The correspondence principle in quantum field theory and quantum gravity

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In this talk, I discuss the fate of the correspondence principle beyond quantum mechanics, i.e. in quantum field theory and quantum gravity, in connection with the intrinsic limitations of the human ability to observe the external world

I claim that the best correspondence principle is made of

unitarity
locality
renormalizability

combined with fundamental local symmetries and the requirement of having a finite number of fields

- Quantum gravity is identified in an essentially unique way

- The gauge interactions are uniquely identified in form

- Instead, the matter sector remains basically unrestricted

- The major prediction is the violation of causality at small distances

Bohr's correspondence principle is a guideline for the selection of theories in quantum mechanics

Is it well posed? Useful? Necessary? Redundant?

The quantization understands that a quantum theory is not built from scratch, but instead guessed from another theory, typically a classical one, which is later quantized

So, there must be some sort of correspondence between the two

... or we are doomed ...

We do have a size!

Our eyes perceive up to a certain resolution, which is $3 \cdot 10^{-4}$ radians

We can define a direct perception range (DPR)

For the rest, we must rely on indirect measurements and observations

In the long run, indirect perceptions may introduce and propagate errors, particularly in connection with the notions of time and causality

The human brain can process an image perceived for around 10^{-3}s (being optimistic) direct perception time resolution (DPTR)

Basically, we see the world at around 1000fps

Very unlikely causality breaks down right below the DPTR

Instruments allow us to resolve time intervals that are way shorter than the DPTR

- There exist $5{\cdot}10^{12}$fps cameras, which can capture light in motion

- The shortest time interval ever measured is about 10^{-18}s

- There are elementary particles with mean lifetimes of about 10^{-25}s

We build our instruments on the work hypothesis that the validity of the laws of nature can be extended below the DPTR

When everything works as expected, we get an a-posteriori validation of the assumption

This allows us to conclude that, indeed, causality holds well below the DPTR

Yet, having checked that it extends to, say, one billionth of a billionth of the DPTR is still not enough to prove that it holds for arbitrarily short time intervals or large energies

Eventually, it may break down

So, the question is: what is principle, what is universal, what is absolute?

The only honest reply is: NOTHING

That is why we may need a correspondence principle, with no guarantee that there exists a satisfactory one

Classically, we can "turn on the light"

Luckily, when the distances we explore are not too small, the laws of nature keep a similarity, or correspondence, with the classical laws

However, we expect that exploring smaller and smaller distances, the correspondence will become weaker and weaker

The first descent to smaller distances is quantum field theory
The second descent is quantum gravity

Our thought is shaped by our interactions with the environment that surrounds us, so it is a "classical" thought

When we apply it to the rest of the universe, we assume that our knowledge is "universal", which is far from justified

The indeterminacy principle proves that the principles suggested by our classical experiences were not principles

And that there is no principle that can be trusted to the very end

In such a situation we might not have much more at our disposal than some sort of "correspondence", even if we know in advance that it is doomed to fade away eventually

Summarizing, the reason why a correspondence principle may be useful in quantum field theory and quantum gravity is rooted in how the quantization works, since the right quantum theory must be identified by starting from a non quantum theory

The environment we wish to explore is so different from the environment we are placed in, that a correspondence between the two may be all we can get

The ground is slippery, so we must be as conservative as possible

Principles

unitarity

locality

renormalizability

Fundamental symmetry requirements

- Global Lorentz invariance
- General covariance
- Local Lorentz invariance
- Gauge invariance

Analyticity

Regionwise analyticity

Causality

- Macrocausality
- Microcausality

Yang-Mills theory

$$S_{\rm YM} = -\frac{1}{4}\int \mathrm{d}^4x\sqrt{-g}F^a_{\mu\nu}F^{a\mu\nu}$$

D=4

unitarity ✓

locality ✓

(strict) **renormalizability** ✓

Fundamental symmetry requirements

- Global Lorentz invariance ✓
- ~~General covariance~~
- ~~Local Lorentz invariance~~
- Gauge invariance ✓

Analyticity ✓

Regionwise analyticity

Causality ✓

- Macrocausality ✓
- Microcausality ✓

The theory is <u>unique</u> in form

(and $D=4$ is uniquely predicted)

if the checked principles are assumed

BUT

THE GAUGE GROUP IS NOT

PREDICTED

why $SU(2) \times SU(3) \times U(1)$? Why no $SU(37)$, $SU(19)$?

Matter sector of the standard model

We basocally have no clue about why it is what it is

Only Known restrictions :

- low energy limit
- cancellation of gauge anomalies

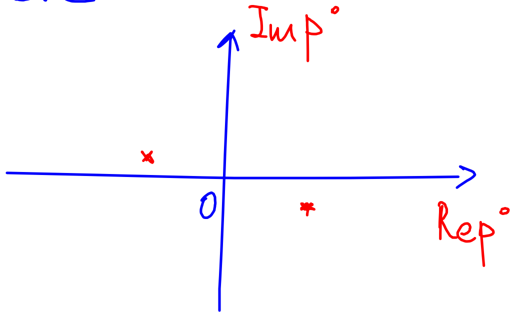
Correspondence fading away already ?

Quantum Gravity

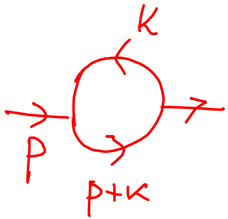
Feynman prescription:

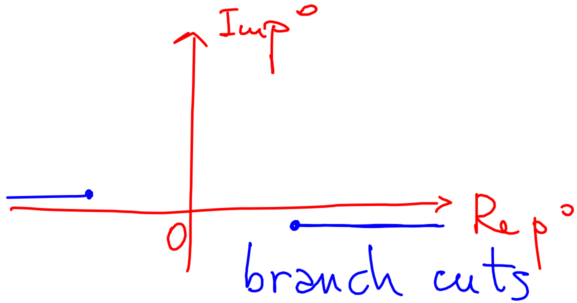
$$\frac{1}{p^2 - m^2 + i\epsilon} \quad : \quad \text{physical particle}$$

$$-\frac{1}{p^2 - m^2 + i\epsilon} \quad : \quad \text{ghost}$$



Bubble diagram:





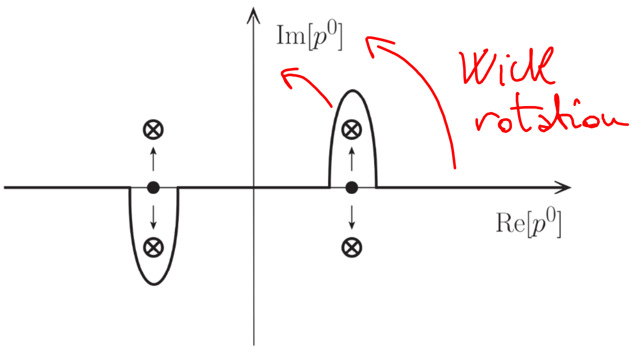
D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity, arXiv:1809.05037 [hep-th].

A new prescription can quantize the pole of the propagator as a fakeon, i.e. a fake particle

$$\frac{1}{p^2-m^2} \longrightarrow \frac{p^2-m^2}{(p^2-m^2)^2} \longrightarrow \frac{p^2-m^2}{(p^2-m^2)^2+\varepsilon^4} \xrightarrow[\varepsilon\to 0]{}$$

in a new sense!

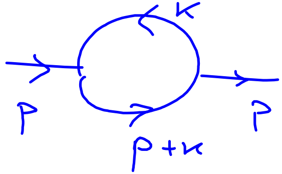
(New type of mathematical distributions?)

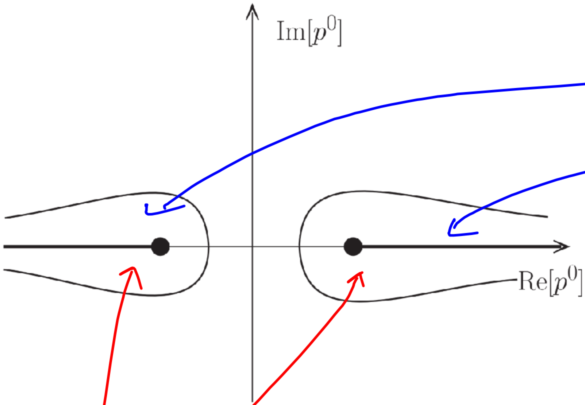


The poles of $\frac{1}{p^2-m^2}$ are split into pairs

The integration path is the one following from the Wick rotation

Bubble diagram :



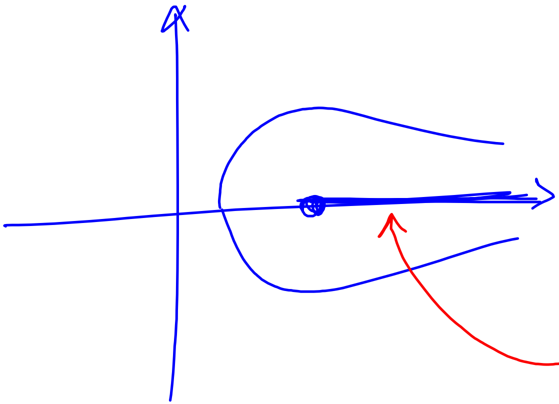


extended regions instead of branch cuts

inside analyticity & Lorentz invariance are violated

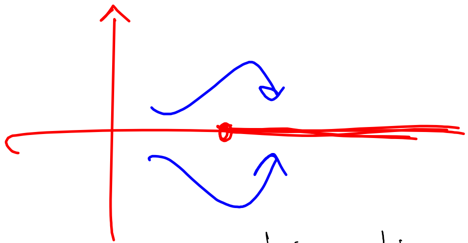
The region can be shrinked onto the branch cuts by deforming the integration domain on the loop <u>space</u> momentum

How to calculate with the new prescription:

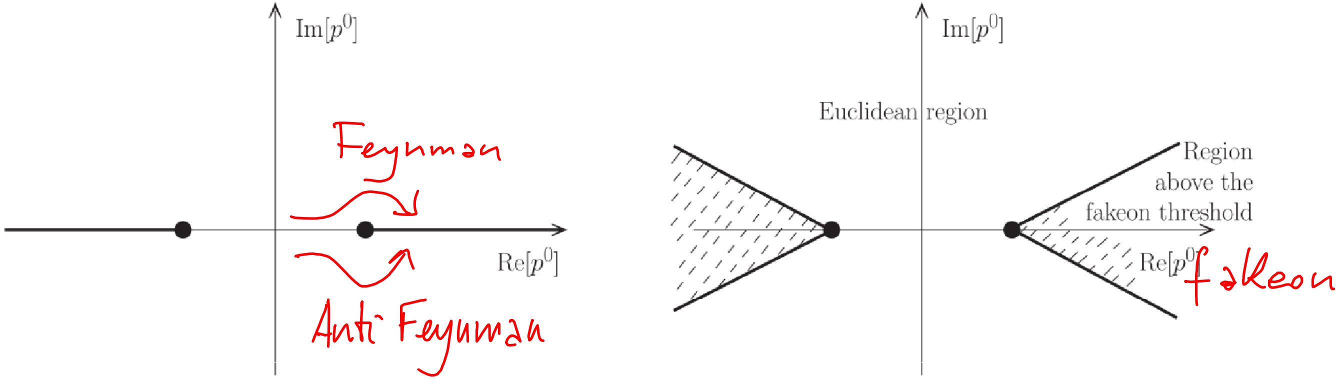


Go inside the region, calculate the integral, then deform the domain to shrink the region

After this, the result is analytic & Lorentz invariant and coincides with the arithmetic average of the two analytic continuations that circumvent the branch point



average continuation



Analyticity versus regionwise analyticity

Analyticity ensures that it is sufficient to calculate an amplitude, or a loop diagram, in any open subset of the space $\mathcal{P}$ of the complexified external momenta to derive it everywhere in $\mathcal{P}$ by means of the analytic continuation. It holds if the theory contains only physical particles.

Regionwise analyticity is the generalization of analyticity that holds when the theory contains fakeons in addition to physical particles. The space $\mathcal{P}$ is divided into disjoint regions of analyticity. It is sufficient to calculate an amplitude, or a loop diagram, in any open set of the Euclidean region to derive it everywhere in $\mathcal{P}$ by means of a nonanalytic operation, called average continuation. The average continuation is the arithmetic average of the two analytic continuations that circumvent a branch point the discontinuity disappears, because the operation is symmetric under reflections with respect to the real axis, so it cannot generate an imaginary part. Thanks to this, the fakeon can be projected away from the physical spectrum V.

$D=4$ interim classical action (unprojected)

$$S_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right]$$

UNIQUE!

$\alpha > 0 \quad \xi > 0 \quad \zeta > 0$

It propagates the graviton, a massive scalar ϕ and a massive spin-2 fakeon $\chi_{\mu\nu}$ $\quad T_\chi < 0$!

Finalized classical action: solve the $\chi_{\mu\nu}$ field equations by means of the fakeon prescription & insert the solution back into S_{QG} $\quad M_\chi \sim \frac{\zeta}{\alpha}$

The final corrected equations of General Relativity look like

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle ,$$

where $\langle \quad \rangle$ is an average that is sensitive to a little bit of future ($E \gg$ mass of fakeon)

Toy model

$$\mathcal{L} = \frac{m}{2}\left(v^2 - \tau^2 a^2\right) - V(x,t)$$

$$v = \dot{x} \qquad a = \dot{v}$$

$$ma = \langle F \rangle = \int_{-\infty}^{+\infty} dt' \, \frac{\operatorname{sen}\left(\frac{|t-t'|}{\tau}\right)}{2\tau} F(t')$$

$$\longrightarrow F(t) \quad \text{for } \tau \to 0$$

$S_{QG} + S_{\text{Standard Model}}$:

unitarity ✓

locality ✓

(proper) **renormalizability** ✓

dimensionless gauge couplings

Fundamental symmetry requirements ✓

- Global Lorentz invariance ✓
- General covariance ✓
- Local Lorentz invariance ✓
- Gauge invariance ✓

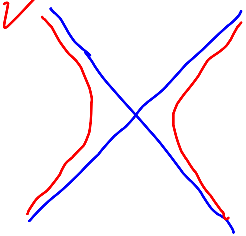
~~Analyticity~~

Regionwise analyticity ✓

Causality

- Macrocausality ✓
- ~~Microcausality~~

$\Gamma_X < 0$!



$D = \text{even} \geqslant 6$:

$$S^D_{\mathrm{YM}} = -\frac{1}{4}\int \mathrm{d}^D x\sqrt{-g}\left[F^a_{\mu\nu}P_{(D-4)/2}(\mathcal{D}^2)F^{a\mu\nu} + \mathcal{O}(F^3)\right]$$

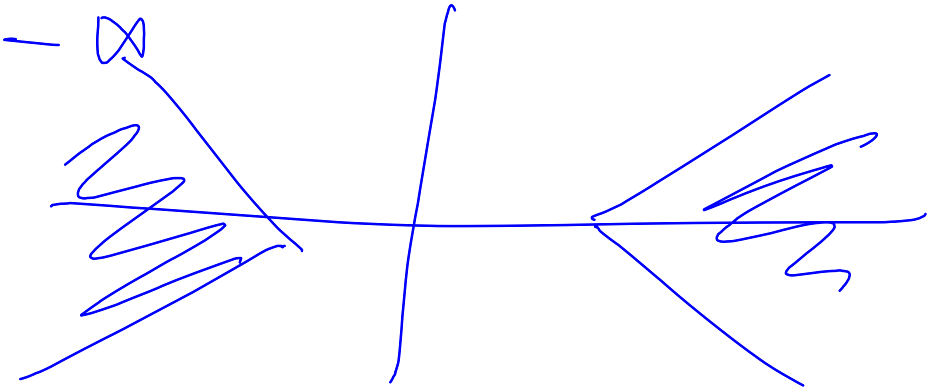
$P_n(x)$ is a real polynomial of degree n in x and $\mathcal{D}$ is the covariant derivative

$$S^D_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\left[2\Lambda_C + \zeta R + R_{\mu\nu}\mathcal{P}_{(D-4)/2}(\mathcal{D}^2)R^{\mu\nu}\right.$$
$$\left. + R\mathcal{P}'_{(D-4)/2}(\mathcal{D}^2)R + \mathcal{O}(R^3)\right]$$

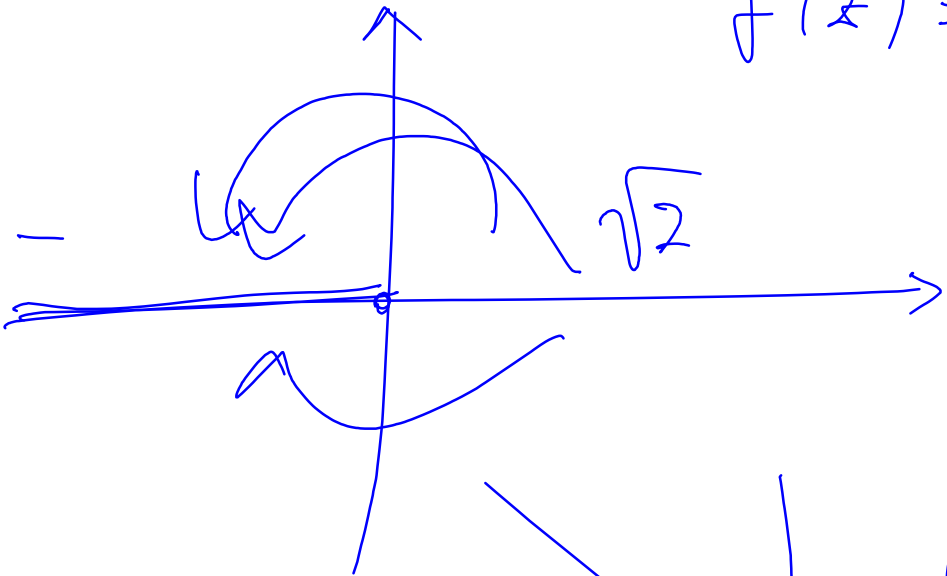
$$R + R_{\mu\nu}\Box R^{\mu\nu} + \cdots$$

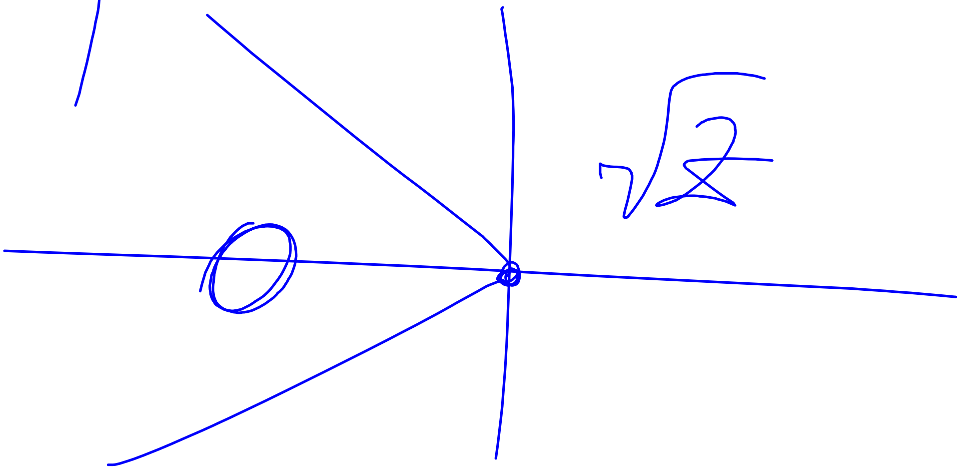
$$\frac{1}{E - m + i\frac{\Gamma}{2}} \qquad G(t) = \theta(t)\, e^{-\Gamma t/2}$$

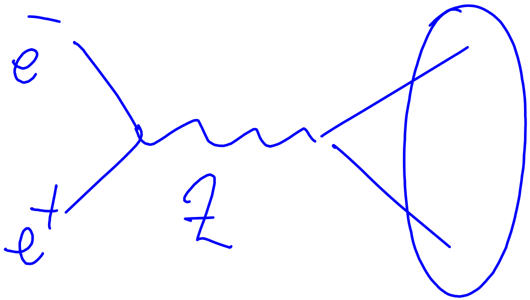
$$\varphi(t) = \int_{-\infty}^{+\infty} G(t - t')\, J(t')\, dt'$$



$f(z) = \sqrt{z}$







$$SS^\dagger = 1$$

Im