution

IN MINKOWSKI :



Here $\dfrac{p^2-m^2}{(p^2-m^2)^2+\varepsilon^4} =$

= principal value :

NOT GOOD

Indeed, working directly in Minkowski spacetime

$\boxed{\text{the locality of counterterms is violated}}$

U. Aglietti and D. Anselmi, Inconsistency of Minkowsky higher-derivative theories, EPJC 77 (2017) 84
and aXiv:1612.06510 [hep-th]

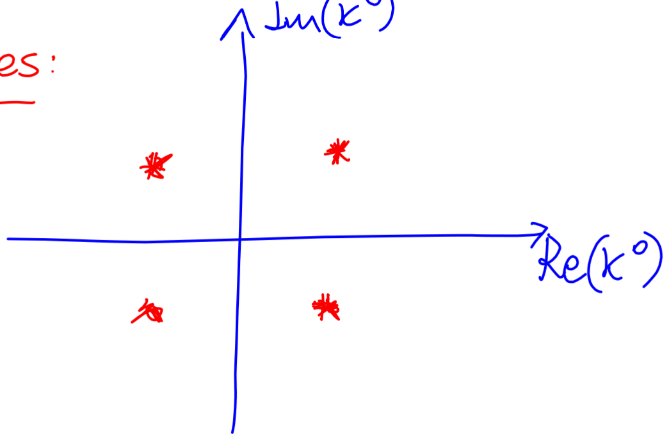
Consider the HD propagator

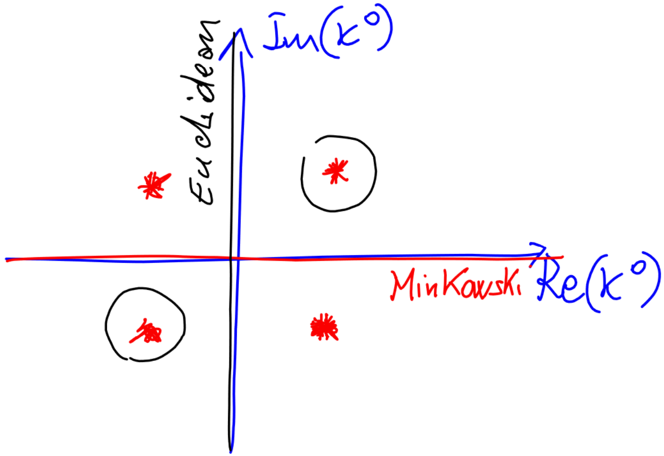
$$S(k) = \frac{1}{(k^2)^2 + M^4} = \frac{1}{(k^2 + iM^2)(k^2 - iM^2)}$$

$k^0 \in \mathbb{C}$

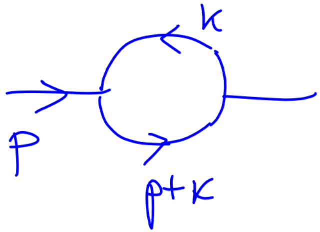
$\vec{k} \in \mathbb{R}^3$

Poles:



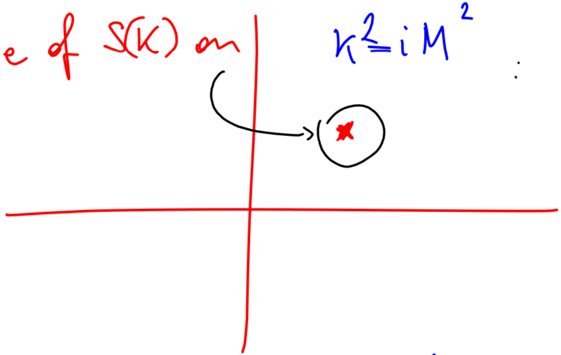


Bubble diagram



$$= \int_{\text{Mink}} \frac{dk^0}{2\pi} \int \frac{d^3k}{(2\pi)^3}\, S(k)\, S(p+k)$$

Residue of $S(k)$ on $k^2 = iM^2$:



$$S(k) \sim \frac{1}{\omega} \sim \frac{1}{|\vec{k}|}$$

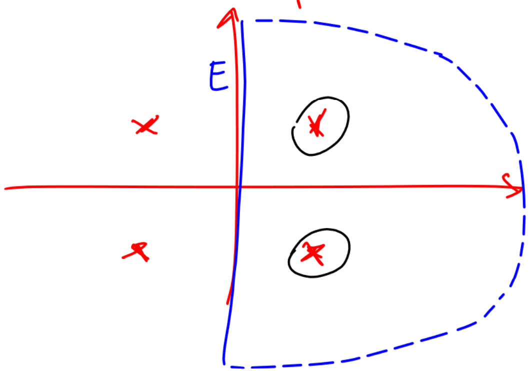
for $|\vec{k}|$ large

$$S(p+k) = \left.\frac{1}{(p^2+k^2+2p\cdot k)^2 + M^4}\right|_{k^2 = iM^2} = \frac{1}{(p^2+iM^2+2p\cdot k)^2 + M^4} \sim$$

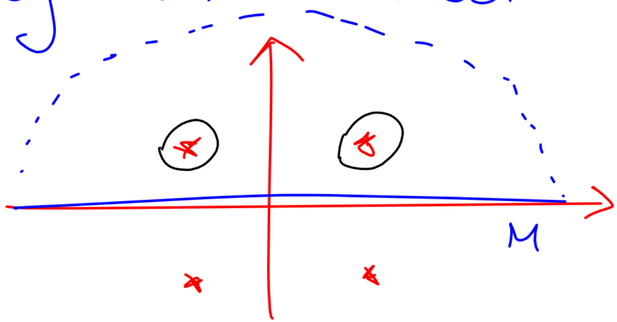
$$\sim \frac{1}{4(p\cdot k)^2}$$: depressed power counting

+ IR singularity : nonlocal divergent parts, such as $\frac{1}{p^2} \ln \Lambda^2_{UV}$

The nonlocal divergences cancel summing the residues as prescribed by the Euclidean theory:



but they do not cancel in the Minkowski case



So, the Wick rotation of the Euclidean version of the theory is the only hope to define the theory properly

However, the Wick rotation is not analytic

D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, arXiv:1703.04584 [hep-th].

and leads to the Lee-Wick models

T.D. Lee and G.C. Wick, Negative metric and the unitarity of the S-matrix, Nucl. Phys. B 9 (1969) 209;

T.D. Lee and G.C. Wick, Finite theory of quantum electrodynamics, Phys. Rev. D 2 (1970) 1033.

Moreover, the Lee-Wick models were not formulated in a complete way

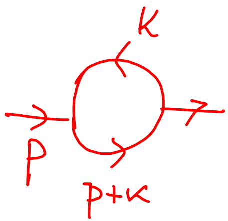
N. Nakanishi, Lorentz noninvariance of the complex-ghost relativistic field theory, Phys. Rev. D 3, 811 (1971).

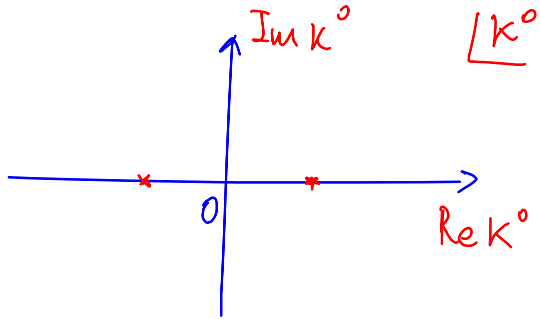
R.E. Cutkosky, P.V Landshoff, D.I. Olive, J.C. Polkinghorne, A non-analytic S matrix, Nucl. Phys. B12 (1969) 281.

Violations of Lorentz invariance and analyticity could not be removed in non ambiguous ways

1 usual case:

Bubble diagram:





$$S(k) = \frac{1}{k^2 - m^2}$$ (no prescription)

$$\int \frac{dk^0}{2\pi} \int \frac{d^3\vec{k}}{(2\pi)^3} \, S(k)\, S(p+k) = \int \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{k}, p)$$

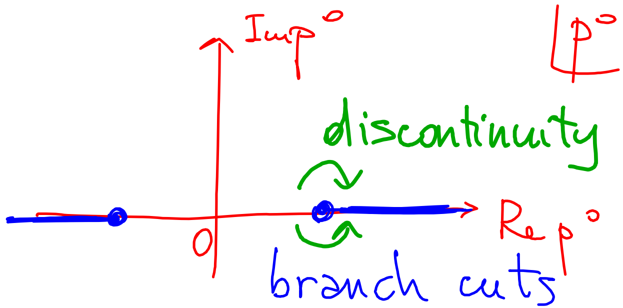
Mink. or Eucl. $\qquad \mathbb{R}^3$

The integrand $f(\vec{k}, \vec{p})$ is singular for

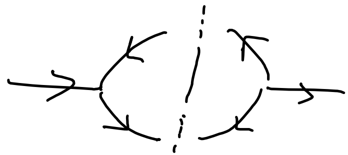
$$|p^0| = \sqrt{\vec{k}^2 + m^2} + \sqrt{(\vec{p} - \vec{k})^2 + m^2} = \omega_{\vec{k}} + \omega_{\vec{p}-\vec{k}}$$

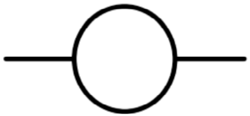
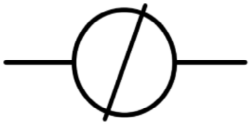
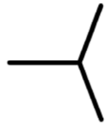
[2nd propagator on shell: $(p-k)^2 = (p^0 - k^0)^2 + (\vec{p} - \vec{k})^2 = m^2$
with residue of 1st propagator: $k^0 = \omega_{\vec{k}}$]

Locations of the singularities:



The singularity of $f(\vec{k}, p)$ is associated with the physical process predicted by the optical theorem

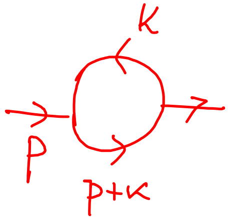


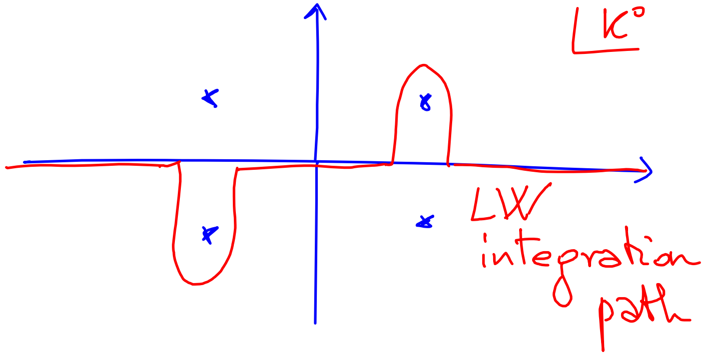
$2\mathrm{Im}\left[(-i)\right.$  $\left.\right] =$  $= \int d\Pi_f \left|\right.$  $\left.\right|^2$

discontinuity

2 our case:

Bubble diagram:





$$S(k) = \frac{1}{(k^2)^2 + M^4}$$

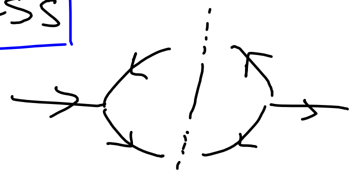
$$\int_{LW} \frac{dk^0}{2\pi} \int_{\mathbb{R}^3} \frac{d^3\vec{k}}{(2\pi)^3} S(k)\, S(p+k) = \int_{\mathbb{R}^3} \frac{d^3\vec{k}}{(2\pi)^3} f(\vec{k}, p)$$

$f(\vec{k}, \vec{p})$ is singular for

$$|p^0| = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(p-\vec{k})^2 \pm iM^2} = \omega_{\vec{k}} + \omega_{\vec{p}-\vec{k}}$$

singularity is associated with $\boxed{\text{an unphysical process}}$

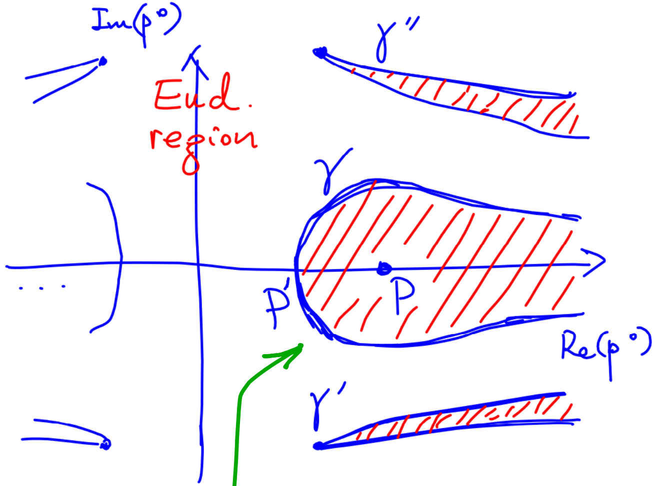
(complex masses are not acceptable)



Singularities of $f(\vec{k}, p)$:

$p^0 \in \mathbb{C} \quad \vec{p} \in \mathbb{R}^3$

$$p^0 = \sqrt{\vec{k}^2 \pm iM^2} + \sqrt{(\vec{p}-\vec{k})^2 \pm iM^2}$$



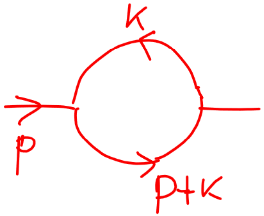
Inside γ the result is NOT analytic & NOT Lorentz invariant

Beyond Lee-Wick:

$$\int_{\mathbb{R}^3} \frac{d^3k}{(2\pi)^3} f(\vec{k}, p)$$

The problem is here!

Prescription needs to be completed!!!



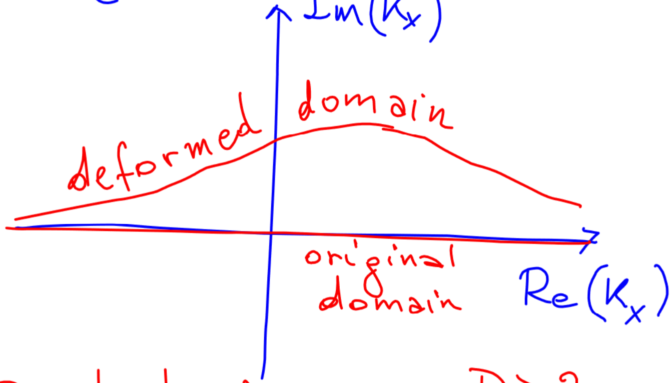
$$= \int_{LW} \frac{dk^0}{2\pi} \int_{\cancel{\mathbb{R}^3}\ D_3} \frac{d^3k}{(2\pi)^3} S(k)\, S(p+k)$$

$D_3 \rightarrow$ deformed domain on the loop (complexified) space momenta

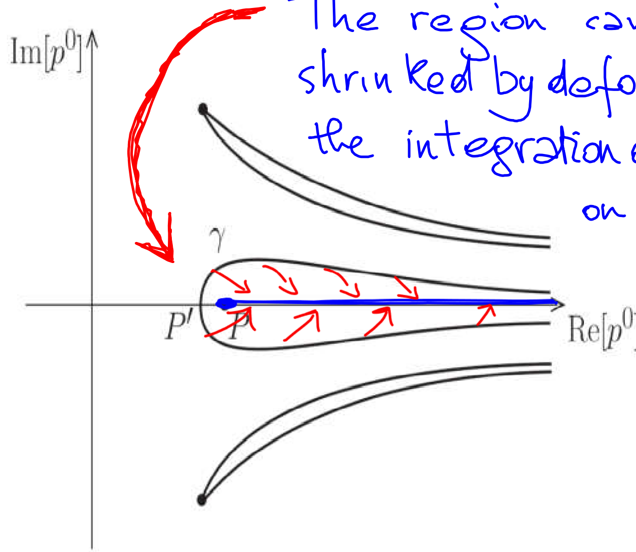
D. A., Fakeons and Lee-Wick models,
J. High Energy Phys. 02 (2018) 141 and arXiv:1801.00915 [hep-th]

Example:

$D = 2$

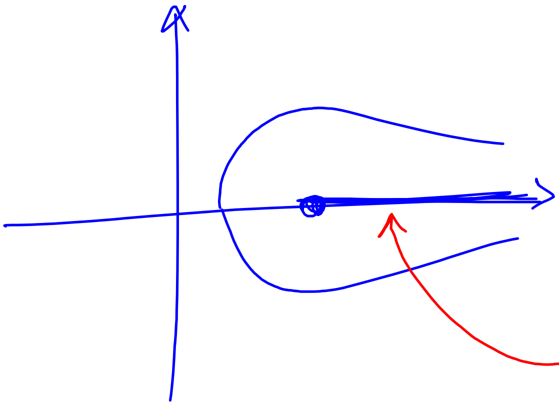


Similarly in every $D \geq 2$



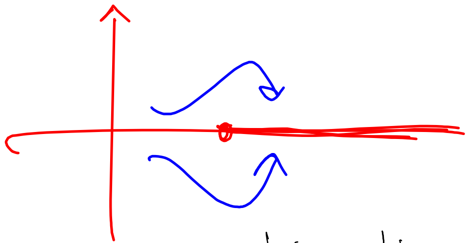
The region can be shrinked by deforming the integration domain on $\vec{k}$

How to calculate with the new prescription:

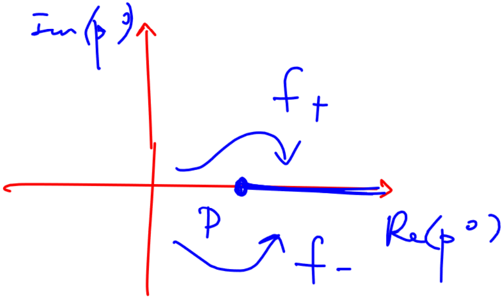


Go inside the region, calculate the integral, then deform the domain to shrink the region

After this, the result is analytic & Lorentz invariant and coincides with the arithmetic average of the two analytic continuations that circumvent the branch point



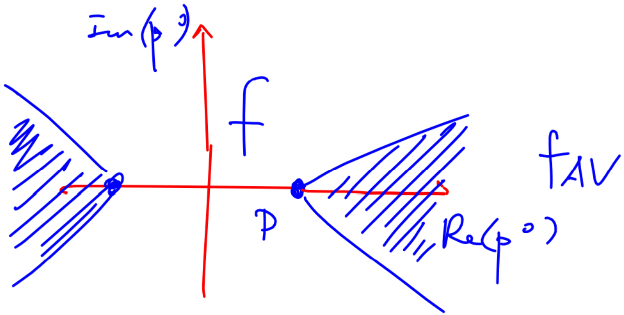
The result is the arithmetic average of the two analytic continuations

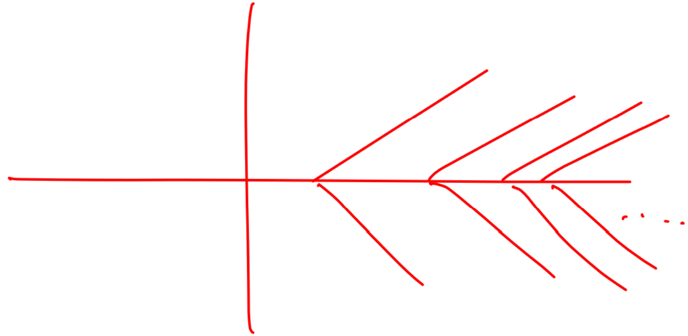


Result: $f_{AV} = \dfrac{f_+ + f_-}{2}$

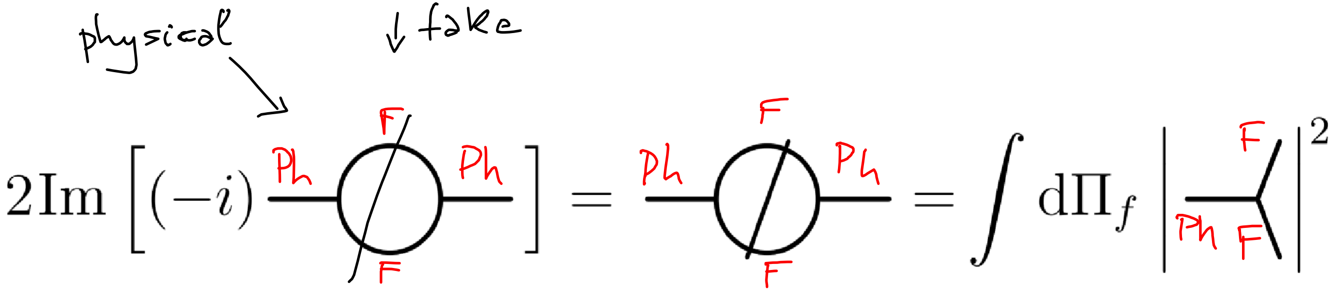
Analyticity & Lorentz invariance are recovered.

The complex p^0 plane is divided into disjoint regions of analyticity





f_{AV} has no absorptive part, so the optical theorem holds:



$S^\dagger_{ik} S_{kj} = \delta_{ij}$

$\| \; 0$ (because the amplitude is real)

$\| \; 0$ (because I project the fakeon away from the spectrum)

$\mathrm{Im}\,\mathcal{M} = 0 \Rightarrow$ we can consistently

project away the final states f, f'

of squared masses $\pm i M^2$ from the physical space

Projection:

do $\underline{\text{not}}$ use $a_f^\dagger$ of the fake particle f

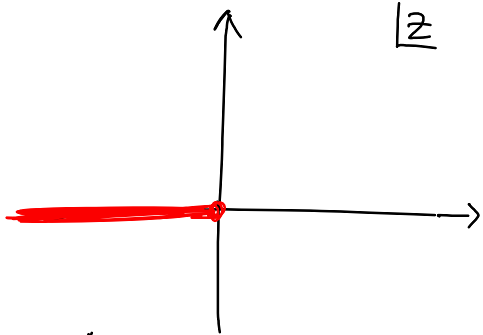
$$(a_f^\dagger)^{n_f}(a_{ph}^\dagger)^{n_{ph}}|0\rangle \quad : \quad \text{total Fock space } W$$

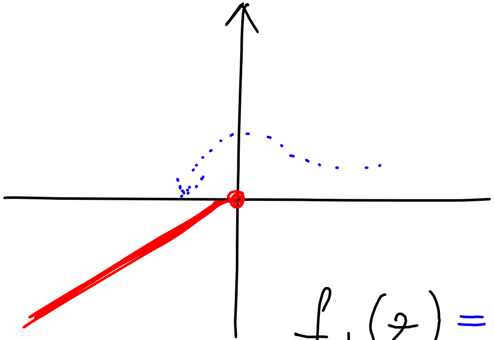
$$(a_{ph}^\dagger)^{n_{ph}}|0\rangle \quad : \quad \text{projected Fock space } V$$

The free Hamiltonian is bounded from below in V

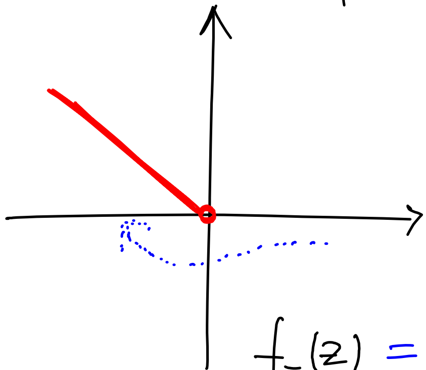
Average continuation

$f(z) = \ln z$:



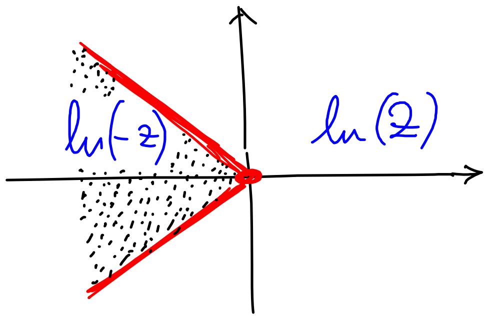


$f_+(z) = \ln(z + i\epsilon)$

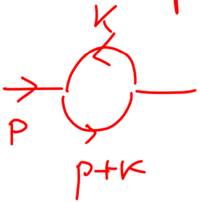


$f_-(z) = \ln(z - i\epsilon)$

$$f_{AV}(z) = \frac{1}{2}\left(\ln(z+i\epsilon) + \ln(z-i\epsilon)\right) = \frac{1}{4}\ln z^2$$



Example: massless scalar $\frac{1}{p^2}$



$\longrightarrow \ln(-p^2)$

$\longrightarrow$ physical scalar (Feynman): $\ln(-p^2+i\epsilon)$

$\longrightarrow$ fakeon: $\frac{1}{2}\ln(-p^2)^2$

We can cure the ghosts of

$$S_{\mathrm{HD}} = -\frac{\mu^{-\varepsilon}}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right]$$

by turning them into fakeons

Eliminating the higher derivatives by means of extra fields, the action of quantum gravity can also be written as (at vanishing cosmological constant)

$$\mathcal{S}_{\rm QG}(g,\phi,\chi,\Phi)=S_{\rm H}(g)+S_{\chi}(g,\chi)+S_{\phi}(\tilde{g},\phi)+S_{\rm m}(\tilde{g}e^{\kappa\phi},\Phi)$$

Standard model

where $\tilde{g}_{\mu\nu}=g_{\mu\nu}+2\chi_{\mu\nu}$ and

$$S_{\rm H}(g)=-\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R,\qquad S_{\phi}(g,\phi)=\frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_{\mu}\phi\nabla^{\mu}\phi-\frac{m_{\phi}^2}{\kappa^2}\left(1-e^{\kappa\phi}\right)^2\right]$$

$$S_{\chi}(g,\chi)=S_{\rm H}(\tilde{g})-S_{\rm H}(g)-2\int\chi_{\mu\nu}\frac{\delta S_{\rm H}(\tilde{g})}{\delta g_{\mu\nu}}+\frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu}-\chi^2)\Big|_{g\rightarrow\tilde{g}}.$$

$\chi_{\mu\nu}$ MUST be quantized as a fakeon

wrong sign!

Graviton multiplet : $\{h_{\mu\nu}, \phi, \chi_{\mu\nu}\}$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$$

fluctuation of the metric

massive scalar

spin-2 fakeon of mass m_χ

Fakeon width :

$$\Gamma_\chi = -\alpha_\chi C m_\chi \qquad C = \frac{N_s + 6N_f + 12N_v}{120}$$

$$\alpha_\chi = \left(\frac{m_\chi}{M_{Pl}}\right)^2$$

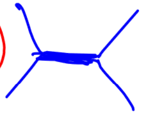
$\Gamma_\chi < 0$: causality is violated by $\chi_{\mu\nu}$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th] JHEP 11(2018) 21

$$-\frac{i}{p^2-m^2+im\Gamma}$$ dressed fakeon propagator

$(\chi_{\mu\nu})$

$\mathrm{Im}\Big[(-i)$  $\Big] = \dfrac{-m\Gamma}{(p^2-m^2)^2+m^2\Gamma^2} \geq 0$ ok

Causality:

$$\frac{i}{E-m+i\frac{\Gamma}{2}} \longrightarrow \mathrm{sgn}(t)\,\theta(\Gamma t)\, e^{-imt-\frac{\Gamma t}{2}}$$

$\Gamma<0$ picks the future instead of the past

The final, corrected equations of General Relativity have the form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle ,$$

where $\langle \dots \rangle$ is an average that picks up a little bit of "future"

Causality is violated for $E \gtrsim m_\chi$

mass of the fakeon

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{\kappa^2}{\zeta} \left[\mathrm{e}^{3\kappa\phi} f T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) + f T_{\phi}^{\mu\nu}(\tilde{g}, \phi) + T_{\mu\nu}^{\chi}(g, \chi) \right],$$

$$-\widetilde{\nabla}_{\mu} \widetilde{\nabla}^{\mu} \phi - \frac{m_{\phi}^2}{\kappa} \left(\mathrm{e}^{\kappa\phi} - 1 \right) \mathrm{e}^{\kappa\phi} = \frac{\kappa \ \ \mathrm{e}^{3\kappa\phi}}{3\zeta} T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) \tilde{g}_{\mu\nu},$$

$$\frac{1}{\sqrt{-g}} \frac{\delta S_{\chi}(g, \chi)}{\delta \chi_{\mu\nu}} = \mathrm{e}^{3\kappa\phi} f T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) + f T_{\phi}^{\mu\nu}(\tilde{g}, \phi), \qquad (*)$$

where $T_A^{\mu\nu}(g) = -(2/\sqrt{-g})(\delta S_A(g)/\delta g_{\mu\nu})$ are the energy-momentum tensors ($A = \mathfrak{m}$, ϕ, χ) and $f = \sqrt{\det \tilde{g}_{\rho\sigma} / \det g_{\alpha\beta}}$. $\quad \tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}$

(Classical) fakeon projection:

Solve (*) with half the sum of the retarded and advanced potentials and insert the solution $\langle \chi \rangle$ into the other two equations

Interim classical action

$\downarrow$

Quantization

$\downarrow$

Classicization

$S_{QG} + S_{SM}$: local, but unprojected (think of a gauge-fixed action)

THE THEORY

Γ, S matrix

True classical limit: nonlocal and non causal at high energies

$E \gtrsim$ fakeon mass

$S_{QG} + S_{SM}\big|_{\text{projected}}$

Toy model.

$$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(v^2 - \tau^2 a^2) - V(x,t)$$

τ = constant $\quad v = \dot{x} \quad a = \ddot{x}$ $\quad V(x,t) = -xF_{\mathrm{ext}}(t)$

Once the fakeon is projected away, the equations of motion $m(a + \tau^2 \ddot{a}) = F_{\mathrm{ext}}$ turn into

$$ma = \frac{1}{1+\tau^2 \frac{d^2}{dt^2}} F_{ext} \qquad m\left(1+\tau^2 \frac{d^2}{dt^2}\right) a = F_{ext}$$

$$ma(t) = \int_{-\infty}^{\infty} \mathrm{d}u G_F(u,\tau) F_{\mathrm{ext}}(t-u) \equiv \langle F_{\mathrm{ext}}(t)\rangle$$

where $\quad G_F(u,\tau) = \dfrac{\sin(|u|/\tau)}{2\tau},$

$$\lim_{\tau\to 0} G_F(u,\tau) = \delta(u)$$

$S_{QG} + S_{\text{Standard Model}}$:

unitarity

locality

(proper) **renormalizability**

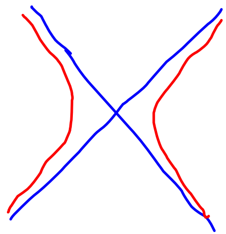
dimensionless gauge couplings

Fundamental symmetry requirements
- Global Lorentz invariance
- General covariance
- Local Lorentz invariance
- Gauge invariance

Analyticity

Regionwise analyticity

Causality
- Macrocausality
- Microcausality



$\langle F \rangle = ma$, $\Gamma_x < 0$!

- Quantum gravity is identified in an essentially unique way

- The gauge interactions are uniquely identified in form

- Instead, the matter sector remains basically unrestricted

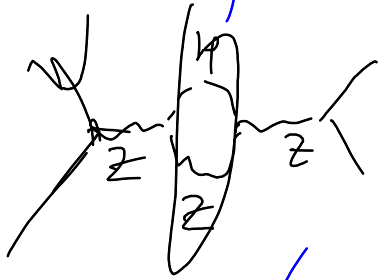
- The major prediction of quantum gravity is the violation of causality at small distances

- The classical limit is not described by the starting classical action

- The mass of the fakeon $\chi_{\mu\nu}$ could be much smaller than the Planck mass

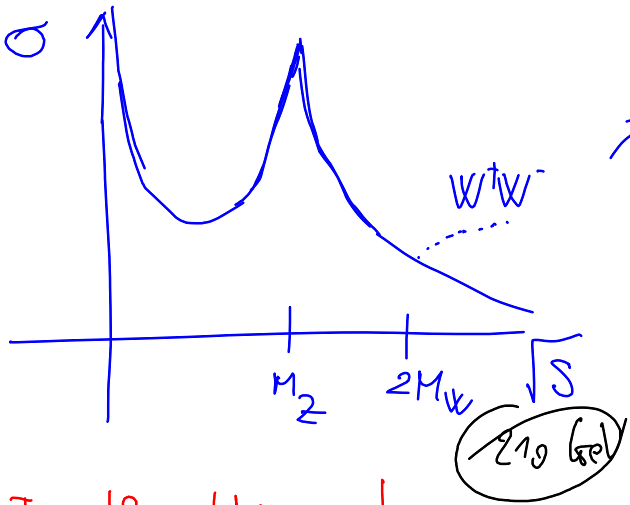
- The violations of microcausality and the other departures from General Relativity could be detectable in a foreseeable future

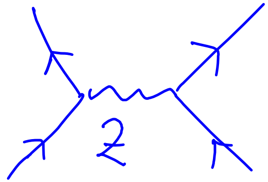
- At least one fakeon is predicted to exist, the spin-2 field $\chi_{\mu\nu}$

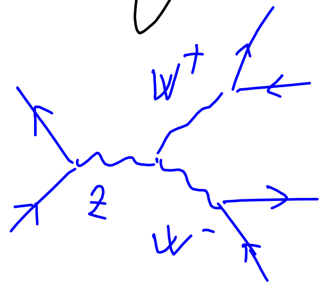


- The scalar field ϕ could be a fakeon or not

- Are there any other fakeons in nature?



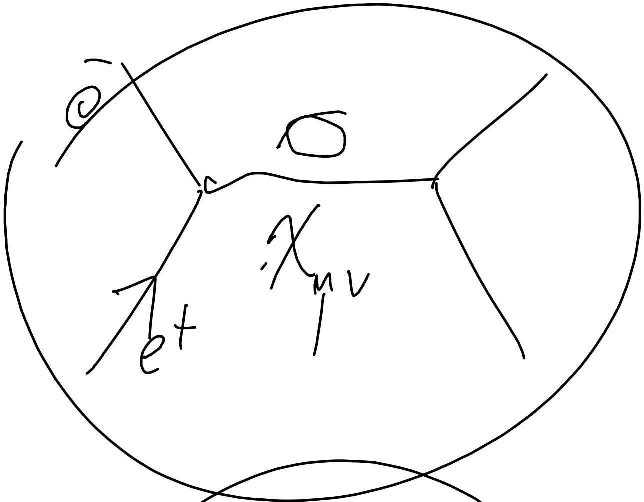


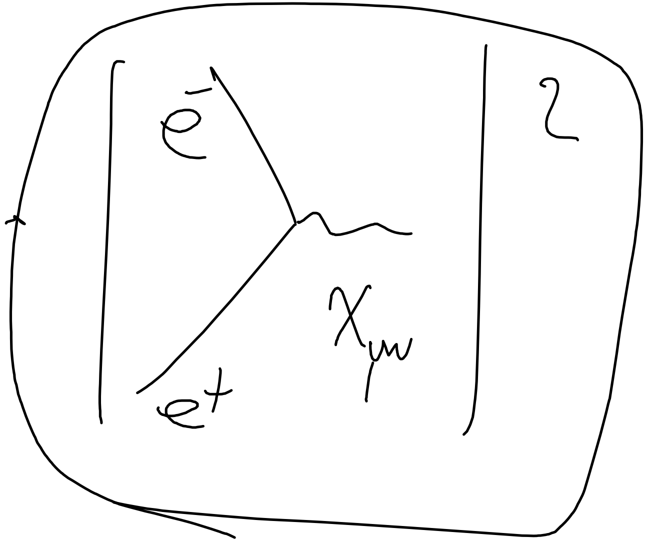


Fakeons cannot be observed directly, but affect the imaginary parts of amplitudes

Is the Higgs boson a fakeon? We do not know, yet

D. A., On the nature of the Higgs boson, arXiv: 1811.02600 [hep-ph]


e⁻
χ_{μν}
e⁺


+
e⁻
χ_{μν}
e⁺
2