Standard Model Without Elementary Scalars And High Energy Lorentz Violation

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Abstract

If Lorentz symmetry is violated at high energies, interactions that are usually non-renormalizable can become renormalizable by weighted power counting. Recently, a CPT invariant, Lorentz violating extension of the Standard Model containing two scalar-two fermion interactions (which can explain neutrino masses) and four fermion interactions (which can explain proton decay) was proposed. In this paper we consider a variant of this model, obtained suppressing the elementary scalar fields, and argue that it can reproduce the known low energy physics. In the Nambu–Jona-Lasinio spirit, we show, using a large N_c expansion, that a dynamical symmetry breaking takes place. The effective potential has a Lorentz invariant minimum and the Lorentz violation does not reverberate down to low energies. The mechanism generates fermion masses, gauge-boson masses and scalar bound states, to be identified with composite Higgs bosons. Our approach is not plagued by the ambiguities of approaches based on non-renormalizable vertices. The low-energy effective action is uniquely determined and predicts relations among parameters of the Standard Model.

1 Introduction

Lorentz symmetry is a basic ingredient of the Standard Model of particle physics. However, the possibility that it might be violated at very high energies is still open [1, 2] and has inspired several investigations about the new physics that could emerge, at low and high energies [3].

In quantum field theory, the violation of Lorentz symmetry at high energies allows us to renormalize otherwise non-renormalizable interactions [4, 5, 6], such as two scalar-two fermion vertices and four fermion vertices. Terms with higher space derivatives modify the dispersion relations and generate propagators with improved ultraviolet behaviors. A "weighted" power counting, which assigns different weights to space and time, allows us to prove that the theory is renormalizable and consistent with (perturbative) unitarity, namely that no counterterms with higher time derivatives are generated.

Using these tools, we have recently proposed [7] a Standard Model extension with the following properties: it is CPT invariant, but Lorentz violating at high energies, it is unitary and renormalizable by weighted power counting; it contains the vertex $(LH)^2/\Lambda_L$, which gives Majorana masses to the neutrinos after symmetry breaking, but no right-handed neutrinos, nor other extra fields; it contains four fermion vertices, which can explain proton decay. The scale $\Lambda_L \sim 10^{14}$GeV is interpreted as the scale of Lorentz violation. Below that scale, Lorentz symmetry is recovered.

The model has two "weighted" dimensions, which means that at high energies its power counting resembles the one of a two-dimensional quantum field theory. In particular, only the four fermion vertices are strictly renormalizable, while the gauge and Higgs interactions are super-renormalizable. This means that at energies $\gtrsim \Lambda_L$ all gauge bosons and the Higgs field become free and decouple, and what remains is a (Lorentz violating) four fermion model in two weighted dimensions. It is then natural to inquire what physical effects are induced, at lower energies, by a dynamical symmetry breaking mechanism, in the Nambu–Jona-Lasinio spirit [8]. If we suppress the elementary scalar field, we obtain a model that is candidate to reproduce the observed low energy physics, predict relations among otherwise independent parameters, and possibly predict new physics detectable at LHC.

Adapting an old suggestion due to Nambu [9], Miransky *et al.* [10] and Bardeen *et al.* [11] to our case, we explore the following scenario. When gauge interactions are switched off, the dynamical symmetry mechanism produces fermion condensates $\langle \bar{q}q \rangle$. The effective potential can be calculated in the large N_c limit and has a Lorentz invariant (local) minimum, which gives masses to the fermions. Massive scalar bound states (composite Higgs bosons) emerge, together with Goldstone bosons [12]. At a second stage, gauge interactions are switched back on, so the Goldstone bosons associated with the breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_Q$ are "eaten" by the $W^{\pm}$ and Z bosons, which become massive.

The low-energy effective action is Lorentz invariant and uniquely determined. It predicts relations among parameters of the Standard Model. The naivest predictions are obtained in the leading order of the large N_c expansion, with gauge interactions switched off, and considering just the top and bottom quarks. In this simplified situation neutral composite Higgs bosons have masses $\sim 2m_t$ and $\sim 2m_b$, and charged Higgs bosons have masses $\sim \sqrt{2}m_t$. The ordinary single-Higgs situation can be retrieved choosing the four fermion vertices appropriately, namely squaring the Yukawa coupling to the Higgs field. More complicate formulas relate m_t to the W- and Z-masses. The leading order of the $1/N_c$ expansion carries a large theoretical error, say 50%. Curiously, the relation between m_t and the Fermi constant turns out to be in "too-good" agreement with the experimental value.

The Nambu–Jona-Lasinio mechanism induces low-energy physics from otherwise highly suppressed interactions. Since our model is Lorentz violating at high energies, we can worry that the Lorentz violation might be reverberated down to low energies. We show that this does not happen, since the minimum of the effective potential is Lorentz invariant and no Lorentz violating interactions are drawn down to low energies.

The advantage of our approach with respect to ordinary Nambu–Jona-Lasinio approaches is that our model is renormalizable, so its high energy behavior is given, and depends on a certain finite set of free parameters. The predictivity of non-renormalizable approaches [10, 11] is questionable [13]. For example in ref. [14], the complete set of independent Standard Model parameters was generated introducing other non-renormalizable terms, besides the usual four fermion vertices. Our model, on the contrary, has an unambiguous high-energy behavior, and can be used to justify results previously obtained in uncertain theoretical frameworks. With respect to other approaches to composite Higgs bosons, such as technicolor [15], or the introduction of extra heavy gauge bosons to renormalize four fermion vertices [13], it has the advantage of being conceptually more economic.

The paper is organized as follows. In section 2 we recall the model of ref. [7], present a variant with a simplified gauge sector and introduce the scalarless model. In section 3 we study the dynamical symmetry breaking in Lorentz violating four fermion models. In section 4 we study the phenomenological consequences of this mechanism in our scalarless model, in particular the generation of fermion masses, bound states, gauge-boson masses, and so on. In section 5 we study Goldstone's theorem in Lorentz violating theories. Section 6 contains our conclusions. In Appendix A we show that a suitable weight rearrangement simplifies the gauge-field sector of our model (but produces new vertices in the matter sector). In Appendix B we prove certain mathematical relations that are used in the paper. We work in Minkoswki spacetime and Wick rotate to Euclidean space when necessary.

2 The model

We assume that invariance under rotations in preserved. We decompose coordinates x^μ as $(\hat{x}^\mu, \bar{x}^\mu)$, where $\hat{x}^\mu$, or simply $\hat{x}$, denotes the time component (keeping an index is useful to use the dimensional regularization), and $\bar{x}^\mu$ denote the space components. Similarly, we decompose the space time index μ as $(\hat{\mu}, \bar{\mu})$, the partial derivative ∂_μ as $(\hat{\partial}_\mu, \bar{\partial}_\mu)$, and gauge vectors A_μ as $(\hat{A}_\mu, \bar{A}_\mu)$. The Lorentz violating theory is renormalizable by weighted power counting [4, 5, 6] in đ$= 1+3/n$ "weighted dimensions", where energy has weight 1 and the space components of momenta have weight $1/n$. Scalar propagators have weight -2 and fermion propagators have weight -1. Details on gauge fields are given in the Appendix.

The "Standard-Extended Model" of ref. [7] has $n = 3$ and therefore weighted dimension 2. The lagrangian of its simplest version reads

$$\mathcal{L} = \mathcal{L}_Q + \mathcal{L}_{\mathrm{kin}f} + \mathcal{L}_H + \mathcal{L}_Y - \frac{\bar{g}^2}{4\Lambda_L}(LH)^2 - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g \bar{D}\bar{F}\,(\bar{\chi}_I \bar{\gamma} \chi_I) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3, \qquad (2.1)$$

where

$$\begin{aligned}
\mathcal{L}_Q &= \frac{1}{4}\sum_G \left(2F^G_{\hat{\mu}\bar{\nu}}\eta^G(\bar{\Upsilon})F^G_{\hat{\mu}\bar{\nu}} - F^G_{\bar{\mu}\bar{\nu}}\tau^G(\bar{\Upsilon})F^G_{\bar{\mu}\bar{\nu}}\right),\\
\mathcal{L}_{\mathrm{kin}f} &= \sum_{a,b=1}^{3}\sum_{I=1}^{5} \bar{\chi}_I^a\, i\left(\delta^{ab}\hat{\not{D}} - \frac{b_0^{Iab}}{\Lambda_L^2}\bar{\not{D}}^3 + b_1^{Iab}\bar{\not{D}}\right)\chi_I^b,\\
\mathcal{L}_H &= |\hat{D}_{\hat{\mu}}H|^2 - \frac{a_0}{\Lambda_L^4}|\bar{D}^2\bar{D}_{\bar{\mu}}H|^2 - \frac{a_1}{\Lambda_L^2}|\bar{D}^2H|^2 - a_2|\bar{D}_{\bar{\mu}}H|^2 - \mu_H^2|H|^2 - \frac{\lambda_4\bar{g}^2}{4}|H|^4,\\
\mathcal{L}_Y &= -\bar{g}\Omega_i H^i + \mathrm{h.c.}, \qquad \Omega_i = \sum_{a,b=1}^{3} Y_1^{ab}\bar{L}^{ai}\ell_R^b + Y_2^{ab}\bar{u}_R^a Q_L^{bj}\varepsilon^{ji} + Y_3^{ab}\bar{Q}_L^{ai}d_R^b, \qquad (2.2)
\end{aligned}$$

i,j are $SU(2)_L$ indices, $\chi_1^a = L^a = (\nu_L^a, \ell_L^a)$, $\chi_2^a = Q_L^a = (u_L^a, d_L^a)$, $\chi_3^a = \ell_R^a$, $\chi_4^a = u_R^a$ and $\chi_5^a = d_R^a$. Moreover, $\nu^a = (\nu_e, \nu_\mu, \nu_\tau)$, $\ell^a = (e, \mu, \tau)$, $u^a = (u, c, t)$ and $d^a = (d, s, b)$. The sum $\sum_G$ is over the gauge groups $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$, and the last three terms of (2.1) are symbolic. Finally, $\bar{\Upsilon} \equiv -\bar{D}^2/\Lambda_L^2$, where Λ_L is the scale of Lorentz violation, and η^G, τ^G are polynomials of degree 2 and 4, respectively. Gauge anomalies cancel out exactly as in the Standard Model [7]. The "boundary conditions" such that Lorentz invariance is recovered at low energies are that b_1^{Iab} tend to δ^{ab} and a_2, η^G and τ^G tend to 1 (four such conditions can be trivially fulfilled normalizing the gauge fields and the space coordinates $\bar{x}$).

The dispersion relations are modified, because propagators contain higher powers of the space components of momenta. This improves their ultraviolet behaviors and makes the theory renormalizable. Since the weight of a scalar field vanishes in đ$= 2$ a constant $\bar{g}$ of weight $1/2$ is attached to the scalar legs to ensure renormalizability. The gauge coupling g has weight 1. The weights of all other parameters are determined so that each lagrangian term has weight 2 (=đ). We have neutrino masses $\sim v^2/\Lambda_L$, v being the Higgs vev, assuming that all other parameters involved in the vertex $(LH)^2/\Lambda_L$ are of order 1. Reasonable estimates of the neutrino masses (a fraction of eV) give $\Lambda_L \sim 10^{14}$GeV.

An alternative model can be obtained rearranging the weight assignments as explained in Appendix A, which allows us to simplify the gauge sector. Specifically, we replace η^G with unity and τ^G with a polynomial of degree 2, which we denote by τ'^G. In a suitable "Feynman" gauge the gauge-field propagator becomes reasonably simple to be used in practical computations. The price is a more complicated Higgs sector, because g and $\bar{g}$ get a lower weight (1/3). The simplest version of the alternative model (see the Appendix for details) has lagrangian

$$\mathcal{L}' = \mathcal{L}'_Q + \mathcal{L}_{\text{kin}f} + \mathcal{L}'_H + \mathcal{L}_Y - \frac{\bar{g}^2}{4\Lambda_L}(LH)^2 - \sum_{I=1}^{5} \frac{1}{\Lambda_L^2} g\bar{D}\bar{F}\,(\bar{\chi}_I \bar{\gamma}\chi_I) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3$$
$$-\frac{1}{\Lambda_L^2} g\bar{g}\bar{\psi}\psi\bar{F}H - \frac{1}{\Lambda_L^2}\left(\bar{g}^3\bar{\psi}\psi H^3 + \bar{g}^2\bar{\psi}\bar{D}\psi H^2 + \bar{g}\bar{\psi}\bar{D}^2\psi H\right) - \frac{1}{\Lambda_L^4}\left(g\bar{D}^2\bar{F} + g^2\bar{F}^2\right)H^\dagger H, \quad (2.3)$$

where

$$\mathcal{L}'_Q = \frac{1}{4}\sum_G \left(2F^G_{\hat{\mu}\bar{\nu}}F^G_{\hat{\mu}\bar{\nu}} - F^G_{\bar{\mu}\bar{\nu}}\tau'^G(\bar{\Upsilon})F^G_{\bar{\mu}\bar{\nu}}\right),$$
$$\mathcal{L}'_H = \mathcal{L}_H - \frac{\lambda_4^{(3)}\bar{g}^2}{4\Lambda_L^2}|H|^2|\bar{D}_{\bar{\mu}}H|^2 - \frac{\lambda_4^{(2)}\bar{g}^2}{4\Lambda_L^2}|H^\dagger\bar{D}_{\bar{\mu}}H|^2 - \frac{\bar{g}^2}{4\Lambda_L^2}\left[\lambda_4^{(1)}(H^\dagger\bar{D}_{\bar{\mu}}H)^2 + \text{h.c.}\right] - \frac{\lambda_6\bar{g}^4}{36\Lambda_L^2}|H|^6,$$

Scalarless model Our scalarless Standard-Extended Model reads

$$\mathcal{L}_{\text{noH}} = \mathcal{L}'_Q + \mathcal{L}_{\text{kin}f} - \sum_{I=1}^{5}\frac{1}{\Lambda_L^2}g\bar{D}\bar{F}\,(\bar{\chi}_I\bar{\gamma}\chi_I) + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{g}{\Lambda_L^2}\bar{F}^3, \quad (2.4)$$

and is obtained suppressing the Higgs field in (2.3). Obviously, the gauge anomalies of (2.4) still cancel. We see that the simplification is considerable.

If we suppress the Higgs field in (2.1), the only difference is that $\mathcal{L}_Q$ appears instead of $\mathcal{L}'_Q$ in (2.4). We keep the simpler model (2.4), but the conclusions of this paper do not depend on this choice.

At high energies gauge and Higgs fields become free and decouple, because their interactions are super-renormalizable, so all theories (2.1), (2.3) and (2.4) become a four fermion model in two weighted dimensions, with lagrangian

$$\mathcal{L}_{\text{4f}} = \sum_{a,b=1}^{3}\sum_{I=1}^{5}\bar{\chi}_I^a\, i\left(\delta^{ab}\hat{\partial\!\!\!/} - \frac{b_0^{Iab}}{\Lambda_L^2}\bar{\partial\!\!\!/}^{\,3} + b_1^{Iab}\bar{\partial\!\!\!/}\right)\chi_I^b + \frac{Y_f}{\Lambda_L^2}\bar{\psi}\psi\bar{\psi}\psi. \quad (2.5)$$

We have kept also the terms multiplied by b_1^{Iab}, since they are necessary to recover Lorentz invariance at low energies.

Our purpose is to investigate whether (2.4) can describe the known low-energy physics by means of a dynamical symmetry breaking mechanism triggered by the four fermion vertices, where some quark-antiquark bilinears acquire expectation values.

Let us list the candidate condensates. Observe that left- and right-handed spinors transform in the same way under spatial rotations. Thus, the most general fermionic bilinears that are scalars under spatial rotations are

$$(\psi^\dagger_{1R}\psi_{2L}), \qquad (\psi^{c\dagger}_{1L}\psi_{2L}), \qquad (\psi^{c\dagger}_{1R}\psi_{2R}), \tag{2.6}$$

and their Hermitian conjugates, which are Lorentz invariant, plus

$$(\psi^\dagger_{1L}\psi_{2L}), \qquad (\psi^\dagger_{1R}\psi_{2R}), \qquad (\psi^{c\dagger}_{1R}\psi_{2L}), \qquad (\psi^\dagger_{1R}\psi^c_{2L}), \tag{2.7}$$

which violate both Lorentz symmetry and CPT. We see that every fermion condensate or mass term that violates Lorentz symmetry violates also CPT. Thus, at low energies the dynamical symmetry breaking can either preserve Lorentz symmetry, or break it together with CPT. We show that the effective potential has a Lorentz invariant minimum.

Consider now the four fermion vertices. The Fierz identity can be used to convert the structure $(\psi_1^\dagger\sigma_i\psi_2)(\psi_3^\dagger\sigma_i\psi_4)$ into the structure $(\psi_1^\dagger\psi_2')(\psi_3^\dagger\psi_4')$. Thus, the most general $U(1)_L\times U(1)_R$-invariant, rotationally invariant four fermion interactions are

$$(\psi^\dagger_{1L}\psi_{2R})(\psi^\dagger_{3R}\psi_{4L}), \qquad (\psi^\dagger_{1L}\psi_{2L})(\psi^\dagger_{3L}\psi_{4L}), \qquad (\psi^\dagger_{1R}\psi_{2R})(\psi^\dagger_{3R}\psi_{4R}), \qquad (\psi^\dagger_{1L}\psi_{2L})(\psi^\dagger_{3R}\psi_{4R}). \tag{2.8}$$

The Lorentz invariant combinations are

$$(\psi^\dagger_{1L}\psi_{2R})(\psi^\dagger_{3R}\psi_{4L}), \qquad (\psi^\dagger_{1L}\psi^c_{2L})(\psi^{c\dagger}_{3L}\psi_{4L}), \qquad (\psi^\dagger_{1R}\psi^c_{2R})(\psi^{c\dagger}_{3R}\psi_{4R}), \qquad (\psi^\dagger_{1L}\psi^c_{2R})(\psi^{\dagger c}_{3L}\psi_{4R}).$$

All combinations are CPT invariant. Lorentz violating four fermion vertices remain highly suppressed, while the Lorentz invariant ones determine interactions of the low energy effective theory.

At energy scales much smaller than Λ_L the low-energy effective theory resembles a Standard Model with one or more Higgs doublets. However, the masses of composite Higgs bosons, as well as their self-couplings and couplings to quarks and gauge fields, are not free, but unambiguously determined by the model (2.4).

Predictivity The ordinary Nambu–Jona-Lasinio framework [8] makes use of non-renormalizable interactions. The dynamical symmetry breaking in scalarless models was studied in ref.s [10, 11]. The predictivity of this approach was questioned in ref. [14], where it was shown that the unknown high-energy physics, duly parametrized, can add enough extra parameters to the low-energy effective action, and make it completely equivalent to the Standard Model (with elementary

Higgs field), equipped with all its free constants. The virtue of our approach is that the high-energy physics of our model is unambiguous, encoded in (2.5). Since (2.5), as well as (2.4), (2.1) and (2.3), are renormalizable by weighted power counting, we do not need to consider other sectors of unknown physics beyond them. In particular, (2.4) does not contain the interactions used in [14] to show the predictivity loss. Thus, our model is predictive, and actually provides a viable renormalizable environment for the Nambu–Jona-Lasinio mechanism.

On the other hand, we have a new source of worry. The dynamical symmetry breaking is a non-perturbative mechanism to generate low-energy effects from otherwise suppressed high-energy interactions. In our model (2.4) this mechanism is triggered by four fermion vertices, which are renormalizable only thanks to the Lorentz violation. The scale of Lorentz violation Λ_L cannot be treated as a cut-off in an otherwise renormalizable Lorentz invariant theory (in that case, it would be possible to completely recover Lorentz symmetry at low energies in an obvious way). The dynamical symmetry breaking might reverberate the Lorentz violation down to low energies. If that happened, our scalarless model (2.4) would be in trouble. One of our goals is to prove that the violation of Lorentz symmetry remains highly suppressed even when the dynamical symmetry breaking takes place. Crucial for the proof is the fact, noted above, that Lorentz violating fermion condensates violate also CPT. In some sense, this raises the price of low-energy Lorentz violation enough to disfavour it.

CPT In this paper, as in [7], we assume exact CPT invariance. While (in local, Hermitian) theories a CPT violation implies also the violation of Lorentz symmetry, the converse is not true, in general, except for special subclasses of terms, such as the fermionic bilinears (2.6) and (2.7). Thus, we have to introduce two a priori different energy scales, a scale of Lorentz violation Λ_L, and a scale of CPT violation $\Lambda_{\rm CPT}$, with $\Lambda_{\rm CPT} \geqslant \Lambda_L$. Our estimate $\Lambda_L \sim 10^{14}$GeV is obtained without using the recent bounds on Lorentz violation suggested by the analysis of γ-ray bursts [16], which claim that the first correction

$$c(E) \sim c\left(1 - \frac{E}{\bar{M}}\right) \tag{2.9}$$

to the velocity of light involves an energy scale $\bar{M} \geqslant 1.3{\cdot}10^{18}$GeV. In the realm of local perturbative quantum field theory, a dispersion relation giving (2.9) must contain odd powers of the energy, therefore it must also violate CPT. Thus, we are lead to interpret the results of [16] as bounds on $\Lambda_{\rm CPT} = \bar{M}$ rather than Λ_L. It is conceivable that there exists an energy region $\Lambda_L \leqslant E \leqslant \Lambda_{\rm CPT}$ where Lorentz symmetry is violated but CPT is still conserved. Assuming $\Lambda_{\rm CPT} \geqslant M_{Pl}$, we expect that this region spans at least four or five orders of magnitude. This argument justifies our assumption of CPT invariance.

3 Dynamical symmetry breaking

In this section we illustrate the dynamical symmetry breaking in a simple Lorentz violating four fermion model. We show that there exists a Lorentz invariant minimum, and that the Lorentz violation remains highly suppressed. In the next section we derive phenomenological consequences for the scalarless model (2.4).

We consider the model

$$\mathcal{L}_q = \sum_{I=1}^{N} \bar{\psi}_I \left(\gamma^\mu i \left(\hat{\partial}_\mu + \bar{\partial}_\mu - \bar{\partial}_\mu \frac{\bar{\partial}^2}{\Lambda_L^2}\right) - M\right)\psi_I - V_2(M), \tag{3.1}$$

in the leading order of the large N expansion. We have introduced real auxiliary fields $\rho_\pm$ and a complex auxiliary field τ, such that, in the basis $\psi = (\psi_L, \psi_R)$,

$$M = \left(\begin{array}{cc} \tau & \rho_+ - \rho_- \\ \rho_+ + \rho_- & \bar{\tau} \end{array}\right), \tag{3.2}$$

and

$$V_2(M) = \frac{\Lambda_L^2}{\lambda^2}|\tau|^2 + \frac{\Lambda_L^2}{2g_+^2}\rho_+^2 + \frac{\Lambda_L^2}{2g_-^2}\rho_-^2 + \frac{\Lambda_L^2}{g_{+-}^2}\rho_+\rho_-.$$

The four fermion vertices are obtained integrating $\rho_\pm$ and τ out. We keep only the combinations of the form $(\psi_{1I}^\dagger\psi_{2I})(\psi_{3J}^\dagger\psi_{4J})$, which contribute to scalar intermediate channels in the leading order.

We could introduce also parameters b_{0R}, b_{0L} and b_{1R}, b_{1L} in front of $\bar{\partial\!\!\!/}^{\,3}$ and $\bar{\partial\!\!\!/}$, as in (2.5). However, the ψ self-energy receives no renormalization to the lowest order. Thus, in our approximation b_{1R} and b_{1L} must be set equal to 1, to have Lorentz invariance at low energies. We also make the simplifying assumption $b_{0R} = b_{0L} \equiv b_0$ and reabsorb b_0 inside Λ_L. The model (3.1) is renormalizable as it stands in the leading order.

A non-vanishing τ vacuum expectation value gives the fermions a Dirac mass. On the other hand, non-trivial $\rho_\pm$ expectation values correspond to Lorentz and CPT violating mass terms of the form $\bar{\psi}_I\gamma^0\psi_I$ and $\bar{\psi}_I\gamma^0\gamma_5\psi_I$.

The M-effective potential $V(M)$ can be calculated assuming that $\rho_\pm$ and τ are constants. The leading contributions come from fermion loops with $\rho_\pm$ and τ external legs. We have

$$V(M) = V_2(M) + iN\int \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\det(-\gamma^\mu p'_\mu + M),$$

having defined

$$\hat{p}'^\mu = \hat{p}^\mu, \qquad \bar{p}'^\mu = \bar{p}^\mu\left(\frac{\bar{p}^2}{\Lambda_L^2} + 1\right). \tag{3.3}$$

Using invariance under rotations, we can orient $\bar{p}'^\mu$ along the z direction. Then we find

$$\det\left(-\gamma^\mu p'_\mu + M\right) = T_+T_-, \qquad T_\pm \equiv |\tau|^2 - (\hat{p} - \rho_+)^2 + (|\bar{p}'| \pm \rho_-)^2.$$

Splitting the integral as the sum of two integrals, one for each contribution $T_\pm$, and translating $\hat{p}$, we find, after the Wick rotation,

$$V(M) = V_2(M) + v(\rho_-, \tau) + v(-\rho_-, \tau), \qquad v(\rho, \tau) = -N \int^\Lambda \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln\left(|\tau|^2 + \hat{p}^2 + (|\bar{p}'| + \rho)^2\right).$$

The integral is divergent and regulated with a cut-off $\Lambda \gg \Lambda_L$. Observe that the corrections to $V_2(M)$ are ρ_+-independent.

We first work at $\rho_- = 0$, find the tentative minimum and later prove that it does remain a minimum once ρ_- is switched on. Rescaling the momentum p to $p\Lambda_L$ (and the cut-off Λ to Λ/Λ_L) and defining $|\sigma|^2 = |\tau|^2/\Lambda_L^2$, we find, up to an irrelevant additive constant,

$$V(0, \tau) = \Lambda_L^2 \frac{|\tau|^2}{\lambda^2} + 2N\Lambda_L^4 v(|\sigma|^2),$$

where the function v is defined in Appendix B, formula (B.1). Differentiating once with respect to τ, using (B.6) and subtracting the logarithmic divergence (which amounts to replace the cut-off Λ with the dynamical scale μ), we obtain the *gap equation*

$$\Lambda_{\mathrm{RG}}^3 = \frac{\Lambda_L^2}{2}\sqrt{\langle|\tau|^2\rangle}\exp(12\pi^2 \bar{v}'(\langle|\sigma|^2\rangle)) > 0, \qquad \Lambda_{\mathrm{RG}} = \mu \exp\left(-\frac{2\pi^2}{\lambda^2 N}\right), \tag{3.4}$$

for the non-trivial vacuum expectation value $\langle|\sigma|^2\rangle$, where $\bar{v}'$ is the finite function defined in (B.7). Since Λ_{RG} is a free parameter, $\langle|\sigma|^2\rangle$ cannot be determined, and equation (3.4) just relates $\langle|\sigma|^2\rangle$ and Λ_{RG}. Choosing Λ_{RG} appropriately, the gap equation has any solution $\langle|\sigma|^2\rangle$ we like. This arbitrariness is going to disappear from every other physical quantity. Observe that the τ-sector is asymptotically free in the large N expansion.

The ratio $\langle|\sigma|\rangle = \langle|\tau|\rangle/\Lambda_L$ between the fermion mass $\langle|\tau|\rangle$ and the scale of Lorentz violation is very small. Typical values are $\langle|\tau|\rangle \sim m_t \sim 171$GeV and $\Lambda_L \sim 10^{14}$GeV, so $\langle|\sigma|\rangle \sim 10^{-12}$. Thus, it is meaningful to expand for $\langle|\sigma|^2\rangle \ll 1$, which can be done with the help of formula (B.9). The gap equation becomes

$$\frac{\Lambda_{\mathrm{RG}}^2}{\Lambda_L^2} - 1 \sim \frac{1}{2}\frac{\langle|\tau|^2\rangle}{\Lambda_L^2} \ln \frac{\Lambda_L^2}{\langle|\tau|^2\rangle} \sim 10^{-22},$$

which exhibits a fine-tuning problem, associated with the quadratic divergences arising for large Λ_L. Nevertheless, this problem is isolated to the gap equation, since all other quantities we are going to work with depend on Λ_L only logarithmically.

Expanding around $|\tau|^2 = \langle|\tau|^2\rangle$, the potential at vanishing $\rho_\pm$ reads

$$V(0, \tau) \sim 2N|\tau - \langle\tau\rangle|^2 \langle|\tau|^2\rangle v''(\langle|\sigma|^2\rangle), \tag{3.5}$$

which is finite and positive (see (B.4)). This proves that $|\tau|^2 = \langle|\tau|^2\rangle$ is indeed a minimum at $\rho_\pm = 0$. For $\langle|\sigma|^2\rangle \ll 1$ we have, using (B.8),

$$V(0, \tau) \sim |\tau - \langle\tau\rangle|^2 \frac{N\langle|\tau|^2\rangle}{8\pi^2} \ln \frac{\Lambda_L^2}{\langle|\tau|^2\rangle}.$$

Finally, we switch the fields ρ back on and prove that $|\tau|^2 = \langle|\tau|^2\rangle$, $\rho_\pm = 0$ remains a minimum. The first derivative of $V(\rho,\tau)$ with respect to ρ, calculated at $\rho = 0$, must necessarily vanish, since ρ is CPT odd. The same is true for the second derivatives with respect to ρ and τ. On the other hand, the second derivative of $v(\rho,\tau)$ with respect to ρ, calculated at $\rho = 0$, is finite and g-independent (and actually, "small"). Precisely, we find, using (B.10),

$$\left.\frac{\partial^2 v}{\partial \rho^2}\right|_{\mathrm{min}} = -4N\langle|\tau|^2\rangle v''(\langle|\sigma|^2\rangle) < 0. \tag{3.6}$$

Although negative, this is a finite quantity, independent of g_-, so it can always be beaten by $V_2(M)$, choosing the coupling g_- sufficiently small. Under this assumption, $|\tau|^2 = \langle|\tau|^2\rangle$, $\rho_\pm = 0$ does remain a minimum after switching $\rho_\pm$ on.

Using (B.8) again, the effective potential around the minimum reads for $\langle|\tau|^2\rangle \ll \Lambda_L^2$,

$$V(\rho,\tau) \sim \frac{\Lambda_L^2}{2g_+^2}\rho_+^2 + \frac{\Lambda_L^2}{2g_-^2}\rho_-^2\left(1 - \frac{g_-^2 N}{2\pi^2}\frac{\langle|\tau|^2\rangle}{\Lambda_L^2}\ln\frac{\Lambda_L^2}{\langle|\tau|^2\rangle}\right) + \frac{\Lambda_L^2}{g_{+-}^2}\rho_+\rho_- + |\tau - \langle\tau\rangle|^2\frac{N\langle|\tau|^2\rangle}{8\pi^2}\ln\frac{\Lambda_L^2}{\langle|\tau|^2\rangle}. \tag{3.7}$$

We see that the coefficient of ρ_-^2 receives a very small correction, of the order 10^{-23}, even taking $g_L^2 N$ and $g_-^2 N$ of order 1. The Lorentz invariant minimum could be spoiled only if $g_-^2 N$ had an inordinate value.

Before proceeding, a comment is in order. We have so far worked in the large N expansion, concentrating on the leading order. In the case of the Standard Model, we are going to expand for large number of colors N_c [11]. The reason is that, because of the intrinsically non-perturbative nature of minima such as the one encoded in (3.4), we have control on them only in an expansion of this type. For example, (3.4) implies, using (B.9),

$$-\frac{1}{2\lambda^2 N} = v'(\langle|\sigma|^2\rangle) = -\frac{1}{4\pi^2}\ln\mu + \cdots .$$

The left-hand side of this expression is singular in the ordinary perturbative expansion ($\lambda \ll 1$), but regular in the large N expansion ($\lambda N^2 \sim 1$). At finite N, higher order corrections contain arbitrarily high powers of $\lambda^2 \ln\mu$ ("leading logs"), plus powers of $\lambda^2\ln\mu$ multiplied by one extra factor of λ^2 ("next-to-leading logs"), and so on. The resummation of leading logs involves only one-loop results, and can be easily done. Useful references for such kind of resummations applied to the Coleman-Weinberg mechanism are for example [17, 18]. However, it is known [17] that in general the so-corrected potential does not exhibit the nice features of the one calculated in the large N expansion. Most of the times the minimum suggested by a one-loop truncation (which cannot be trusted unless it is combined with a large N expansion or a dimensional transmutation [19]) is spoiled by the resummation, depending on the model. Although we can generically expect that the exact potential will have a non-trivial minimum in most theories, here we want to have some explicit control on the vacuum, for example check that it preserves Lorentz symmetry. At present we can answer our questions only in the large N expansion.

4 Masses and bound states in scalarless model

In this section we show how the masses of quarks and gauge bosons, as well as composite Higgs bosons, emerge in the scalarless model (2.4). We start with the t-b model

$$\mathcal{L}_q = \sum_{I=1}^{N_c} \bar{\psi}_I \left(\Gamma^\mu i \left(\hat{\partial}_\mu + \bar{\partial}_\mu - \bar{\partial}_\mu \frac{\bar{\partial}^2}{\Lambda_L^2} \right) - M \right) \psi_I - V_2(M), \tag{4.1}$$

where

$$M = \begin{pmatrix} \tau & \rho_R \\ \rho_L & \tau^\dagger \end{pmatrix}, \quad \psi = \begin{pmatrix} Q_L^i \\ Q_R^k \end{pmatrix},$$

$Q = (t, b)$, $i, j, \ldots$ are indices of $SU(2)_L$ or $SU(2)_R$, depending on the case, $(\Gamma^\mu)^{ij}_{\alpha\beta} = \gamma^\mu_{\alpha\beta}\delta^{ij}$, τ, ρ_R and ρ_L are 2×2 matrices of fields, and ρ_R, ρ_L are Hermitian. The most general $SU(2)_L \times U(1)_Y \times U(1)_B \times U(1)_A$-invariant quadratic potential is

$$\Lambda_L^{-2} V_2(M) = \mathrm{tr}[\tau\tau^\dagger C] + \frac{1}{2g_L^2}\mathrm{tr}[\rho_L^2] + \frac{1}{2g_L'^2}(\mathrm{tr}[\rho_L])^2 + g_R^{kl}\mathrm{tr}[\rho_L]\rho_R^{kl} + \frac{1}{2} g_{RR}^{klmn} \rho_R^{kl}\rho_R^{mn}, \tag{4.2}$$

where C_{ij}, g_L, g_L', g_R^{kl} and g_{RR}^{klmn} are constants, C_{ij} and g_R^{kl} are diagonal and g_{RR}^{klmn} are non-vanishing only for $k = l, m = n$ and $k = n, l = m$. Although we do not assume any "custodial" $SU(2)_R$-invariance, which is indeed violated by $V_2(M)$, note that $\mathcal{L}_q + V_2(M)$ is invariant under $U(2)_L \times U(2)_R$ (transforming τ, ρ_R and ρ_L appropriately), therefore so is the one-loop correction to the effective potential.

We want to prove that

$$\tau = \begin{pmatrix} m_t & 0 \\ 0 & m_b \end{pmatrix} \equiv \tau_0, \qquad \rho_L = \rho_R = 0. \tag{4.3}$$

is a minimum, where $m_t > m_b$ can be identified with the top and bottom masses, respectively, and are related to the C entries (see below). The vacuum (4.3) breaks $SU(2)_L \times U(1)_Y \times U(1)_B \times U(1)_A$ to $U(1)_Q \times U(1)_B$.

Again, we first work at $\rho_L = \rho_R = 0$, find the tentative minimum of the effective potential and later prove that it remains a minimum when ρ_L and ρ_R are switched back on. At $\rho_R = \rho_L = 0$ the determinant of $-\gamma^\mu p'_\mu + M$ is a Lorentz invariant polynomial of the four-vector p'_μ and can be easily calculated first at $\bar{p}'_\mu = 0$, then replacing $\hat{p}^2$ with $p'_\mu p'^\mu$. We find

$$\det(-\gamma^\mu p'_\mu + M) = \left[(p'^2)^2 - p'^2 t_1 + \frac{1}{2}(t_1^2 - t_2) \right]^2, \qquad t_1 = \mathrm{tr}[\tau\tau^\dagger], \qquad t_2 = \mathrm{tr}[\tau\tau^\dagger\tau\tau^\dagger].$$

It is easy to prove the inequalities

$$t_1^2 \geqslant t_2 \geqslant \frac{1}{2} t_1^2.$$

The second inequality follows from $\mathrm{tr}[\mathrm{N}^2] \geqslant \mathrm{tr}[\mathrm{N}]^2/2$, where N is any Hermitian 2×2 matrix. We find the one-loop effective potential

$$V(\tau) = \Lambda_L^2 \mathrm{tr}[\tau\tau^\dagger C] + 2N_c\Lambda_L^4 \left(v(r_+) + v(r_-)\right),$$

where

$$r_\pm = \frac{1}{2\Lambda_L^2}\left(t_1 \pm \sqrt{2t_2 - t_1^2}\right) \geqslant 0,$$

and $v(r)$ is defined in formula (B.1). The minimum (4.3) has $r_+ = m_t^2/\Lambda_L^2 \equiv r_t$, $r_- = m_b^2/\Lambda_L^2 \equiv r_b$. Taylor-expanding $v(r)$ around (4.3), we can write, neglecting additive constants,

$$\begin{aligned} V(\tau) &= \Lambda_L^2 V_a(\tau) + \Lambda_L^4 V_b(\tau), \\ V_a(\tau) &= \mathrm{tr}[\tau\tau^\dagger C] + 2\Lambda_L^2 N_c \left(r_+ v'(r_t) + r_- v'(r_b)\right), \\ V_b(\tau) &= N_c(r_+ - r_t)^2 v''(r_t) + N_c(r_- - r_b)^2 v''(r_b). \end{aligned}$$

Setting the first derivatives of $V_a(\tau)$ to zero gives the gap equations

$$c_t = -2N_c v'(r_t), \qquad c_b = -2N_c v'(r_b), \tag{4.4}$$

where $C = \mathrm{diag}(c_t, c_b)$. Since c_t and c_b are free parameters, they can always be chosen so that the gap equations have solutions. Now, (4.3) is a minimum of $V_b(\tau)$, because $v''(r) > 0$. Moreover, expanding $V_a(\tau)$ around (4.3), we find

$$V_a(\tau) \sim 2N_c \frac{v'(r_t) - v'(r_b)}{m_t^2 - m_b^2} |m_t\tau_{21} + m_b\bar\tau_{12}|^2 \geqslant 0.$$

The coefficient is positive because of (B.5). Thus, (4.3) is a minimum of the effective potential at $\rho_L = \rho_R = 0$. There are of course flat directions $m_t\delta\tau_{21} + m_b\delta\bar\tau_{12} = 0$ corresponding to the charged Goldstone bosons (see below).

When ρ_L and ρ_R are switched on, we can proceed as in the previous section. The first derivatives of the effective potential around (4.3) still vanish, by CPT invariance, as well as the second derivatives with respect to one τ-entry and one ρ-entry. On the other hand, the second derivatives with respect to ρ-entries are finite, and can always be made positive choosing the arbitrary constants g_L, g_L', g_R^{kl} and g_{RR}^{klmn} in (4.2) appropriately. The reason why the second ρ-derivatives have no UV divergences is that in the corresponding integrals $\gamma^\mu p'_\mu$ is sandwiched between two γ^0's. Then, using (B.11) with $k = 2$, we have, in Euclidean space,

$$\begin{aligned} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\gamma^0\gamma^\mu p'_\mu \gamma^0\gamma^\nu p'_\nu}{(p'^2 + m_1^2)(p'^2 + m_2^2)} &= \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\hat p^2 - \bar p'^2 - 2\hat p\gamma^0\gamma^{\bar\nu}\bar p'_{\bar\nu}}{(p'^2 + m_1^2)(p'^2 + m_2^2)} \\ &= \int_0^1 \mathrm{d}x \left[m_1^2 x + m_2^2(1-x)\right] \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{(p'^2 + m_1^2 x + m_2^2(1-x))^2} < \infty. \end{aligned} \tag{4.5}$$

Thus, we have proved that (4.3) is indeed a minimum of the effective potential. Because of the arbitrariness of c_t and c_b, the top and bottom masses cannot be predicted. However, they can be related to other known quantities.

Bound states We write $\tau = \tau_0 + \eta$. The contributions $\Gamma_{\eta\eta}$ and $\Gamma_{\rho\rho}$ to the generating functional Γ that are quadratic in η and ρ give the dynamically generated propagators of such fields, from which we can read the bound states. We expect four massless scalars in the τ-sector, which are the Goldstone bosons associated with the broken generators of $SU(2)_L \times U(1)_Y$ and $U(1)_A$. Using the solutions (4.4) to the gap equations, every Λ-divergences cancel out and both $\Gamma_{\eta\eta}$ and $\Gamma_{\rho\rho}$ are given by finite integrals. If we are interested in the low-energy limit with respect to Λ_L, then we can view our (finite, but Lorentz violating) integrals as usual Lorentz invariant integrals regulated by the cut-off Λ_L. Such a cut-off is Lorentz violating, but invariant under translations, spatial rotations and CPT. Using power counting, it can be easily checked that the Λ_L-divergences are linear or logarithmic. However, linear divergences are absent by CPT and rotational invariance. On the other hand, logarithmic divergences do not depend on the regulator. In particular, they are Lorentz invariant, as are the finite parts. Thus, to study the large Λ_L limit we can regulate our integrals in the most convenient way, e.g. integrating suitable Lorentz invariant integrands over momenta $p \leqslant \Lambda_L$. Specifically, we can perform the replacement

$$\int \frac{\mathrm{d}^4 p}{(2\pi)^4} f(p, \Lambda_L) \to \int^{\Lambda_L} \frac{\mathrm{d}^4 p}{(2\pi)^4} f(p, \infty)$$

and use a symmetric integration to kill linear divergences.

We first calculate the leading contributions to $\Gamma_{\eta\eta}$. Using the tricks mentioned above, we find, in momentum space,

$$\Gamma_{\eta\eta} = N_c \sum_{i,j=t,b} \left\{ 2\eta_{ij}(p)\bar{\eta}_{ij}(-p)(p^2 f'_{ij} - m_j^2 f_{ij}) - m_i m_j f_{ij} \left[\eta_{ij}(p)\eta_{ji}(-p) + \bar{\eta}_{ij}(p)\bar{\eta}_{ji}(-p) \right] \right\},$$

the functions $f_{ij}(p^2)$ and $f'_{ij}(p^2)$ being defined in (B.12). Studying the poles of the effective η-propagators we find:

1) two neutral massive bound states of squared masses

$$2m_t^2 \frac{f_{tt}}{f'_{tt}} = 4m_t^2, \qquad 2m_b^2 \frac{f_{bb}}{f'_{bb}} = 4m_b^2;$$

2) two neutral and two charged massless states, which are the Goldstone bosons;

3) two charged massive bound states of squared masses

$$f_{tb} \left(\frac{m_t^2}{f'_{bt}} + \frac{m_b^2}{f'_{tb}} \right) \sim 2m_t^2.$$

We have used (B.13) for the approximate values.

The ρ–self-energies do not give bound states at low energies. Using tricks similar to those leading to (4.5) we find

$$\Gamma_{\rho\rho} = -\frac{\Lambda_L^2}{2g_L^2}\mathrm{tr}[\rho_L^2] - \frac{\Lambda_L^2}{2g_L'^2}(\mathrm{tr}[\rho_L])^2 - \Lambda_L^2 g_R^{kl}\mathrm{tr}[\rho_L]\rho_R^{kl} - \frac{\Lambda_L^2}{2}g_{RR}^{klmn}\rho_R^{kl}\rho_R^{mn} +$$
$$+N_c\sum_{i,j=t,b}\left[\left(\rho_L^{ij}\rho_L^{ji}+\rho_R^{ij}\rho_R^{ji}\right)\left(2\mathbf{p}^2 f_{ij}'' + m_i^2 f_{ij}' + m_j^2 f_{ji}'\right) - 2\rho_L^{ij}\rho_R^{ji}m_i m_j f_{ij}\right].$$

The spatial squared momentum $\mathbf{p}^2$ appears instead of p^2, which signals the absence of bound states. Moreover, no gap equation reabsorbs the $\mathcal{O}(\Lambda_L^2)$-terms of $V_2(M)$. This means that all corrections to $V_2(M)$ are negligible at low energies, so even if some ρ-bound state existed, it would have masses of the order $\sim \Lambda_L$. We conclude that the Lorentz violation does not reverberate down to low energies.

Masses of the gauge bosons As usual, when gauge interactions are switched on the three Goldstone bosons $\phi^\pm$ and ϕ^0 associated with the broken generators of $SU(2)_L\times U(1)_Y$ are "eaten" by the gauge bosons $W^\pm$ and Z, which become massive. To see how that happens in our case, we proceed as follows. We first calculate the leading contributions to the W-η, Z-η and A-η two-point functions. They are given by the diagrams constructed with one fermion loop, one vertex $\bar{\psi}M\psi$ and one vertex

$$g(W_\mu^+ J_+^\mu + W_\mu^- J_-^\mu) + \tilde{g}Z_\mu J_Z^\mu + eA_\mu J_{\mathrm{em}}^\mu, \tag{4.6}$$

where $\tilde{g}=\sqrt{g^2+g'^2}$. We find

$$\Gamma_{A\eta} = -gN_c(\partial^\mu\phi^-)W_\mu^+ - gN_c(\partial^\mu\phi^+)W_\mu^- - N_c\tilde{g}(\partial^\mu\phi^0)Z_\mu, \tag{4.7}$$

where

$$\phi^+ = i\sqrt{2}(m_t f_{tb}'\eta_{tb} - m_b f_{bt}'\bar{\eta}_{bt}), \qquad \phi^0 = \frac{i}{2}(m_t f_{tt}(\eta_{tt}-\bar{\eta}_{tt}) - m_b f_{bb}(\eta_{bb}-\bar{\eta}_{bb})),$$

and $\phi^- = \bar{\phi}^+$. This result identifies the bosons $\phi^\pm$ and ϕ^0. Then we search for constants f_W and f_Z such that these Goldstone bosons disappear from the difference $\Gamma_{\eta\eta}' = \Gamma_{\eta\eta} - \Gamma_{\phi\phi}$, where

$$\Gamma_{\phi\phi} = \frac{N_c}{f_W}(\partial_\mu\phi^+)(\partial^\mu\phi^-) + \frac{N_c}{2f_Z}(\partial_\mu\phi^0)(\partial^\mu\phi^0).$$

We find

$$f_W = m_t^2 f_{tb}' + m_b^2 f_{bt}', \qquad f_Z = \frac{1}{2}(m_t^2 f_{tt} + m_b^2 f_{bb}).$$

Next, we determine the linearized gauge transformations of the Goldstone bosons, demanding that $\Gamma_{\phi\phi}+\Gamma_{A\eta}$ be invariant up to $\mathcal{O}(A)$, where A denotes a generic gauge field. We find

$$\delta W_\mu^\pm = \partial_\mu C^\pm, \qquad \delta Z_\mu = \partial_\mu C^0, \qquad \delta\phi^\pm = gf_W C^\pm, \qquad \delta\phi^0 = \tilde{g}f_Z C^0. \tag{4.8}$$

Finally, we add A^2-terms to have gauge invariance at the linearized level. In total, the relevant quadratic contributions $\Delta_2\Gamma$ to the Γ functional read

$$\Delta_2\Gamma = \Gamma'_{\eta\eta} + \frac{N_c}{f_W}(\partial_\mu\phi^+ - gf_W W^+_\mu)(\partial^\mu\phi^- - gf_W W^{\mu-}) + \frac{N_c}{2f_Z}(\partial_\mu\phi^0 - \tilde{g}f_Z Z_\mu)(\partial^\mu\phi^0 - \tilde{g}f_Z Z^\mu).$$

Choosing the unitary gauge-fixing $\phi^\pm = \phi^0 = 0$, we can read the gauge-boson squared masses

$$m_W^2 = N_c g^2 f_W \sim \frac{N_c g^2}{32\pi^2} m_t^2 \ln\frac{\Lambda_L^2}{m_t^2}, \qquad m_Z^2 = N_c\tilde{g}^2 f_Z \sim \frac{\tilde{g}^2}{g^2} m_W^2. \tag{4.9}$$

The first formula can be converted into a relation between the Fermi constant and the mass of the top quark, namely

$$\frac{1}{G_F} = \frac{N_c m_t^2}{4\pi^2\sqrt{2}} \ln\frac{\Lambda_L^2}{m_t^2}. \tag{4.10}$$

Using our estimated value $\Lambda_L = 10^{14}$GeV, we find $m_t = 171.6$GeV. This “too-good” agreement has no simple explanation, as far as we know, also taking into account that from a quantitative point of view our rough large N_c approximation contains a good 50% margin of error[1].

The quantities f_W and f_Z are related in a non-straightforward way. We find

$$\rho \equiv \frac{\tilde{g}^2 m_W^2}{g^2 m_Z^2} = \frac{f_W}{f_Z} = 2\frac{m_t^2 f'_{tb} + m_b^2 f'_{bt}}{m_t^2 f_{tt} + m_b^2 f_{bb}}.$$

The explicit computation for $\Lambda_L \gg m_t \gg m_b$ shows that the values of f_W and f_Z are actually close. Indeed, the relation $\rho \sim 1$ is fulfilled not only when there is an approximate custodial symmetry (which would imply approximately equal quark masses), but also in the opposite situation, namely when one quark mass is much larger than the other one. In this case the deviations from $\rho = 1$ are, at low energies, just those predicted by the usual Standard Model results, as already noted in [11].

The vertices of the effective action can be derived calculating diagrams with one fermion loop and more external A and η legs. Once the gap equations (4.4) are used, every other contribution is convergent and unambiguously determined. In principle, using the large N_c expansion we can calculate the effective action with the desired precision.

We have shown that a forth Goldstone boson is associated with the breaking of $U(1)_A$. This boson becomes massive because of the $U(1)_A$-anomaly. On the other hand, quark masses, which are not due to an explicit symmetry breaking, do not contribute to the mass of this boson.

[1] Call “1” the leading order of the large N_c expansion. Resumming powers of $1/3$ from 1 to infinity we get $1/2$, so, generically speaking, a “1” could be anything between $1/2$ and $3/2$.

Composite Higgs bosons So far, we have assumed that all η-entries were independent, which amounts to have two independent composite Higgs doublets. The situation with a single composite Higgs doublet can be retrieved choosing

$$\tau = \frac{m_t}{v}\sqrt{2}\begin{pmatrix} H_2 & -H_1 \\ \kappa\bar{H}_1 & \kappa\bar{H}_2 \end{pmatrix}, \qquad \kappa = \frac{m_b}{m_t}, \tag{4.11}$$

where $v/\sqrt{2}$ is the Higgs vev. Substituting (4.11) in $\Gamma_{\tau\tau}$ we find the three Goldstone bosons associated with the breaking of $SU(2)_L \times U(1)_Y$, plus one neutral massive scalar with squared mass

$$4\frac{m_t^4 f_{tt} + m_b^4 f_{bb}}{m_t^2 f_{tt} + m_b^2 f_{bb}} \sim 4m_t^2,$$

to be identified with the composite Higgs scalar. Its mass is $2m_t$, as originally suggested by Nambu [9]. This value is far from the expected Higgs mass, but taking into account of our 50% margin of error, the final, exact formula could still give $m_H \sim m_t$, which would be compatible with present expectations.

More generally, we can introduce three doublets for each family (if there exist no right-handed neutrinos): two for the quarks and one for the leptons. Actually, to allow for mixing among families we can just take the Yukawa vertices $\mathcal{L}_Y$ and promote every product YH to an independent field τ:

$$\mathcal{L}_{\tau\psi\psi} + \mathcal{L}_{\tau\tau} = -\sum_{a,b=1}^{3}\left(\bar{L}^{ai}\tau_\ell^{ab,i}\ell_R^b + \bar{Q}_R^{ai}\tau_q^{ab,ij}Q_L^{bj} + \text{h.c.}\right) + V_{\ell q}(\tau_\ell, \tau_q),$$

where V_2 is the most general quadratic polynomial compatible with the symmetries of the theory. To generate Majorana masses for neutrinos we need extra four fermion vertices encoded in [20]

$$\mathcal{L}'_{\tau LL} + \mathcal{L}'_{\tau\tau} = -\sum_{a,b=1}^{3}(\bar{L}^c)^{ai}\varepsilon^{ij}\tau_L^{ab,jk}L^{bk} + \text{h.c.} + V'_{\ell\ell}(\tau_L).$$

We can add analogous Lorentz violating terms containing fields ρ, but we know that we can neglect such terms both for the search of the minimum of the potential and to derive the induced low-energy effective action.

Note that in the lepton sector we have no analogue of the large N_c expansion to justify our arguments. We still expect, however, that the minimum exists and the dynamical symmetry breaking takes place.

5 Goldstone theorem in Lorentz violating theories

In the previous section we have used the large N_c expansion to prove the dynamical symmetry breaking and study bound states, among which the Goldstone bosons. However, the Goldstone theorem is an exact result, that can be derived without making use of expansions or approximations. In this section we show how to generalize it to Lorentz violating theories. We assume that the theory becomes Lorentz invariant at large distances. We do not need to assume that the scale of symmetry breaking is much smaller than the scale of Lorentz violation.

Let $\omega(x)$ denote a generic field operator and $J_\mu = (J^{\hat{\mu}}, J^{\bar{\mu}})$ the conserved current associated with a continuous global symmetry. Current conservation $\partial_\mu J^\mu = \partial_{\hat{\mu}} J^{\hat{\mu}} + \partial_{\bar{\mu}} J^{\bar{\mu}} = 0$ still implies

$$\frac{\mathrm{d}}{\mathrm{d}t}[Q(t), \omega(0)] = 0, \qquad Q(t) = \int \mathrm{d}^3 x \, J^0(t, \mathbf{x}). \tag{5.1}$$

Indeed, the last term of the equality

$$0 = \int \mathrm{d}^3 x [\partial_\mu J^\mu(x), \omega(0)] = \frac{\mathrm{d}}{\mathrm{d}t}[Q(t), \omega(0)] + \int_{S^\infty} \mathrm{d}\mathbf{s} \cdot [\mathbf{J}(t, \mathbf{x}), \omega(0)]$$

is equal to zero, because for *large* space-like separations the commutator $[\mathbf{J}(t, \mathbf{x}), \omega(0)]$ vanishes. This property holds also in our Lorentz violating theories, since they become Lorentz invariant at large distances, if the vacuum is Lorentz invariant. For generic space-like separations a commutator does not need to vanish.

The symmetry is spontaneously broken if the commutator $[Q(t), \omega(0)]$ has a non-vanishing expectation value u. Then, inserting a complete set of intermediate states $|n\rangle$ and using translational invariance we have

$$u = \sum_n (2\pi)^3 \delta^{(3)}(\mathbf{p}_n) \left[\mathrm{e}^{-iE_n t} \langle 0|J_0(0)|n\rangle \langle n|\omega(0)|0\rangle - \mathrm{e}^{iE_n t} \langle 0|\omega(0)|n\rangle \langle n|J_0(0)|0\rangle \right].$$

Since u is constant, because of (5.1), there must exist a state $|\bar{n}\rangle$ such that $E_{\bar{n}} = \mathbf{p}_{\bar{n}} = 0$ and

$$\langle 0|J_0(0)|\bar{n}\rangle \neq 0, \qquad \langle \bar{n}|\omega(0)|0\rangle \neq 0.$$

Now, consider the case of $SU(2)_L \times U(1)_Y$ spontaneously broken to $U(1)_Q$. We have composite fields $\omega^{\pm,0}$ and Goldstone bosons $\phi^{\pm,0}$, such that

$$\langle 0|\omega^{\pm,0}(0)|\phi^{\pm,0}\rangle \neq 0, \qquad \langle 0|J_0^{\pm,0}(0)|\phi^{\pm,0}\rangle \neq 0, \tag{5.2}$$

where $J_\mu^{\pm,0}$ are the currents associated with the broken generators. The form of $\langle 0|J_0^{\pm,0}(0)|\phi^{\pm,0}\rangle$ is no longer constrained by Lorentz invariance. Instead, we have, in momentum space,

$$\langle 0|J_\mu^{\pm,0}(0)|\phi^{\pm,0}(p)\rangle = i f_{\pm,0}(\hat{p}_\mu + \zeta_{\pm,0}\bar{p}_\mu), \tag{5.3}$$

where $f_{\pm,0}$ and $\zeta_{\pm,0}$ may depend on $\bar{p}^2$. Current conservation implies $\hat{p}^2 - \zeta_{\pm,0}\bar{p}^2 = 0$ on shell, which determines the ϕ-kinetic terms. It also eliminates the $\hat{p}^2$-dependence and ensures that $\zeta_{\pm,0}$ are real. The conservation of electric charge implies $f_+ = f_-^*$, $\zeta_+ = \zeta_- \equiv \zeta$ and that f_0 is also real. The effective lagrangian incorporating such pieces of information reads in the quadratic approximation

$$\mathcal{L}_{\rm eff} = (\hat{\partial}_\mu \phi^+ - g\hat{W}^+_\mu f_+)(\hat{\partial}_\mu \phi^- - g f_- \hat{W}^-_\mu) - (\bar{\partial}_\mu \phi^+ - g\bar{W}^+_\mu f_+)\zeta(\bar{\partial}_\mu \phi^- - g f_- \bar{W}^-_\mu)$$
$$+\frac{1}{2}(\hat{\partial}_\mu \phi^0 - \tilde{g}\hat{Z}_\mu f_0)^2 - \frac{1}{2}(\bar{\partial}_\mu \phi^0 - \tilde{g}\bar{Z}_\mu f_0)\zeta_0(\bar{\partial}_\mu \phi^0 - \tilde{g} f_0 \bar{Z}_\mu).$$

We have the linearized gauge symmetry

$$\delta W^\pm_\mu = \partial_\mu C^\pm, \qquad \delta Z_\mu = \partial_\mu C^0, \qquad \delta\phi^\pm = g f_\pm C^\pm, \qquad \delta\phi^0 = \tilde{g} f_0 C^0. \tag{5.4}$$

Choosing the gauge-fixing $\phi^{\pm,0} = 0$, we find the W and Z mass terms

$$\mathcal{L}_{\rm m} = g^2(\hat{W}^+_\mu f_+ f_- \hat{W}^-_\mu - \bar{W}^+_\mu f_+ \zeta f_- \bar{W}^-_\mu) + \frac{1}{2}\tilde{g}^2(\hat{Z}_\mu f_0^2 \hat{Z}_\mu - \bar{Z}_\mu f_0 \zeta_0 f_0 \bar{Z}_\mu),$$

as in the Proca version of Lorentz violating gauge theories [5].

Observe that at this level, our construction is unable to relate the W and Z masses. At low energies, when Lorentz symmetry is restored ($\zeta = \zeta_0 = 1$) and $f_\pm$, f_0 can be taken to be constant, we have

$$\mathcal{L}_{\rm m} = g^2 f_+ f_- W^+_\mu W^{-\mu} + \frac{1}{2}\tilde{g}^2 f_0^2 Z_\mu Z^\mu, \tag{5.5}$$

so on general grounds we are unable to predict $\rho = 1$, actually

$$\rho = \frac{f_+ f_-}{f_0^2}, \tag{5.6}$$

In the Standard Model, $\rho = 1$ follows from $f_+ f_- = f_0^2 = v^2/4$. In the model of the previous section, it follows from explicit calculation in the large N_c expansion, for quarks with very different masses.

When there exists a symmetry $SU(2)_R$, for example a "custodial" symmetry [21], the right-handed quarks can be collected into $SU(2)_R$ doublets, and the quark sector becomes $SU(2)_L \times SU(2)_R$-invariant. If the vacuum preserves $SU(2)_{L+R}$, (5.3) is modified to

$$\langle 0|J^{\pm,0}_\mu(0)|\phi^{\pm,0}(p)\rangle = i\tilde{f}(\hat{p}_\mu + \tilde{\zeta}\bar{p}_\mu), \tag{5.7}$$

with unique $\tilde{f}$ and $\tilde{\zeta}$ for $W^\pm$ and Z. Then $\rho = 1$ follows (at the tree level).

6 Conclusions

The Standard-Extended Model (2.1) proposed in ref. [7], and its variants, such as (2.3), contain interactions that are non-renormalizable by ordinary power counting, but are renormalizable by weighted power counting, thanks to the high-energy Lorentz violation. An interesting feature of those models is that they become very simple at high energies ($\gtrsim 10^{14}$GeV), because all gauge and Higgs interactions, being super-renormalizable, disappear. There survives a four fermion model in two weighted dimensions, which admits a dynamical symmetry breaking. In spite of the fact that such a four fermion model is Lorentz violating, the dynamically generated vacuum and the low energy effective action are Lorentz invariant. We have therefore focused our attention on the scalarless variant (2.4) of the model of [7], which contains no elementary scalar field. In the large N_c expansion we have seen that the dynamical symmetry breaking generates composite massive Higgs bosons and gives masses to fermions and gauge bosons. The model is predictive, in the sense that it does not contain the ambiguities of previous approaches, which relied on the non-renormalizable Nambu–Jona-Lasinio mechanism, and is candidate to explain the observed low energy physics. The leading order of the large N_c expansion, with gauge interactions switched off, does not allow us to make very precise quantitative predictions, although the relation (4.10) between the Fermi constant and the top mass turns out to be mysteriously right.

A step forward towards more precise predictions is to include the effects of the RG flow from energies $\sim m_t$ to Λ_L, and study the condition of compositeness at energies $\sim \Lambda_L$ [11]. However, in our Lorentz violating theories the RG flow is considerably different from the usual one: it coincides with the usual one at energies $\sim m_t$, since the low energy theory (with composite Higgs bosons included) is renormalizable by ordinary power counting; on the other hand, it changes completely as we move to energies $\sim \Lambda_L$, because there, gauge interactions do not run. Work is in progress in this direction.

What we have done in this paper is not only to describe low energy effects of high energy Lorentz violations, but also show that they can be consistent with low energy Lorentz invariance. This fact was not obvious a priori.

Our mechanism can of course take place also in the Higgsed models (2.1) and (2.3), if the four fermion vertices are chosen appropriately. There, its effects sum to those of the elementary Higgs doublet. It can also be applied to Standard Model extensions that contain new types of fermions, interacting by four fermion vertices, such as those considered in ref.s [22].

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Appendix A: Simplification of the pure gauge sector

In this appendix we describe a weight reassignment that is useful to simplify the gauge sector.

Indeed, the quadratic gauge field lagrangian $\mathcal{L}_Q$ generates an involved propagator [5]. Recall that in Lorentz violating gauge theories the BRST symmetry is the same as usual,

$$\begin{aligned} &sA_\mu^a = D_\mu^{ab}C^b = \partial_\mu C^a + gf^{abc}A_\mu^b C^c, \qquad sC^a = -\frac{g}{2}f^{abc}C^bC^c, \\ &s\bar{C}^a = B^a, \qquad sB^a = 0, \qquad s\psi^i = -gT_{ij}^a C^a\psi^j, \end{aligned}$$

etc., where B^a are Lagrange multipliers for the gauge-fixing. The most convenient gauge-fixing is

$$\mathcal{L}_{\mathrm{gf}} = s\Psi, \qquad \Psi = \bar{C}^a\left(-\frac{\lambda}{2}B^a + \mathcal{G}^a\right), \qquad \mathcal{G}^a \equiv \hat{\partial}\cdot\hat{A}^a + \zeta\left(\bar{\upsilon}\right)\bar{\partial}\cdot\bar{A}^a, \tag{A.1}$$

where λ is a dimensionless, weightless constant, $\bar{\upsilon} \equiv -\bar{\partial}^2/\Lambda_L^2$ and ζ is a polynomial of degree $n-1$. The total gauge-fixed action is

$$\mathcal{S} = \int \mathrm{d}^d x\left(\mathcal{L}_Q + \mathcal{L}_I + \mathcal{L}_{\mathrm{gf}}\right), \tag{A.2}$$

where $\mathcal{L}_I$ collects the terms that are at least cubic in the field strength.

The gauge-field propagator can be worked out from the free subsector of (A.2), after integrating B^a out, which amounts to add $(\mathcal{G}^a)^2/(2\lambda)$ to the quadratic lagrangian $\mathcal{L}_Q$[2]. We find, in Euclidean space,

$$\langle A(k)\ A(-k)\rangle = \begin{pmatrix} \langle\hat{A}\hat{A}\rangle & \langle\hat{A}\bar{A}\rangle \\ \langle\bar{A}\hat{A}\rangle & \langle\bar{A}\bar{A}\rangle \end{pmatrix} = \begin{pmatrix} u & r\hat{k}\bar{k} \\ r\bar{k}\hat{k} & v\bar{\delta} + t\bar{k}\bar{k} \end{pmatrix}, \tag{A.3}$$

where

$$u = \frac{\lambda\hat{k}^2 + \frac{\zeta^2}{\eta}\bar{k}^2}{D^2(1,\zeta)}, \qquad r = \frac{\lambda - \frac{\zeta}{\eta}}{D^2(1,\zeta)}, \qquad v = \frac{1}{D(\eta,\tau)}, \qquad t = \frac{\left(\eta\lambda + \frac{\tau}{\eta} - 2\zeta\right)\hat{k}^2 + \left(\tau\lambda - \zeta^2\right)\bar{k}^2}{D(\eta,\tau)D^2(1,\zeta)},$$

and $D(x,y) \equiv x\hat{k}^2 + y\bar{k}^2$. Now η, τ and ζ, as well as x and y, are functions of $\bar{k}^2/\Lambda_L^2$. The ghost propagator is $1/D(1,\zeta)$.

If $\eta \neq 1$ the propagator is not regular [5, 6], because for $\hat{k}$ large $\langle\bar{A}\bar{A}\rangle$ behaves like $1/(\eta\hat{k}^2)$ and η depends only on $\bar{k}^2$. Thus the $\hat{k}$-integrals may contain "spurious" subdivergences. A separate power-counting analysis is necessary to show that such subdivergences are absent under certain conditions [6], fulfilled by the model (2.1).

An alternative solution, which avoids the problem from the start, is to set $\eta = 1$, in which case the propagator clearly becomes regular. We show that this choice is consistent with renormalization, if combined with a rearrangement of the weight assignments and other choices, such as

[2]With respect to the $\mathcal{L}_Q$ of (2.2), the most general quadratic gauge field lagrangian [5, 6] can contain another polynomial $\xi(\Upsilon)$. Here we set it to zero, since the weight reassignment excludes it anyway from the theory with simplified gauge sector.

restricting τ to be a polynomial of degree $n-1$ (which we then denote by τ'). The new quadratic gauge field lagrangian reads

$$\mathcal{L}'_Q = \frac{1}{2} F^2_{\hat{\mu}\bar{\nu}} - \frac{1}{4} F_{\bar{\mu}\bar{\nu}} \tau'(\bar{\Upsilon}) F_{\bar{\mu}\bar{\nu}} \tag{A.4}$$

and admits a nice diagonal propagator

$$\begin{pmatrix} \langle \hat{A}\hat{A}\rangle & \langle \hat{A}\bar{A}\rangle \\ \langle \bar{A}\hat{A}\rangle & \langle \bar{A}\bar{A}\rangle \end{pmatrix} = \frac{1}{D(1,\tau')} \begin{pmatrix} \tau' & 0 \\ 0 & \bar{\delta} \end{pmatrix}, \tag{A.5}$$

in the "Feynman" gauge

$$\zeta = \lambda = \tau'. \tag{A.6}$$

After integrating B out, the propagator (A.5) is obtained adding

$$\frac{1}{2}(\mathcal{G}^a)\frac{1}{\tau'}(\mathcal{G}^a) \tag{A.7}$$

to $\mathcal{L}'_Q$. Note that (A.7) is non-local, because the gauge condition $\lambda = \tau'$ contained in (A.6) implies that the constant λ is replaced with a function of $\bar{k}^2/\Lambda_L^2$. This replacement is legitimate, since the action (A.2) is local before integrating B out, and B is non-propagating (see (A.1)).

The consistency of (A.4) is explained by a simple weight rearrangement, with gauge field components acquiring higher weights and the gauge coupling acquiring a lower weight, such that the product gA maintains the same weight. Denoting weights with square brakets, we have, by covariance, $[g\hat{A}] = [\hat{\partial}] = 1$ and $[g\bar{A}] = [\bar{\partial}] = 1/n$. The field strength is split into $\tilde{F}_{\mu\nu} \equiv F_{\hat{\mu}\bar{\nu}}$ and $\bar{F}_{\mu\nu} \equiv F_{\bar{\mu}\bar{\nu}}$. The kinetic lagrangian $\mathcal{L}'_Q$ contains $\tilde{F}^2$, so $\tilde{F}$ must have weight $\text{đ}/2$. Since $[\tilde{F}] = [\bar{\partial}] + [\hat{A}] = [\hat{\partial}] + [\bar{A}]$, we have

$$[\hat{A}] = \frac{\text{đ}}{2} - \frac{1}{n}, \qquad [\bar{A}] = \frac{\text{đ}}{2} - 1, \qquad [\tilde{F}] = \frac{\text{đ}}{2}, \qquad [\bar{F}] = \frac{\text{đ}}{2} - 1 + \frac{1}{n}. \tag{A.8}$$

The weight of the gauge coupling is

$$[g] = 1 + \frac{1}{n} - \frac{\text{đ}}{2}. \tag{A.9}$$

Observe that $[g] > 0$ in four dimensions, for $n > 1$, where gauge interactions are always super-renormalizable. We also find $[\zeta] = [\lambda] = [\tau'] = 2 - 2/n$, which implies that τ' must be of order $n-1$ and makes the gauge choice (A.6) renormalizable.

The quadratic terms of the ghost Lagrangian contain $\bar{C}\hat{\partial}^2 C$ and λB^2, which have weight đ, so we have the weight assignments

$$[C] = [\bar{C}] = \frac{\text{đ}}{2} - 1, \qquad [s] = \frac{1}{n}, \qquad [B] = \frac{\text{đ}}{2} - 1 + \frac{1}{n}. \tag{A.10}$$

In our case ($\bar{d}= 2$, $n = 3$) we have $[g] = 1/3$. By covariance, the coupling $\bar{g}$ attached to the scalar legs must satisfy $[\bar{g}] \leq [g]$ [6], so we lower $[\bar{g}]$ from $1/2$ to $1/3$. All other weights are unchanged. Then the most general lagrangian is (2.3) plus the extra terms

$$\begin{array}{l}\bar{g}^6 H^8, \quad \bar{g}^4 \bar{D}^2 H^6, \quad \bar{g}^2 \bar{D}^4 H^4, \quad g\bar{g}^2 \bar{D}^2 \bar{F} H^4, \quad g\bar{D}^4 \bar{F} H^2, \quad g^2 \bar{g}^2 \bar{F}^2 H^4, \quad g^2 \bar{D}^2 \bar{F}^2 H^2, \quad g^3 \bar{F}^3 H^2, \\ g^2 \bar{F}^4, \qquad g\bar{D}^2 \bar{F}^3, \qquad g\bar{\varepsilon}\tilde{F}\bar{D}^2 H^2, \qquad g^2 \bar{\varepsilon}\tilde{F}\bar{F} H^2, \qquad \bar{\varepsilon}\tilde{F}\bar{D}^2 \bar{F}, \qquad g\bar{\varepsilon}\bar{F}\tilde{F}\bar{F}, \qquad \bar{\varepsilon}\bar{F}\hat{D}\bar{D}\bar{F}, \\ g\bar{g}\bar{\psi}\psi\bar{F}H, \qquad \bar{g}\bar{\psi}\psi\bar{D}^2 H, \qquad \bar{g}^2 \bar{\psi}\psi\bar{D}H^2, \qquad \bar{g}^3 \bar{\psi}\psi H^3,\end{array} \tag{A.11}$$

and those obtained suppressing some fields and/or derivatives, where $\bar{\varepsilon}$ is the ε-tensor with three space indices. The extra terms (A.11) can be consistently dropped, because they are not generated back by renormalization.

The two models (2.1) and (2.3) correspond to the basic weight assignments $[s] = 1$ and $[s] = 1/n$, respectively. All intermediate situations $[s] = k/n$, $k = 1, \ldots n$ are actually allowed, with suitable weight reassignments. Observe that the construction of this Appendix is possible only because spacetime is broken into space and time. Indeed, other types of breakings are disfavored [5, 6].

Appendix B: Useful mathematical formulas

In this appendix we collect a number of mathematical formulas and relations that are used in the paper.

Define the function

$$v(r) = -\int^{\Lambda/\Lambda_L} \frac{\mathrm{d}^4 p}{(2\pi)^4} \ln(p''^2 + r), \tag{B.1}$$

where $r > 0$, the integral is in Euclidean space,

$$\hat{p}''^\mu = \hat{p}^\mu, \qquad \bar{p}''^\mu = \bar{p}^\mu(\bar{p}^2 + 1) \tag{B.2}$$

and Λ is a UV cut-off. We want to study the Taylor expansion

$$v(r) = v(r_0) + (r - r_0)v'(r_0) + \frac{1}{2}(r - r_0)^2 v''(r_0) + \mathcal{O}((r - r_0)^3) \tag{B.3}$$

of this function in the neighborhood of a generic point r_0. Observe that the second derivative

$$v''(r_0) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{(p''^2 + r_0)^2} \tag{B.4}$$

is convergent and strictly positive. On the other hand, the first derivative is logarithmically divergent. By weighted power counting, its divergent part is independent of r_0. We have

$$\frac{v'(r_0) - v'(r_1)}{r_0 - r_1} = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{1}{(p''^2 + r_0)(p''^2 + r_1)} > 0. \tag{B.5}$$

Adding and subtracting $1/(\hat{p}^2+(\bar{p}^2)^3+r_0)$ to the integrand of $v'(r_0)$ we can also write

$$v'(r_0)=\bar{v}'(r_0)-\frac{1}{12\pi^2}\ln\frac{2\Lambda^3}{\Lambda_L^3\sqrt{r_0}}, \tag{B.6}$$

up to $\mathcal{O}(1/\Lambda)$, where

$$\bar{v}'(r_0)=\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\bar{p}^2(2\bar{p}^2+1)}{(p''^2+r_0)(\hat{p}^2+(\bar{p}^2)^3+r_0)} \tag{B.7}$$

is finite and positive.

Now we approximate the expansion (B.3) for $r_0\ll 1$. To study the right-hand side of (B.4) we note that the integral diverges logarithmically for small r_0, so it is sufficient to look for the corresponding logarithm. We find

$$v''(r_0)\sim-\frac{1}{16\pi^2}\ln r_0, \qquad \text{for } r_0\ll 1. \tag{B.8}$$

Integrating this expression, we also find

$$v'(r_0)=-\int^{\Lambda/\Lambda_L}\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{1}{p''^2+r_0}\sim-\frac{1}{16\pi^2}\left(2\ln\frac{\Lambda^2}{\Lambda_L^2}+r_0\ln r_0\right). \tag{B.9}$$

up to $\mathcal{O}(1/\Lambda)$. The arbitrary constant can be determined calculating $v'(0)$.

Another useful integral is

$$I_1=\int\frac{\mathrm{d}^4p}{(2\pi)^4}\frac{\hat{p}^2-\bar{p}''^2}{(p''^2+r_0)^2}=r_0v''(r_0)>0. \tag{B.10}$$

This formula is proved using the identity

$$\int_{-\infty}^{+\infty}\frac{\mathrm{d}\hat{p}}{2\pi}\frac{\hat{p}^2-a_kx}{(\hat{p}^2+x)^k}=0, \qquad a_k=\frac{\Gamma\left(k-\frac{3}{2}\right)}{2\Gamma\left(k-\frac{1}{2}\right)}, \qquad x>0, \qquad k>\frac{3}{2}, \tag{B.11}$$

for $k=2$.

Next, define the functions

$$\left(f_{ij},f'_{ij},f''_{ij}\right)(p^2)=\frac{1}{(4\pi)^2}\int_0^1\mathrm{d}x\,(1,x,x(1-x))\left[\ln\frac{\Lambda_L^2}{m_i^2x+m_j^2(1-x)-p^2x(1-x)}-1\right], \tag{B.12}$$

where i,j can have the values t and b. While f_{ij} is clearly symmetric, f'_{ij} satisfies

$$f'_{ij}+f'_{ji}=f_{ij}.$$

In the range $0\leqslant p\leqslant m_t$ the functions (B.12) do not depend on p very much and can be treated as constants, calculated for $p=0$. Using $m_b\ll m_t\ll\Lambda_L$, we have

$$f_{ii}\sim\frac{1}{(4\pi)^2}\ln\frac{\Lambda_L^2}{m_i^2}, \qquad f_{tb}=f_{bt}\sim f_{tt}, \qquad f'_{ij}\sim\frac{1}{2}f_{ij}, \qquad f''_{ij}\sim\frac{1}{6}f_{ij}. \tag{B.13}$$

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