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Weighted Power Counting And Lorentz Violating Gauge Theories. I: General Properties

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Abstract

We construct local, unitary gauge theories that violate Lorentz symmetry explicitly at high energies and are renormalizable by weighted power counting. They contain higher space derivatives, which improve the behavior of propagators at large momenta, but no higher time derivatives. We show that the regularity of the gauge-field propagator privileges a particular spacetime breaking, the one into into space and time. We then concentrate on the simplest class of models, study four dimensional examples and discuss a number of issues that arise in our approach, such as the low-energy recovery of Lorentz invariance.

1 Introduction

Lorentz symmetry has been verified in many experiments with great precision [1]. However, different types of arguments have lead some authors to suggest that it could be violated at very high energies [2, 3, 4]. This possibility has raised a considerable interest, because, if true, it would substantially affect our understanding of Nature. The Lorentz violating extension of the Standard Model [3] contains a large amount of new parameters. Bounds on many of them, particularly those belonging to the power-counting renormalizable subsector, are available. Their updated values are reported in ref. [5].

In quantum field theory, the classification of local, unitarity, polynomial and renormalizable models changes dramatically if we do not assume that Lorentz invariance is exact at arbitrarily high energies [6, 7]. In that case, higher space derivatives are allowed and can improve the behavior of propagators at large momenta. A number of theories that are not renormalizable by ordinary power counting become renormalizable in the framework of a "weighted power counting" [6], which assigns different weights to space and time, and ensures that no term containing higher time derivatives is generated by renormalization, in agreement with unitarity. Having studied scalar and fermion theories in ref.s [6, 7], here we begin the study of gauge theories, focusing on the simplest class of models. The investigation is completed in a second paper [8], to which we refer as paper II, which contains the classification of renormalizable gauge theories.

The theories we are interested in must be local and polynomial, free of infrared divergences in Feynman diagrams at non-exceptional external momenta, and renormalizable by weighted power counting. We find that in the presence of gauge interactions the set of renormalizable theories is more restricted than in the scalar-fermion framework. Due to the particular structure of the gauge-field propagator, Feynman diagrams are plagued with certain spurious subdivergences. We are able to prove that they cancel out when spacetime is broken into space and time, and certain other restrictions are fulfilled.

A more delicate physical issue is the low-energy recovery of Lorentz symmetry. Once Lorentz symmetry is violated at high energies, its low-energy recovery is not guaranteed, because renormalization makes the low-energy parameters run independently. One possibility is that the Lorentz invariant surface is RG stable [9], otherwise a suitable fine-tuning must be advocated.

In other domains of physics, such as the theory of critical phenomena, where Lorentz symmetry is not a fundamental requirement, certain scalar models of the types classified in ref. [6] have already been studied [10] and have physical applications.

The paper is organized as follows. In section 2 we review the weighted power counting for scalar-fermion theories. In section 3 we extend it to Lorentz violating gauge theories and define the class of models we focus on in this paper. We study the conditions for renormalizability, absence of infrared divergences in Feynman diagrams and regularity of the propagator. In section 4 we prove that the theories are renormalizable to all orders, using the Batalin-Vilkovisky formalism. In section 5 we study four dimensional examples and the low-energy recovery of Lorentz invariance. In section 6 we discuss strictly renormalizable and weighted scale invariant theories. In section 7 we study the Proca Lorentz violating theories, and prove that they are not renormalizable. Section 8 contains our conclusions. In appendix A we classify the quadratic terms of the gauge-field lagrangian and in appendix B we derive sufficient conditions for the absence of spurious subdivergences.

2 Weighted power counting

In this section we briefly review the weighted power counting criterion of refs. [6, 7]. The simplest framework to study Lorentz violations is to assume that the d-dimensional spacetime manifold $M = \mathbb{R}^d$ is split into the product $\hat{M} \times \bar{M}$ of two submanifolds, a $\hat{d}$-dimensional submanifold $\hat{M} = \mathbb{R}^{\hat{d}}$, containing time and possibly some space coordinates, and a $\bar{d}$-dimensional space submanifold $\bar{M} = \mathbb{R}^{\bar{d}}$, so that the d-dimensional Lorentz group $O(1, d-1)$ is broken to a residual Lorentz group $O(1, \hat{d}-1) \times O(\bar{d})$. In this paper we study renormalization in this simplified framework. The generalization to the most general breaking is done in paper II.

The partial derivative ∂ is decomposed as $(\hat{\partial}, \bar{\partial})$, where $\hat{\partial}$ and $\bar{\partial}$ act on the subspaces $\hat{M}$ and $\bar{M}$, respectively. Coordinates, momenta and spacetime indices are decomposed similarly. Consider a free scalar theory with (Euclidean) lagrangian

$$\mathcal{L}_{\text{free}} = \frac{1}{2}(\hat{\partial}\varphi)^2 + \frac{1}{2\Lambda_L^{2n-2}}(\bar{\partial}^n\varphi)^2, \qquad (2.1)$$

where Λ_L is an energy scale and n is an integer > 1. Up to total derivatives it is not necessary to specify how the $\bar{\partial}$'s are contracted among themselves. The coefficient of $(\bar{\partial}^n\varphi)^2$ must be positive to have a positive energy in the Minkowskian framework. The theory (2.1) is invariant under the weighted rescaling

$$\hat{x} \rightarrow \hat{x}\ \mathrm{e}^{-\Omega}, \qquad \bar{x} \rightarrow \bar{x}\ \mathrm{e}^{-\Omega/n}, \qquad \varphi \rightarrow \varphi\ \mathrm{e}^{\Omega(\text{đ}/2-1)}, \qquad (2.2)$$

where $\text{đ} = \hat{d} + \bar{d}/n$ is the "weighted dimension". Note that Λ_L is not rescaled.

The interacting theory is defined as a perturbative expansion around the free theory (2.1). For the purposes of renormalization, the masses and the other quadratic terms can be treated perturbatively, since the counterterms depend polynomially on them. Denote the "weight" of an object $\mathcal{O}$ by $[\mathcal{O}]$ and assign weights to coordinates, momenta and fields as follows:

$$[\hat{x}] = -1, \qquad [\bar{x}] = -\frac{1}{n}, \qquad [\hat{\partial}] = 1, \qquad [\bar{\partial}] = \frac{1}{n}, \qquad [\varphi] = \frac{\text{đ}}{2} - 1, \qquad (2.3)$$

while Λ_L is weightless. Polynomiality demands that the weight of φ be strictly positive, so we assume $\text{đ} > 2$.

We say that $P_{k,n}(\hat{p},\bar{p})$ is a weighted polynomial in $\hat{p}$ and $\bar{p}$, of degree k, where k is a multiple of $1/n$, if $P_{k,n}(\xi^n\hat{p},\xi\bar{p})$ is a polynomial of degree kn in ξ. A diagram G with L loops, V vertices and I internal legs gives an integral of the form

$$\mathcal{I}_G(k)=\int\frac{\mathrm{d}^{dL}p}{(2\pi)^d}\prod_{i=1}^{I}\mathcal{P}_i(p,k)\prod_{j=1}^{V}\mathcal{V}_j(p,k),$$

where p are the loop momenta, k are the external momenta, $\mathcal{P}_i(p,k)$ are the propagators and $\mathcal{V}_j(p,k)$ are the vertices. The momentum integration measure d^dp has weight đ. The propagator is equal to 1 divided by a weighted polynomial of degree 2. We can assume that, as far as their momentum dependence is concerned, the vertices are weighted monomials of certain degrees δ_j. Rescaling k and as $(\hat{k},\bar{k})\rightarrow(\lambda\hat{k},\lambda^{1/n}\bar{k})$, the integral $\mathcal{I}_G(k)$ rescales with a factor equal to its total weight $\omega(G)$. By locality, the divergent part of $\mathcal{I}_G(k)$ is a weighted polynomial of degree $\omega(G)$. Assume that the lagrangian contains all vertices that have weights not greater than đ and only those. This bound excludes terms with higher time derivatives. Then we find

$$\omega(G)\leq \text{đ}-E_s\frac{\text{đ}-2}{2}, \tag{2.4}$$

where E_s is the number of external scalar legs. Formula (2.4) and đ>2 ensure that every counterterm has a weight not larger than đ, therefore it can be subtracted renormalizing the fields and couplings of the lagrangian, and no new vertex needs to be introduced.

The lagrangian terms of weight đ are strictly renormalizable, those of weights smaller than đ super-renormalizable and those of weights greater than đ non-renormalizable. The weighted power counting criterion amounts to demand that the theory contains no parameter of negative weight.

Simple examples of renormalizable theories are the φ^4, đ$=4$ models

$$\mathcal{L}_{\text{đ}=4}=\frac{1}{2}(\widehat{\partial}\varphi)^2+\frac{1}{2\Lambda_L^{2(n-1)}}(\overline{\partial}^n\varphi)^2+\frac{\lambda}{4!\Lambda_L^{d-4}}\varphi^4 \tag{2.5}$$

and the φ^6, đ$=3$ models

$$\mathcal{L}_{\text{đ}=3}=\frac{1}{2}(\widehat{\partial}\varphi)^2+\frac{1}{2\Lambda_L^{2(n-1)}}(\overline{\partial}^n\varphi)^2+\frac{1}{4!\Lambda_L^{2(n-1)}}\sum_\alpha\lambda_\alpha\left[\overline{\partial}^n\varphi^4\right]_\alpha+\frac{\lambda_6}{6!\Lambda_L^{2(n-1)}}\varphi^6. \tag{2.6}$$

where $\left[\overline{\partial}^n\varphi^4\right]_\alpha$ denotes a basis of inequivalent terms constructed with n derivatives $\overline{\partial}$ acting on four φ's[1]. Only the strictly-renormalizable terms have been listed in (2.5) and (2.6). It is straightforward to complete the actions adding the super-renormalizable terms, which are those that contain fewer derivatives and/or φ-powers.

[1]Because of $O(\overline{d})$-invariance, these exist no such terms if n is odd.

The considerations just recalled are easily generalized to fermions. The weight of a fermion field is $(\text{đ}-1)/2$, so polynomiality is ensured, because đ is necessarily greater than 1 (if $d > 1$). Formula (2.4) becomes

$$\omega(G) \leq \text{đ} - E_s \frac{\text{đ}-2}{2} - E_f \frac{\text{đ}-1}{2},$$

where E_f is the number of external fermionic legs.

Our investigation focuses on the theories that do contain higher space derivatives ($n \geq 2$). Indeed, the theories with $n = 1$, which can be either Lorentz invariant or Lorentz violating, obey the usual rules of power counting.

3 Lorentz violating gauge theories

Having decomposed the partial derivative operator as $\partial = (\hat{\partial}, \bar{\partial})$, the gauge field has to be decomposed similarly. We write $A' = (\hat{A}', \bar{A}') \equiv gA = g(\hat{A}, \bar{A})$, where g is the gauge coupling and $A_\mu = A_\mu^a T^a$, with T^a anti-Hermitian. The covariant derivative is decomposed as

$$D = (\hat{D}, \bar{D}) = (\hat{\partial} + \hat{A}', \bar{\partial} + \bar{A}'). \tag{3.1}$$

With the weight assignments

$$[\hat{A}'] = [\hat{D}] = 1, \qquad [\bar{A}'] = [\bar{D}] = \frac{1}{n},$$

the decomposition (3.1) is compatible with the weighted rescaling. The field strength is split into three sets of components, namely

$$\hat{F}_{\mu\nu} \equiv F_{\hat{\mu}\hat{\nu}}, \qquad \tilde{F}_{\mu\nu} \equiv F_{\hat{\mu}\bar{\nu}}, \qquad \bar{F}_{\mu\nu} \equiv F_{\bar{\mu}\bar{\nu}}. \tag{3.2}$$

Since the kinetic lagrangian must contain $(\hat{\partial}\hat{A})^2$, the weight of $\hat{A}$ is $\text{đ}/2 - 1$, hence $[g] = 2 - \text{đ}/2$. We can read $[\bar{A}]$ from $[\tilde{F}] = [\bar{\partial}] + [\hat{A}] = [\hat{\partial}] + [\bar{A}]$. In summary,

$$[\hat{A}] = \frac{\text{đ}}{2} - 1, \qquad [\bar{A}] = \frac{\text{đ}}{2} - 2 + \frac{1}{n}, \qquad [\hat{F}] = \frac{\text{đ}}{2}, \qquad [\tilde{F}] = \frac{\text{đ}}{2} - 1 + \frac{1}{n}, \qquad [\bar{F}] = \frac{\text{đ}}{2} - 2 + \frac{2}{n}. \tag{3.3}$$

Since the weight of g cannot be negative, we must have

$$\text{đ} \leq 4. \tag{3.4}$$

In this paper we focus on the "$1/\alpha$ theories", namely those that have a lagrangian of the form

$$\mathcal{L} = \frac{1}{\alpha} \mathcal{L}_r(gA, g\varphi, g\psi, g\bar{C}, gC, \lambda). \tag{3.5}$$

Here C and $\bar{C}$ are the ghosts and antighosts, φ are the scalar fields and ψ are the fermions. Moreover, the reduced lagrangian $\mathcal{L}_r$ depends polynomially on g and the other parameters λ, and $[\lambda] \geq 0$. The renormalizability of the structure (3.5) is easy to prove (see (3.24)).

When $\bar{d}=4$ the gauge coupling g is weightless and the theory can always be written in the $1/\alpha$ form with a suitable redefinition of parameters. Instead, when $\bar{d}<4$ the $1/\alpha$ theories are a small subset of the allowed theories. For example, in $d=4$ the weights of $\mathcal{L}_r$, $\hat{\partial}$, $g\hat{A}$, $g\varphi$, $g\bar{C}$, gC and $g\psi$ coincide with their dimensions in units of mass, and only the weights of $\bar{\partial}$ and $g\bar{A}$, which are equal to $1/n$, differ from their dimensions. Then the lagrangian contains just the usual power-counting renormalizable terms, plus the terms that can be constructed with $\bar{D}$, $\bar{F}$ and $\tilde{F}$. The form of the lagrangian does not change in $d\neq 4$. Polynomiality is always ensured.

Even if the $1/\alpha$ theories are not particularly interesting from the physical point of view, it is convenient to start from them, because the simplified structure (3.5) allows us to illustrate the basic properties of Lorentz violating gauge theories without unnecessary complicacies. The most general case is studied in paper II.

Observe that the theories (3.5) cannot contain higher time derivatives, as desired. Indeed, by $O(1,\hat{d}-1)$-invariance a term with three $\hat{\partial}$'s in $\mathcal{L}_r$ must contain at least another $\hat{\partial}$, or a $g\hat{A}$, or a fermion bilinear such as $g^2\bar{\psi}\hat{\gamma}\psi$. However, the weights of $\hat{\partial}^4$ and $g\hat{\partial}^3\hat{A}$ are already equal to four, so no other leg can be attached to such objects, and the weight of $g^2\hat{\partial}^3\bar{\psi}\hat{\gamma}\psi$ is equal to six.

It is convenient to write the action

$$\mathcal{S}_0=\int \mathrm{d}^d x\,(\mathcal{L}_Q+\mathcal{L}_I)\equiv \mathcal{S}_Q+\mathcal{S}_I, \tag{3.6}$$

as the sum of two gauge-invariant contributions, the quadratic terms $\mathcal{S}_Q$ plus the vertex terms $\mathcal{S}_I$. By "quadratic terms" we mean the terms constructed with two field strengths and possibly covariant derivatives. By "vertex terms" we mean the terms constructed with at least three field strengths, and possibly covariant derivatives.

In Appendix A we prove that, up to total derivatives, the quadratic part $\mathcal{L}_Q$ of the lagrangian reads (in the Euclidean framework)

$$\mathcal{L}_Q=\frac{1}{4}\left\{F_{\hat{\mu}\hat{\nu}}^2+2F_{\hat{\mu}\bar{\nu}}\eta(\bar{\Upsilon})F_{\hat{\mu}\bar{\nu}}+F_{\bar{\mu}\bar{\nu}}\tau(\bar{\Upsilon})F_{\bar{\mu}\bar{\nu}}+\frac{1}{\Lambda_L^2}(D_{\hat{\rho}}F_{\bar{\mu}\bar{\nu}})\xi(\bar{\Upsilon})(D_{\hat{\rho}}F_{\bar{\mu}\bar{\nu}})\right\}. \tag{3.7}$$

Here $\bar{\Upsilon}\equiv -\bar{D}^2/\Lambda_L^2$ and η, τ and ξ are polynomials of degrees $n-1$, $2n-2$ and $n-2$, respectively. We have expansions

$$\eta(\bar{\Upsilon})=\sum_{i=0}^{n-1}\eta_{n-1-i}\bar{\Upsilon}^i,\qquad [\eta_j]=\frac{2j}{n}, \tag{3.8}$$

and similar, where η_i are dimensionless constants of non-negative weights.

In momentum space we see that the free action is positive definite if and only if

$$\eta>0,\qquad \tilde{\eta}\equiv\eta+\frac{\bar{k}^2}{\Lambda_L^2}\xi>0,\qquad \tau>0, \tag{3.9}$$

where now η, τ and ξ are functions of $\bar{k}^2/\Lambda_L^2$.

In the parametrization (3.7) the scale Λ_L is a redundant parameter. It is mainly used to match the dimensions in units of mass, so that the other parameters (e.g. the η_j's) can be assumed to be dimensionless, but possibly weightful. The Λ_L-redundancy implies that Λ_L is RG invariant, so its beta function vanishes by definition.

BRST symmetry and gauge fixing The usual BRST symmetry [12]

$$sA_\mu^a = D_\mu^{ab}C^b = \partial_\mu C^a + gf^{abc}A_\mu^b C^c, \qquad sC^a = -\frac{g}{2}f^{abc}C^bC^c,$$
$$s\bar{C}^a = B^a, \qquad sB^a = 0, \qquad s\psi^i = -gT_{ij}^a C^a \psi^j,$$

etc., where B^a are Lagrange multipliers for the gauge-fixing, is automatically compatible with the weighted power counting. The quadratic terms of the ghost Lagrangian contain $\bar{C}\hat{\partial}^2 C$ and B^2, and have weight đ, so we have the weight assignments

$$[C] = [\bar{C}] = \frac{\text{đ}}{2} - 1, \qquad [s] = 1, \qquad [B] = \frac{\text{đ}}{2}. \tag{3.10}$$

The most convenient gauge-fixing $\mathcal{G}^a$ is linear in the gauge potential and gives

$$\mathcal{L}_{\text{gf}} = s\Psi, \qquad \Psi = \bar{C}^a\left(-\frac{\lambda}{2}B^a + \mathcal{G}^a\right), \qquad \mathcal{G}^a \equiv \hat{\partial}\cdot\hat{A}^a + \zeta\left(\bar{\upsilon}\right)\bar{\partial}\cdot\bar{A}^a, \tag{3.11}$$

where λ is a dimensionless, weightless constant, $\bar{\upsilon} \equiv -\bar{\partial}^2/\Lambda_L^2$ and ζ is a polynomial of degree $n-1$. We demand

$$\zeta > 0, \tag{3.12}$$

to include the "Coulomb gauge-fixing" $\bar{\partial}\cdot\bar{A}^a$.

The total gauge-fixed action is finally

$$\mathcal{S} = \int \mathrm{d}^d x\left(\mathcal{L}_Q + \mathcal{L}_I + \mathcal{L}_{\text{gf}}\right) \equiv \mathcal{S}_0 + \mathcal{S}_{\text{gf}}. \tag{3.13}$$

Propagator The (Euclidean) gauge-field propagator can be worked out from the free subsector of (3.13), after integrating B^a out, which amounts to add

$$\frac{1}{2\lambda}(\mathcal{G}^a)^2 \tag{3.14}$$

to $\mathcal{L}_Q$. The result is[2]

$$\langle A(k)\ A(-k)\rangle = \begin{pmatrix} \langle\hat{A}\hat{A}\rangle & \langle\hat{A}\bar{A}\rangle \\ \langle\bar{A}\hat{A}\rangle & \langle\bar{A}\bar{A}\rangle \end{pmatrix} = \begin{pmatrix} u\hat{\delta} + s\hat{k}\hat{k} & r\hat{k}\bar{k} \\ r\bar{k}\hat{k} & v\bar{\delta} + t\bar{k}\bar{k} \end{pmatrix}, \tag{3.15}$$

[2]A similar propagator, in a different context, has already appeared in ref. [11].

with

$$u = \frac{1}{D(1,\eta)}, \qquad s = \frac{\lambda}{D^2(1,\zeta)} + \frac{-\hat{k}^2 + \zeta\left(\frac{\zeta}{\eta} - 2\right)\bar{k}^2}{D(1,\eta)D^2(1,\zeta)}, \qquad r = \frac{\lambda - \frac{\zeta}{\eta}}{D^2(1,\zeta)},$$
$$v = \frac{1}{D(\tilde{\eta},\tau)}, \qquad t = \frac{\lambda}{D^2(1,\zeta)} + \frac{\left(\frac{\tilde{\tau}}{\eta} - 2\zeta\right)\hat{k}^2 - \zeta^2\bar{k}^2}{D(\tilde{\eta},\tau)D^2(1,\zeta)},$$

where

$$D(x,y) \equiv x\hat{k}^2 + y\bar{k}^2, \qquad \tilde{\eta} = \eta + \frac{\bar{k}^2}{\Lambda_L^2}\xi, \qquad \tilde{\tau} = \tau + \frac{\hat{k}^2}{\Lambda_L^2}\xi,$$

and now η, τ, ξ and ζ, as well as x and y, are meant as functions of $\bar{k}^2/\Lambda_L^2$. The ghost propagator is

$$\frac{1}{D(1,\zeta)}. \tag{3.16}$$

A simple gauge choice ("Feynman gauge") is

$$\lambda = 1, \qquad \zeta = \eta. \tag{3.17}$$

Then, both $\langle \hat{A}\bar{A}\rangle$ and s vanish, so

$$u = \frac{1}{D(1,\eta)}, \qquad s = r = 0, \qquad v = \frac{1}{D(\tilde{\eta},\tau)}, \qquad t = \frac{\tilde{\tau} - \eta^2}{\eta D(\tilde{\eta},\tau)D(1,\eta)}. \tag{3.18}$$

Physical degrees of freedom and dispersion relations To study the physical degrees of freedom we choose the Coulomb gauge-fixing

$$\mathcal{G}_{\rm C}^a = \bar{\partial}\cdot\bar{A}^a.$$

It can be reached from the more general gauge-fixing (3.11) taking the limit $\lambda \to \infty$, $\zeta \to \infty$ in (3.11), (3.14) and (3.15), with $\varsigma \equiv \lambda/\zeta^2$ fixed, and rescaling the antighosts and the Lagrange multiplier as $\bar{C}^a \to \bar{C}^a/\zeta$, $B^a \to B^a/\zeta$. The quadratic lagrangian $\mathcal{L}_Q + (\bar{\partial}\cdot\bar{A}^a)^2/(2\varsigma)$ gives the propagators

$$\langle \hat{A}(k)\ \hat{A}(-k)\rangle = \frac{1}{D(1,\eta)}\left(\hat{\delta} + \frac{\hat{k}\hat{k}}{\bar{k}^2\eta}\right) + \frac{\varsigma\hat{k}\hat{k}}{(\bar{k}^2)^2}, \qquad \langle \hat{A}(k)\ \bar{A}(-k)\rangle = \frac{\varsigma\hat{k}\bar{k}}{(\bar{k}^2)^2},$$
$$\langle \bar{A}(k)\ \bar{A}(-k)\rangle = \frac{1}{D(\tilde{\eta},\tau)}\left(\bar{\delta} - \frac{\bar{k}\bar{k}}{\bar{k}^2}\right) + \frac{\varsigma\bar{k}\bar{k}}{(\bar{k}^2)^2}.$$

Writing $\hat{k} = (iE, \hat{\mathbf{k}})$ and studying the poles, we see that the $\bar{A}$-sector propagates $\bar{d}-1$ degrees of freedom with energies

$$E = \sqrt{\hat{\mathbf{k}}^2 + \bar{k}^2\frac{\tau(\bar{k}^2/\Lambda_L^2)}{\tilde{\eta}(\bar{k}^2/\Lambda_L^2)}},$$

while the $\hat{A}$-sector propagates $\hat{d}-1$ degrees of freedom with energies

$$E=\sqrt{\hat{\mathbf{k}}^2+\bar{k}^2\eta(\bar{k}^2/\Lambda_L^2)}.$$

Indeed, the matrix $\hat{\delta}+\hat{k}\hat{k}/(\bar{k}^2\eta)$ has one null eigenvector on the pole, since its determinant is equal to $D(1,\eta)/(\bar{k}^2\eta)$. The residues are positive in the Minkowskian framework. This can be immediately seen using $SO(1,\hat{d}-1)$ invariance to set $\hat{\mathbf{k}}=0$ (at $\bar{k}\neq 0$) and $SO(\bar{d})$ invariance to set all $\bar{k}$-components but one to zero.

Finally, the ghost propagator becomes $1/\bar{k}^2$, which has no pole. In total, the physical degrees of freedom are $d-2$, as expected.

Regularity of the propagator A propagator is regular if it is the ratio

$$\frac{P_r(\hat{k},\bar{k})}{P'_{2s}(\hat{k},\bar{k})} \tag{3.19}$$

of two weighted polynomials of degrees r and $2s$, where r and s are integers, such that the denominator $P'_{2s}(\hat{k},\overline{k})$ is non-negative (in the Euclidean framework), non-vanishing when either $\hat{k}\neq 0$ or $\overline{k}\neq 0$, and has the form

$$P'_s(\hat{k},\bar{k})=\hat{\omega}(\hat{k}^2)^s+\bar{\omega}(\bar{k}^2)^{ns}+\cdots, \tag{3.20}$$

with $\hat{\omega}>0$, $\bar{\omega}>0$, where the dots collect the terms $(\hat{k}^2)^{j-m}(\bar{k}^2)^{mn}$ with $j<s$, $0\leq m\leq j$, and $j=s$, $0<m<s$.

The regularity conditions just stated ensure that: a) the derivatives with respect to $\hat{k}$ improve the large-$\bar{k}$ behavior (because $\bar{\omega}\neq 0$), besides the large-$\hat{k}$ and overall ones; and b) the derivatives with respect to $\bar{k}$ improve the large-$\hat{k}$ behavior (because $\hat{\omega}\neq 0$), besides the large-$\bar{k}$ and overall ones. The overall divergences of the $\hat{k}$-subintegrals are local in $\bar{k}$ and the overall divergences of the $\bar{k}$-subintegrals are local in $\hat{k}$ (once subdiagrams have been inductively subtracted).

In this paper we use the dimensional-regularization technique. We recall [6] that it is necessary to continue both $\hat{d}$ and $\bar{d}$ to complex values, say $\hat{d}-\varepsilon_1$ and $\bar{d}-\varepsilon_2$, respectively. In the framework of the dimensional regularization the absence of $\hat{k}$- and $\bar{k}$-subdivergences is immediate to prove: being local, the $\hat{k}$-subdivergences are killed by the (dimensionally continued) $\bar{k}$-subintegrals and the $\bar{k}$-subdivergences are killed by the $\hat{k}$-subintegrals. More explicitly, at one loop we have integrals of the form

$$\int\frac{\mathrm{d}^{\hat{d}-\varepsilon_1}\hat{k}}{(2\pi)^{\hat{d}}}\left[\int\frac{\mathrm{d}^{\bar{d}-\varepsilon_2}\bar{k}}{(2\pi)^{\bar{d}}}\frac{V(\hat{k},\bar{k};\hat{p},\bar{p})}{\prod_{i=1}^{I}P'_{2s}(\hat{k},\bar{k};\hat{p}_i,\bar{p}_i)}\right],$$

where I denotes the number of propagators, p_i are linear combinations of the external momenta and the numerator collects both the vertices and the polynomials P_r of (3.19). Consider first the integral contained in the square braket. Here $\hat{k}$ can be treated as an external momentum. The regularity of the propagator ensures that differentiating the integrand with respect to $\hat{k}$ (or $\hat{p}_i$, or $\bar{p}_i$) a sufficient number of times the $\bar{k}$-integral becomes convergent. Thus, the divergent part of the $\bar{k}$-integral is a polynomial Q in $\hat{k}$ (and $\hat{p}_i$, $\bar{p}_i$). However,

$$\int \frac{\mathrm{d}^{\hat{d}-\varepsilon_1}\hat{k}}{(2\pi)^{\hat{d}}} Q(\hat{k};\hat{p},\bar{p}) = 0$$

in dimensional regularization, because it is the integral of a polynomial. Thus the (sub)divergence of the $\bar{k}$-integral is killed by the $\hat{k}$-integral. An analogous conclusion holds exchanging the roles of $\hat{k}$ and $\bar{k}$. The arguments can be generalized to higher loops after including the counterterms corresponding to the proper subdiagrams.

In a more general regularization setting the absence of $\hat{k}$- and $\bar{k}$-subdivergences is proved as follows. The overall divergences of the $\hat{k}$-$\overline{k}$-integrals are subtracted, for example, by the first terms of the "weighted Taylor expansion" around vanishing external momenta [6]. When the regularity conditions stated above are fulfilled, those counterterms automatically cure also the $\hat{k}$-subintegrals and the $\overline{k}$-subintegrals. Indeed, the subintegrals cannot behave worse than the full integrals over $\hat{k}$ and $\overline{k}$, because (3.20) ensures that the propagators tend to zero with maximal velocity also in the subintegrals, the loop-integration measures grow less rapidly and the vertices grow not more rapidly than in the $\hat{k}$-$\overline{k}$-integrals.

The scalar and fermion propagators are clearly regular. On the other hand, a propagator of the form

$$\frac{\Lambda_L^{n-1}}{|\hat{k}||\overline{k}|^n}$$

is not, and could generate "spurious subdivergences" when $\hat{k}$ tends to infinity at $\overline{k}$ fixed, or viceversa. The problem appears in certain large N fermion models [7], and becomes crucial whenever gauge fields are present, as we now discuss.

The propagators (3.15) and (3.16) are regular at non-vanishing momenta, because the conditions (3.9) and (3.12) ensure that the denominators are positive-definite in the Euclidean framework. To have the best ultraviolet behaviors we must strengthen those conditions requiring also

$$\eta_0 > 0, \qquad \tau_0 > 0, \qquad \tilde{\eta}_0 = \eta_0 + \xi_0 > 0, \qquad \zeta_0 > 0, \tag{3.21}$$

which we assume from now on. However, attention must be paid to the behaviors of propagators when $\hat{k}$ is sent to infinity at fixed $\overline{k}$, and when $\overline{k}$ is sent to infinity at fixed $\hat{k}$.

The conditions (3.21) ensure that all gauge and ghost propagators are regular in the Feynman gauge (3.17)-(3.18), except $\langle \bar{A}\bar{A} \rangle$, which has $\bar{\omega} \neq 0$, but $\hat{\omega} = 0$, so it is regular when $\overline{k}$ tends to infinity at $\hat{k}$ fixed, but not when $\hat{k}$ tends to infinity at $\overline{k}$ fixed: in that region of momentum space $\langle \bar{A}\bar{A} \rangle$ behaves like $\sim 1/\hat{k}^2$. To ensure that no spurious subdivergences are generated by the $\hat{k}$-subintegrals, we have to perform a more careful analysis, which is done in appendix B and generalized in paper II. The result is that the spurious subdivergences can be proved to be absent for

$$\hat{d} = 1, \qquad đ < 2 + \frac{2}{n}, \qquad d = \text{even}, \qquad n = \text{odd}, \tag{3.22}$$

which we are going to assume from now on, unless explicitly stated. Observe that the case (3.22) is a physically interesting one, since spacetime is split into space and time. In four dimensions, (3.22) are equivalent to just $\hat{d} = 1$, $n =$odd (at $n > 1$).

The absence of spurious subdivergences ensures the locality of counterterms. Consider a diagram G_r equipped with the subtractions that take care of its diverging proper subdiagrams. Differentiating G_r a sufficient number of times with respect to any components $\hat{p}_i$, $\bar{p}_i$ of the external momenta p_i, we can arbitrarily reduce the overall degree of divergence and eventually produce a convergent integral. Therefore, overall divergences are polynomial in all components of the external momenta.

Weighted power counting In the rest of this section we consider renormalizable and super-renormalizable theories. We postpone the analysis of strictly-renormalizable theories, namely the theories that contain only weightless parameters, to section 8.

A generic vertex of (3.5) has the structure

$$\lambda_i g^{n_i - 2} \hat{\partial}^k \bar{\partial}^m \hat{A}^p \bar{A}^q \bar{C}^r C^r \varphi^s \bar{\psi}^t \psi^t, \tag{3.23}$$

where $n_i = p + q + 2r + s + 2t$ and p, q, r, k, m, s and t are integers. Formula (3.23) and analogous expressions in this paper are meant "symbolically", which means that we pay attention to the field- and derivative-contents of the vertices, but not where the derivatives act and how Lorentz, gauge and other indices are contracted.

Every counterterm generated by (3.23) fits into the structure (3.23). Indeed, consider a L-loop diagram with E external legs, I internal legs and v_i vertices of type i. Counting the legs we have $\sum_i n_i v_i = E + 2I = E + 2(L + V - 1)$, so the diagram is multiplied by a product of couplings

$$g^{\sum_i (n_i - 2) v_i} \prod_i \lambda_i^{v_i} = \alpha^L g^{E-2} \prod_i \lambda_i^{v_i}. \tag{3.24}$$

We see that a g^{E-2} factorizes, as expected. Moreover, each loop order carries an additional weight of at least $2[g] = 4 - đ$.

We have already mentioned that when conditions (3.22) are fulfilled we do not need to worry about the $\hat{k}$- and the $\overline{k}$-subintegrals, so we concentrate on the $\hat{k}$-$\overline{k}$-integrals. The positivity conditions (3.9) and (3.12) ensure that the denominators appearing in the Euclidean propagators (3.15) and (3.16) never vanish at non-exceptional momenta. Thus we have to study the integrals only in the ultraviolet and infrared regions. We begin with the ultraviolet behavior. We show that the $1/\alpha$ theories that satisfy (3.4) are renormalizable by weighted power counting, assuming that spurious subdivergences are absent and that gauge (BRST) invariance is preserved. In the next subsection we discuss the infrared behavior, while in the next section we prove the preservation of BRST invariance.

In the analysis of renormalization based on weighted power counting, we can treat the weightful parameters η_i, τ_i, ξ_i and ζ_i, $i > 0$, perturbatively, because the divergent parts of Feynman diagrams depend polynomially on them. Then the propagators are (3.15) and (3.16) with the replacements

$$\eta \rightarrow \eta_0 \left(\frac{\bar{k}^2}{\Lambda_L^2}\right)^{n-1}, \qquad \tau \rightarrow \tau_0 \left(\frac{\bar{k}^2}{\Lambda_L^2}\right)^{2(n-1)}, \qquad \xi \rightarrow \xi_0 \left(\frac{\bar{k}^2}{\Lambda_L^2}\right)^{n-2}, \qquad \zeta \rightarrow \zeta_0 \left(\frac{\bar{k}^2}{\Lambda_L^2}\right)^{n-1}.$$

and every other term is treated as a vertex. Intermediate masses can be added to the denominators, to avoid IR problems, and removed immediately after calculating the divergent parts. Recall that (3.21) are assumed.

The propagators have weights

$$[\langle \hat{A}\hat{A}\rangle] = [\langle C\bar{C}\rangle] = [\langle \varphi\varphi\rangle] = -2, \qquad [\langle \psi\bar{\psi}\rangle] = -1, \qquad [\langle \hat{A}\bar{A}\rangle] = -3 + \frac{1}{n}, \qquad [\langle \bar{A}\bar{A}\rangle] = -4 + \frac{2}{n}. \tag{3.25}$$

Consider the vertex (3.23). Since the weight of (3.23) must be equal to đ, we have the inequality

$$n_i + k + \frac{m+q}{n} - q + t \leq 4. \tag{3.26}$$

The degree of divergence $\omega(G)$ of a diagram G with L loops, $\hat{I}$, $\tilde{I}$ and $\bar{I}$ internal legs of type $\hat{A}\hat{A}$, $\hat{A}\bar{A}$ and $\bar{A}\bar{A}$, respectively, I_{gh}, I_φ and I_ψ internal ghost, scalar and fermion legs, $\hat{E}$, $\bar{E}$, E_{gh}, E_φ and E_ψ external $\hat{A}$-, $\bar{A}$-, ghost, scalar and fermion legs, and v_i vertices (3.23), where i stands for (k, m, p, q, r, s, t), is

$$\omega(G) = L\text{đ} - 2(\hat{I} + I_{\text{gh}} + I_\varphi) - I_\psi - \left(3 - \frac{1}{n}\right)\tilde{I} - \left(4 - \frac{2}{n}\right)\bar{I} + \sum_i v_i \left(k + \frac{m}{n}\right).$$

Using $L = I - V + 1$, (3.26) and the leg-countings

$$\hat{E} + 2\hat{I} + \tilde{I} = \sum_i p v_i, \qquad \bar{E} + 2\bar{I} + \tilde{I} = \sum_i q v_i, \qquad E_{\text{gh}} + 2I_{\text{gh}} = \sum_i 2r v_i,$$
$$E_\varphi + 2I_\varphi = \sum_i s v_i, \qquad E_\psi + 2I_\psi = \sum_i 2t v_i,$$

we get

$$\omega(G) \leq \text{đ} - (\hat{E} + E_{\text{gh}} + E_\varphi)\left(\frac{\text{đ}}{2} - 1\right) - \bar{E}\left(\frac{\text{đ}}{2} - 2 + \frac{1}{n}\right) - E_\psi\left(\frac{\text{đ} - 1}{2}\right) - (4 - \text{đ})\left(L + \frac{E}{2} - 1\right), \tag{3.27}$$

where E is the total number of external legs. The divergent part of G is a weighted polynomial of degree $\omega(G)$ in the external momenta. Recalling (3.24), the counterterms have the form (3.23), precisely

$$\left(\prod_i \lambda_i^{n_i}\right) \alpha^L g^{E-2} \hat{\partial}^K \bar{\partial}^M \hat{A}^{\hat{E}} \bar{A}^{\bar{E}} (\bar{C}C)^{E_{\rm gh}/2} \varphi^{E_\varphi} (\bar{\psi}\psi)^{E_\psi/2}, \qquad \text{with } K + \frac{M}{n} = \omega(G),$$

and therefore are subtracted renormalizing an appropriate coupling λ_i. This proves that the theory is renormalizable by weighted power counting assuming that BRST invariance is preserved and that the spurious subdivergences are absent.

Absence of infrared divergences Now we study the infrared behavior of correlations functions. Correlation functions have to be understood as distributions and calculated off-shell at non-exceptional external momenta. Exceptional configurations of momenta can be reached by analytical continuation. However, the continuation exists if the theory does not contain super-renormalizable vertices and massless fields, otherwise infrared divergences occur in Feynman diagrams at high orders, even off-shell [13]. We work out the conditions under which such problems do not occur in our models.

All theories with a non-trivial super-renormalizable subsector look alike at low energies. Up to terms of higher dimensions, they are described by the lagrangian (for generic $\hat{d}$)

$$\mathcal{L}_{\rm IR} = \frac{1}{4}\left[(F^a_{\hat{\mu}\hat{\nu}})^2 + 2\eta_{n-1}(F^a_{\hat{\mu}\bar{\nu}})^2 + \tau_{2n-2}(F^a_{\bar{\mu}\bar{\nu}})^2\right], \tag{3.28}$$

where η_{n-1} and τ_{2n-2} are constants. Moreover the gauge-fixing becomes $\mathcal{G}^a_{\rm IR} = \hat{\partial}\cdot\hat{A}^a + \zeta_{n-1}\bar{\partial}\cdot\bar{A}^a$. We assume, compatibly with (3.9) and (3.12),

$$\eta_{n-1} > 0, \qquad \tau_{2n-2} > 0, \qquad \zeta_{n-1} > 0.$$

In general, the theory $\mathcal{L}_{\rm IR}$ is Lorentz violating, but has an ordinary power counting, so we can use the results known from ordinary quantum field theory, which tell us that the Feynman diagrams of $\mathcal{L}_{\rm IR}$ have no IR divergences (at non-exceptional external momenta) if and only if

$$d \geq 4. \tag{3.29}$$

Observe that if $d > 4$ the theory $\mathcal{L}_{\rm IR}$ is non-renormalizable, but this fact does not concern our present discussion, since our models are not just $\mathcal{L}_{\rm IR}$, but contain terms of higher dimensionalities that cure the UV behavior.

In strictly renormalizable theories different conditions apply (see section 6).

Massless fields are responsible for another type of IR divergences, those that occur in cross sections of non-confining gauge theories (infrared catastrophe). Such divergences are usually treated with the Bloch-Nordsieck resummation method [14]. This issue is beyond the scope of the present paper, yet we expect that the argument of Bloch and Nordsieck can be adapted also to Lorentz violating theories.

In conclusion, recalling (3.4), (3.22) and (3.29), the conditions to have consistent renormalizable $1/\alpha$ gauge theories with a non-trivial super-renormalizable subsector are

$$d=\text{even}\geq 4,\qquad \hat{d}=1,\qquad \text{đ}<2+\frac{2}{n},\qquad n=\text{odd}. \tag{3.30}$$

In four dimensions they reduce to just $\hat{d}=1$, $n=$odd (for $n>1$).

4 Renormalizability to all orders

So far we have concentrated on the renormalizability of our theories by weighted power counting. It remains to prove that the subtraction of divergences is compatible with gauge invariance. We use the Batalin-Vilkovisky formalism [15]. For simplicity, we concentrate on pure gauge theories and use the minimal subtraction scheme and the dimensional-regularization technique. In particular, the functional integration measure is automatically BRST invariant.

Classical proofs of the renormalizability of (Lorentz invariant) Yang-Mills theories can be found in most textbooks [16]. Complete classifications of the BRST cohomology of local operators [17] and local functionals of arbitrary ghost number [18] are available. The generalization of such classification theorems to Lorentz violating theories appears to be conceptually simple, but technically involved, and is beyond the scope of this paper.

Batalin-Vilkovisky formalism The fields are collectively denoted by $\Phi^i=(A_\mu^a,\overline{C}^a,C^a,B^a)$. Add BRST sources $K_i=(K_a^\mu,K_{\overline{C}}^a,K_C^a,K_B^a)$ for every field Φ^i and extend the action (3.13) as

$$\Sigma(\Phi,K)=\mathcal{S}(\Phi)-\int \mathrm{d}^dx\left[\left(sA_\mu^a\right)K_a^\mu+\left(s\overline{C}^a\right)K_{\overline{C}}^a+\left(sC^a\right)K_C^a+\left(sB^a\right)K_B^a\right], \tag{4.1}$$

From (4.1) we can read the weights of the BRST sources:

$$[K_a^{\hat{\mu}}]=[K_{\overline{C}}^a]=[K_C^a]=\frac{\text{đ}}{2},\qquad [K_a^{\bar{\mu}}]=\frac{\text{đ}}{2}+1-\frac{1}{n},\qquad [K_B^a]=\frac{\text{đ}}{2}-1. \tag{4.2}$$

Define the antiparenthesis

$$(X,Y)=\int \mathrm{d}^dx\left\{\frac{\delta_rX}{\delta\Phi^i(x)}\frac{\delta_lY}{\delta K_i(x)}-\frac{\delta_rX}{\delta K_i(x)}\frac{\delta_lY}{\delta\Phi^i(x)}\right\}. \tag{4.3}$$

BRST invariance is generalized to the identity

$$(\Sigma,\Sigma)=0, \tag{4.4}$$

which is a straightforward consequence of (4.1), the gauge invariance of $\mathcal{S}_0$ and the nilpotency of s. Define also the generalized BRST operator

$$\sigma X \equiv (\Sigma, X), \tag{4.5}$$

which is nilpotent ($\sigma^2 = 0$), because of the identity (4.4). Observe that σ, as well as s, raises the weight by one unit.

The generating functionals Z, W and Γ are defined, in the Euclidean framework, as

$$Z[J,K] = \int \mathcal{D}\Phi \exp\left(-\Sigma(\Phi,K) + \int \Phi^i J_i\right) = \mathrm{e}^{W[J,K]}, \tag{4.6}$$
$$\Gamma[\Phi_\Gamma, K] = -W[J,K] + \int \Phi^i_\Gamma J_i, \qquad \text{where} \qquad \Phi^i_\Gamma = \frac{\delta_r W[J,K]}{\delta J_i}.$$

Below we often suppress the subscript Γ in Φ_Γ. Performing a change of variables

$$\Phi' = \Phi + \theta s \Phi, \tag{4.7}$$

in the functional integral (4.6), θ being a constant anticommuting parameter, and using the identity (4.4), we find

$$(\Gamma, \Gamma) = 0. \tag{4.8}$$

A canonical transformation of fields and sources is defined as a transformation that preserves the antiparenthesis. It is generated by a functional $\mathcal{F}(\Phi, K')$ and reads

$$\Phi^{i\,\prime} = \frac{\delta \mathcal{F}}{\delta K'_i}, \qquad K_i = \frac{\delta \mathcal{F}}{\delta \Phi^i}.$$

The generating functional of the identity transformation is

$$\mathcal{I}(\Phi, K') = \int \mathrm{d}^d x \sum_i \Phi^i K'_i.$$

As usual, renormalizability is proved proceeding inductively. The inductive assumption is that up to the n-th loop included the divergences can be removed redefining the physical parameters α_i and performing a canonical transformation of the fields and BRST sources. Call Σ_n and $\Gamma^{(n)}$ the action and generating functional renormalized up to the n-th loop included. The inductive assumption ensures that Σ_n and $\Gamma^{(n)}$ satisfy (4.4) and (4.8), respectively.

Locality and (4.8) imply that the $(n+1)$-loop divergences $\Gamma^{(n)}_{n+1\ \mathrm{div}}$ of $\Gamma^{(n)}$ are local and σ-closed, namely $\sigma \Gamma^{(n)}_{n+1\ \mathrm{div}} = 0$. We have to work out the most general solution to this condition. First, observe that $\Gamma^{(n)}_{n+1\ \mathrm{div}}$ cannot contain B^a, K^a_B and $K^a_{\overline{C}}$, because the action (4.1) provides no vertices with B^a, K^a_B or $K^a_{\overline{C}}$ on the external legs. In particular, the absence of vertices with B-legs is due to the linearity of the gauge-fixing $\mathcal{G}^a$ (3.11) in the gauge field A. Second, observe that the vertices that contain an antighost $\bar{C}$ contain also a $\hat{\partial}$ or a $\zeta(\bar{v})\,\bar{\partial}$ acting on $\bar{C}$. The vertex containing $\hat{\partial}\overline{C}$ has an identical vertex-partner where $\hat{\partial}\overline{C}$ is replaced by $\hat{K}_A$, while the vertex containing $\zeta(\bar{v})\,\bar{\partial}\overline{C}$ has an identical vertex-partner where $\zeta(\bar{v})\,\bar{\partial}\overline{C}$ is replaced by $\bar{K}_A$. Therefore $\Gamma_{n+1\text{ div}}^{(n)}$ can depend on $\overline{C}$, $\hat{K}_A$ and $\bar{K}_A$ only through the combinations

$$K_a^{\hat{\mu}}+\partial^{\hat{\mu}}\overline{C}^a, \qquad K_a^{\bar{\mu}}+\zeta(\bar{v})\,\partial^{\bar{\mu}}\overline{C}^a.$$

Using the facts just proved, invariance under global gauge transformations and the weighted power counting, we find that in the $1/\alpha$ theories $\Gamma_{n+1\text{ div}}^{(n)}$ has the form

$$\Gamma_{n+1\text{ div}}^{(n)}=\int \mathrm{d}^d x\left[\widetilde{\mathcal{G}}_n(A)+\left(K_a^{\hat{\mu}}+\partial^{\hat{\mu}}\overline{C}^a\right)\left(a_n'\partial_{\hat{\mu}}C^a+h_n^{abc}A_{\hat{\mu}}^b C^c\right)\right.$$
$$\left.+\left(K_a^{\bar{\mu}}+\zeta(\bar{v})\,\partial^{\bar{\mu}}\overline{C}^a\right)\left(b_n'\partial_{\bar{\mu}}C^a+k_n^{abc}A_{\bar{\mu}}^b C^c\right)+e_n g f^{abc}K_C^a C^b C^c\right], \qquad (4.9)$$

where $\widetilde{\mathcal{G}}_n$ depends only on A_μ^a and has weight đ, while a_n', b_n', e_n, h_n^{abc} and k_n^{abc} are weightless constants. Considering the terms proportional to $(\partial^{\hat{\mu}}\overline{C}^a)(\partial_{\hat{\mu}}C^b)C^c$ and $(\partial^{\bar{\mu}}\overline{C}^a)(\partial_{\bar{\mu}}C^b)C^c$ contained in $\sigma\Gamma_{\text{div}}^{(n+1)}=0$, we see that h_n^{abc} and k_n^{abc} must be proportional to f^{abc}. Then (4.9) can be reorganized in the more convenient form

$$\Gamma_{n+1\text{ div}}^{(n)}=\int \mathrm{d}^d x\left[\widetilde{\mathcal{G}}_n(A)+\left(K_a^{\hat{\mu}}+\partial^{\hat{\mu}}\overline{C}^a\right)\left(a_n\partial_{\hat{\mu}}C^a+c_n D_{\hat{\mu}}C^a\right)\right.$$
$$\left.+\left(K_a^{\bar{\mu}}+\zeta(\bar{v})\,\partial^{\bar{\mu}}\overline{C}^a\right)\left(b_n\partial_{\bar{\mu}}C^a+d_n D_{\bar{\mu}}C^a\right)+e_n g f^{abc}K_C^a C^b C^c\right], \qquad (4.10)$$

with new constants a_n, b_n, c_n and d_n. Working out the condition $\sigma\Gamma_{\text{div}}^{(n+1)}=0$ in detail we find $c_n=d_n=2e_n$ and

$$\widetilde{\mathcal{G}}_n=\mathcal{G}_n-a_n\frac{\delta\mathcal{S}_0}{\delta A_{\hat{\mu}}^a}A_{\hat{\mu}}^a-b_n\frac{\delta\mathcal{S}_0}{\delta A_{\bar{\mu}}^a}A_{\bar{\mu}}^a,$$

where $\mathcal{G}_n$ is gauge invariant and $\mathcal{S}_0$ is given by formula (3.6). We have used the property that the gauge-field equations $\delta\mathcal{S}_0/\delta A_{\hat{\mu}}^a$ and $\delta\mathcal{S}_0/\delta A_{\bar{\mu}}^a$ transform covariantly. The result can be collected into the compact form

$$\Gamma_{n+1\text{ div}}^{(n)}=\int \mathrm{d}^d x\left(\mathcal{G}_n+\sigma\mathcal{R}_n\right), \qquad (4.11)$$

with

$$\mathcal{R}_n(\Phi,K)=\int \mathrm{d}^d x\left(-a_n I_1-b_n I_2+c_n I_3\right), \qquad (4.12)$$

where

$$I_1(\Phi,K)=(K_a^{\hat{\mu}}+\partial^{\hat{\mu}}\overline{C}^a)A_{\hat{\mu}}^a, \qquad I_2(\Phi,K)=\left(K_a^{\bar{\mu}}+\partial^{\bar{\mu}}\zeta(\bar{v})\,\overline{C}^a\right)A_{\bar{\mu}}^a, \qquad I_3(\Phi,K)=K_C^a C^a, \qquad (4.13)$$

Now, $\mathcal{G}_n$ is local, gauge-invariant, constructed with A and its derivatives, and has weight đ. Since, by assumption, $\mathcal{S}_0$ contains the full set of such terms, $\mathcal{G}_n$ can be reabsorbed renormalizing the physical couplings α_i contained in $\mathcal{S}_0$. We denote these renormalization constants by Z_{α_i}.

The σ-exact counterterms are reabsorbed with a canonical transformation generated by

$$\mathcal{F}_n(\Phi, K') = \mathcal{I}(\Phi, K') - \mathcal{R}_n(\Phi, K'). \tag{4.14}$$

More explicitly, $\overline{C}^a$, B^a and K_B^a are non-renormalized and the only non-trivial redefinitions are

$$\begin{aligned}
&\hat{A}^a \rightarrow \hat{Z}_{nA}^{1/2}\hat{A}^a, \qquad \bar{A}^a \rightarrow \bar{Z}_{nA}^{1/2}\bar{A}^a, \qquad C^a \rightarrow Z_{nC}^{1/2}C^a, \qquad K_C^a \rightarrow Z_{nC}^{-1/2}K_C^a, \\
&K_a^{\hat{\mu}} \rightarrow \hat{Z}_{nA}^{-1/2}(K_a^{\hat{\mu}} + \partial^{\hat{\mu}}\overline{C}^a) - \partial^{\hat{\mu}}\overline{C}^a, \qquad K_a^{\bar{\mu}} \rightarrow \bar{Z}_{nA}^{-1/2}(K_a^{\bar{\mu}} + \zeta\,(\bar{v})\,\partial^{\bar{\mu}}\overline{C}^a) - \zeta\,(\bar{v})\,\partial^{\bar{\mu}}\overline{C}^a, \\
&K_{\overline{C}}^a \rightarrow K_{\overline{C}}^a + \hat{\partial}\cdot\hat{A}^a(\hat{Z}_{nA}^{1/2} - 1) + \zeta\,(\bar{v})\,\bar{\partial}\cdot\bar{A}^a(\bar{Z}_{nA}^{1/2} - 1),
\end{aligned} \tag{4.15}$$

where

$$\hat{Z}_{nA}^{1/2} = 1 + a_n, \qquad \bar{Z}_{nA}^{1/2} = 1 + b_n, \qquad Z_{nC}^{1/2} = 1 - c_n. \tag{4.16}$$

Call $f(Z_n)$ the map (4.15), where $Z_n = (\hat{Z}_{nA}, \bar{Z}_{nA}, Z_{nC})$. It is straightforward to check that it satisfies the group property

$$f(Z_p) \circ f(Z_q) = f(Z_p Z_q), \qquad Z_p Z_q \equiv (\hat{Z}_{pA}\hat{Z}_{qA}, \bar{Z}_{pA}\bar{Z}_{qA}, Z_{pC}Z_{qC}). \tag{4.17}$$

The $(n+1)$-loop divergences (4.11) are reabsorbed by a map $h(Z_{n\alpha}, Z_n)$ obtained composing the renormalizations of the physical couplings α_i with $f(Z_n)$. Clearly, also h satisfies the group property (4.17). Moreover, the basis (4.13) is h- and f-invariant.

Being a composition of a canonical transformation and redefinitions of the physical couplings, $h(Z_{n\alpha}, Z_n)$ preserves both (4.4) and (4.8). This proves the inductive hypothesis to the order $n+1$, and therefore promotes (4.4) and (4.8) to all orders.

The complete renormalization of divergences is performed by the map

$$h_\infty = \prod_{n=1}^{\infty} h(Z_{n\alpha}, Z_n) = h(Z_\alpha, Z), \qquad Z_\alpha = \prod_{n=1}^{\infty} Z_{n\alpha}, \qquad Z = \prod_{n=1}^{\infty} Z_n.$$

Applying h_∞ to the action Σ, it is easy to prove that the renormalization can be equivalently performed in a more standard multiplicative fashion, namely

$$\begin{aligned}
&\hat{A}^a \rightarrow \hat{Z}_A^{1/2}\hat{A}^a, \qquad \bar{A}^a \rightarrow \bar{Z}_A^{1/2}\bar{A}^a, \qquad C^a \rightarrow Z_C^{1/2}C^a, \qquad \overline{C}^a \rightarrow \hat{Z}_A^{-1/2}\overline{C}^a, \\
&B^a \rightarrow \hat{Z}_A^{-1/2}B^a, \qquad \lambda \rightarrow \lambda\hat{Z}_A, \qquad \zeta \rightarrow \hat{Z}_A^{1/2}\bar{Z}_A^{-1/2}\zeta, \qquad \alpha_i \rightarrow \alpha_i Z_{\alpha_i}.
\end{aligned} \tag{4.18}$$

The renormalization constants of the BRST sources are the reciprocals of the renormalization constants of the fields: $\Phi^i \rightarrow \Phi^i Z_i^{1/2} \Leftrightarrow K_i \rightarrow K_i Z_i^{-1/2}$.

The relatively simple structure of the counterterms is due to the simple form of the gauge fixing (3.11), which is linear in the gauge potential. Had we chosen a non-linear gauge fixing, for example the most general local function $\mathcal{G}^a$ of weight 2 constructed with the gauge potential and its derivatives, there would be vertices with B-external legs, and therefore also counterterms with

B's on the external legs. Then $\mathcal{R}_n$ would be much less constrained, to allow for the most general renormalization of the gauge-fixing parameters contained in $\mathcal{G}^a$.

At the practical level, computations considerably simplify using the background field method [19], which can be applied straightforwardly to our theories.

5 Renormalizable theories

In this section we investigate the four dimensional renormalizable theories and study the low-energy recovery of Lorentz invariance.

We start with the $1/\alpha$ theory with $\hat{d} = 1$, $n = 2$, đ$= 5/2$. Its (Euclidean) lagrangian reads

$$\begin{aligned}\mathcal{L}_{1/\alpha} = &\frac{1}{4}\left[\frac{2\eta_0}{\Lambda_L^2}(D_{\bar\rho}^{ab}F_{\hat\mu\bar\nu}^b)^2 + \frac{\tau_0}{\Lambda_L^4}(\bar D^2 F_{\bar\mu\bar\nu}^a)^2 + \frac{\xi_0}{\Lambda_L^2}(D_{\hat\rho}F_{\bar\mu\bar\nu})(D_{\hat\rho}F_{\bar\mu\bar\nu}) + 2\eta_1(F_{\hat\mu\bar\nu}^a)^2\right.\\ &\left.+\frac{\tau_1}{\Lambda_L^2}(D_{\bar\rho}^{ab}F_{\bar\mu\bar\nu}^b)^2 + \tau_2(F_{\bar\mu\bar\nu}^a)^2\right] + \frac{g}{\Lambda_L^2}f_{abc}\left(\lambda F^a_{\hat\mu\bar\nu}F^b_{\hat\mu\bar\rho} + \lambda' F^a_{\bar\mu\bar\nu}F^b_{\bar\mu\bar\rho}\right)F^c_{\bar\nu\bar\rho}\\ &+\frac{g}{\Lambda_L^4}\sum_j \lambda_j \bar D^2\bar F^3{}_j + \frac{\alpha}{\Lambda_L^4}\sum_k \lambda'_k \bar F^4{}_k, \end{aligned} \tag{5.1}$$

where j labels the independent gauge invariant terms constructed with two covariant derivatives $\bar D$ acting on three field strengths $\bar F$, and k labels the terms constructed with four $\bar F$'s. The last two terms are symbolic, while the rest contains the precise list of allowed terms.

In every super-renormalizable $1/\alpha$ theory we have $\beta_{\Lambda_L} = \beta_{\tau_0} = \beta_{\eta_0} = \beta_{\xi_0} = \beta_\alpha = 0$. Indeed, we know that Λ_L is RG invariant, by construction. So are η_0, τ_0 and ξ_0, because they are weightless. The α-beta function vanishes, because each vertex carries at least a factor of g.

The parameters of the model (5.1) are actually finite. Indeed, we have the weights $[g] = 3/4$, $[\eta_1] = [\tau_1] = [\lambda'] = 1$, $[\tau_2] = 2$, $[\lambda] = [\lambda_j] = [\lambda'_k] = 0$ and by (3.24) every diagram is multiplied by $\alpha^L g^{E-2}$. Therefore, no counterterm can fit into the structure (5.1). However, the model (5.1) has an even n, so it may have spurious subdivergences. Going through the analysis of Appendix B it is possible to show that such subdivergences appear only at three loops.

The low energy limit of (5.1) can be studied taking Λ_L to infinity. We get

$$\mathcal{L}_{\mathrm{IR}} = \frac{1}{4}\left[2\eta_1(F^a_{\hat\mu\bar\nu})^2 + \tau_2(F^a_{\bar\mu\bar\nu})^2\right]. \tag{5.2}$$

Lorentz invariance is recovered, because the redefinition

$$\begin{aligned}&\hat{x}' = \hat{x}, \qquad \hat{A}' = (\eta_1\tau_2)^{1/4}\hat{A}, \qquad \bar{x}' = \eta_1^{1/2}\tau_2^{-1/2}\bar{x}, \qquad \bar{A}' = \eta_1^{-1/4}\tau_2^{3/4}\bar{A},\\ &\alpha' = (\eta_1\tau_2)^{-1/2}\alpha, \qquad \mu' = \mu, \qquad \Lambda'_L = \eta_1^{-1}\tau_2\Lambda_L,\end{aligned} \tag{5.3}$$

converts the low-energy action into the manifestly Lorentz invariant form

$$\mathcal{S}_{\mathrm{IR}} = \int \mathrm{d}^4x\ \mathcal{L}_{\mathrm{IR}} = \int \mathrm{d}^4x'\ \frac{1}{4}(F^a_{\mu\nu}{}')^2. \tag{5.4}$$

In (5.3) we have included the μ- and Λ_L-transformations so the redefinition can be applied also to the high-energy theory, if the remaining couplings are rescaled appropriately. The divergences of the low-energy theory include those due to the $\Lambda_L \to \infty$ limit. Moreover, to have Lorentz invariance at low energies, the (low-energy) subtraction scheme has to be properly adjusted.

Observe that (5.3) is a combination of a gauge-field normalization,

$$A' = \eta_1 \tau_2^{-1/2} A, \qquad \alpha' = \eta_1^{-2}\tau_2 \alpha, \tag{5.5}$$

a usual rescaling,

$$x' = \eta_1 \tau_2^{-1} x, \qquad A' = \eta_1^{-1}\tau_2 A, \qquad \alpha' = \alpha, \qquad \mu' = \eta_1^{-1}\tau_2 \mu, \qquad \Lambda_L' = \eta_1^{-1}\tau_2 \Lambda_L, \tag{5.6}$$

and a weighted rescaling [6],

$$\begin{aligned}
&\hat{x}' = \eta_1^{-1}\tau_2 \hat{x}, \qquad \hat{A}' = \eta_1^{1/4}\tau_2^{-1/4}\hat{A}, \qquad \bar{x}' = \eta_1^{-1/2}\tau_2^{1/2}\bar{x}, \qquad \bar{A}' = \eta_1^{-1/4}\tau_2^{1/4}\bar{A}, \\
&\alpha' = \eta_1^{3/2}\tau_2^{-3/2}\alpha, \qquad \mu' = \eta_1\tau_2^{-1}\mu, \qquad \Lambda_L' = \Lambda_L.
\end{aligned} \tag{5.7}$$

Moreover, note that (5.3) leaves μ unchanged (although it changes Λ_L), therefore it generates no anomalous contributions at high energies. On the other hand, anomalous effects are generated at low energies. Because of (5.6) and (5.7), such effects are the difference between the trace anomaly and the weighted trace anomaly, and correspond to a Λ_L-running with no μ-running. They amount to a low-energy scheme change and are taken into account in the scheme adjustment mentioned above, necessary to restore Lorentz invariance at low energies. The gauge-fixing sector of the theory can remain Lorentz violating with no observable consequence.

In general, to recover Lorentz invariance at low energies we are free to perform just one redefinition, which amounts, ultimately, to a particular $\bar{x}$-rescaling. In the case (5.2) the Lorentz recovery is possible thanks to the simple structure of the theory. Consider a more general situation, for example the theory (5.2) coupled to fermions. Its lagrangian is $\mathcal{L}_{gf} = \mathcal{L}_{1/\alpha} + \mathcal{L}_f$, where

$$\mathcal{L}_f = \bar{\psi}\left(\hat{\not{D}} + \frac{\eta_{0f}}{\Lambda_L}\bar{\not{D}}^2 + \eta_{1f}\bar{\not{D}} + m_f + \frac{\tau_f g}{\Lambda_L} i T^a \bar{F}^a_{\bar{\mu}\bar{\nu}}\sigma^{\bar{\mu}\bar{\nu}}\right)\psi. \tag{5.8}$$

The low-energy lagrangian reads now

$$\mathcal{L}_{\mathrm{IR}\ gf} = \frac{1}{4}\left[2\eta_1 (F^a_{\hat{\mu}\bar{\nu}})^2 + \tau_2 (F^a_{\bar{\mu}\bar{\nu}})^2\right] + \bar{\psi}(\hat{\not{D}} + \eta_{1f}\bar{\not{D}} + m_f)\psi. \tag{5.9}$$

The transformations (5.5), (5.6) and (5.7) extend to fermions as

$$\psi' = \psi, \qquad \psi' = \eta_1^{-3/2}\tau_2^{3/2}\psi, \qquad \psi' = \eta_1^{3/4}\tau_2^{-3/4}\psi,$$

respectively, so

$$\int \mathrm{d}^4 x\ \mathcal{L}_{\mathrm{IR}\ gf} = \int \mathrm{d}^4 x' \ \left[\frac{1}{4}(F^a_{\mu\nu}{}')^2 + \bar{\psi}'\left(\hat{\not{D}}' + \eta_{1f}'\bar{\not{D}}' + m_f\right)\psi'\right],$$

where

$$\eta'_{1f} = \eta_{1f} \left(\frac{\eta_1}{\tau_2}\right)^{1/2}.$$

Therefore, Lorentz invariance cannot be recovered at low energies unless the low-energy couplings are located on the "Lorentz invariant surface"

$$\tau_2 = \eta_{1f}^2 \eta_1. \tag{5.10}$$

If that does not happen the effects of the violation become observable also at low energies.

The Lorentz invariant surface (5.10) is also RG invariant, which means that there exists a low-energy subtraction scheme such that the beta function of $\delta_L \equiv \tau_2 - \eta_{1f}^2 \eta_1$ is proportional to δ_L. RG invariance guarantees that it is not necessary to specify at which low-energy scale the relation (5.10) must hold: if it holds at some low-energy scale it holds at all low-energy scales. Perturbative calculations suggest [9] that in CPT-invariant theories the Lorentz invariant surface might also be RG stable, i.e. the couplings that parametrize the displacement from the surface, such as δ_L, are IR free[3]. When the Lorentz invariant surface is not RG stable, Lorentz invariance can be recovered at low energies only by means of an appropriate fine-tuning. With great accuracy experiments show [5] that the Standard Model is located on the Lorentz invariant surface.

If we look at the issue of Lorentz-invariance recovery from a low-energy viewpoint, it is more natural to consider (5.1) plus (5.8), equipped with (5.10), as a (partial) regularization of the Lorentz-invariant Yang-Mills theory

$$\int \mathrm{d}^4 x \left[\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}\left(\not{D} + m_f\right)\psi\right].$$

In this description, the existence of a Lorentz-preserving subtraction scheme is obvious and the fine-tuning (5.10) appears more natural. Then, however, low-energy Lorentz invariance is assumed from the start. Moreover, we insist that our theories should not be viewed as regularization devices, but as true, fundamental theories, to be experimentally tested.

The model (5.1) is not free of spurious subdivergences, because n is even. The first completely consistent model is thus the $1/\alpha$ theory with $\hat{d} = 1$, $n = 3$, đ$= 2$, which is studied in detail in paper II. Its simplest renormalizable lagrangian is the sum of the quadratic part (3.7) plus $\bar{F}^3$. Other consistent four dimensional solutions to (3.30) exist for every odd $n \geq 3$. The low-energy considerations of this section are very general and apply to the odd-n models with obvious modifications.

[3]Other results [20] give evidence that CPT-violating couplings exhibit the opposite behavior.

6 Strictly renormalizable theories

For the sake of completeness, we investigate strictly renormalizable and weighted scale invariant theories. We recall that there exists a class of subtraction schemes in which no power-like divergences are generated. In those schemes super-renormalizable parameters are not turned on by renormalization. Moreover, such class of schemes is automatically chosen by the dimensional-regularization technique, which we assume in this paper.

In strictly-renormalizable theories the quadratic part $\mathcal{L}_Q$ of the lagrangian must have

$$\eta(\bar{\Upsilon}) = \eta_0 \bar{\Upsilon}^{n-1}, \qquad \tau(\bar{\Upsilon}) = \tau_0 \bar{\Upsilon}^{2(n-1)}, \qquad \xi(\bar{\Upsilon}) = \xi_0 \bar{\Upsilon}^{n-2}.$$

For convenience we can choose a strictly-renormalizable gauge fixing, with $\zeta(\bar{\upsilon}) = \zeta_0 \bar{\upsilon}^{n-1}$. The inequalities (3.21) must hold.

The IR behavior of Feynman diagrams is still dominated by the weighted power counting, so the analysis of potential IR divergences differs from the one of section 3. Now $\eta(0) = \tau(0) = 0$, so the gauge-field propagator contains additional denominators $\sim 1/\bar{k}^{2(n-1)}$ in the $\langle \bar{A}\bar{A} \rangle$-sector. The loop integrals over k and the loop sub-integrals over $\bar{k}$ are IR divergent unless

$$\text{đ} > 4 - \frac{2}{n}, \qquad \bar{d} > 2(n-1), \tag{6.1}$$

respectively. The former condition follows from (3.25). The latter condition and $n \geq 2$ imply $\bar{d} \geq 3$.

We can now distinguish two cases: if đ$= 4$ the gauge coupling is strictly-renormalizable, while if đ< 4 the theory can be strictly-renormalizable only if it is Abelian, but not $1/\alpha$. However, the models with đ$= 4$ do not satisfy (3.22), so we cannot ensure that they are free of subdivergences, even at $\hat{d} = 1$. Moreover, such models exist only in dimensions greater than 6. Indeed, it is easy to see that $n \geq 2$, đ$= 4$ and (3.22) imply

$$\bar{d} = 3n, \qquad d = 1 + 3n \geq 7,$$

and (6.1) are automatically satisfied. The simplest example of this kind is the seven-dimensional theory with $n = 2$, $\bar{d} = 6$. Its lagrangian reads, in symbolic form

$$\mathcal{L} = \mathcal{L}_Q + \frac{\lambda}{\Lambda_L^{7/2}} \tilde{F}^2 \bar{F} + \frac{\lambda'}{\Lambda_L^{11/2}} \bar{D}^2 \bar{F}^3 + \frac{\lambda''}{\Lambda_L^7} \bar{F}^4. \tag{6.2}$$

Other examples of strictly renormalizable theories are the fixed points of the RG flow, which are exactly weighted scale invariant. Following [7], we can attempt to construct four dimensional weighted scale invariant theories in the large N expansion, where N is the number of fermion copies. However, we can easily prove that this attempt fails in the presence of gauge fields. Consider the model with lagrangian

$$\mathcal{L} = \sum_{i=1}^{N} \bar{\psi}_i \left(\hat{\not{D}} + \frac{\bar{\not{D}}^n}{\Lambda_L^{n-1}} \right) \psi_i. \tag{6.3}$$

For simplicity we assume that the gauge field is Abelian, but the argument generalizes straightforwardly to non-Abelian gauge fields. The gauge fields do not have a kinetic term, which is provided by a one-loop diagram. Since the gauge-field propagator is dynamically generated, the regularity conditions (3.22) might have to be replaced by more sophisticated restrictions. We assume (6.1) to ensure that Feynman diagrams are free of IR divergences. We study under which conditions the theory (6.3) is renormalizable in the form (6.3). Observe that if $\bar{d} > 1$ there always exist counterterms of the form

$$\frac{1}{\Lambda_L^{n-1}} \sum_{i=1}^{N} \bar{\psi}_i F_{\bar{\mu}\bar{\nu}} \sigma^{\bar{\mu}\bar{\nu}} \bar{\not{D}}^{n-2} \psi_i, \tag{6.4}$$

that make the theory (6.3) not renormalizable. Then we must require $\bar{d} = 1$, so that the counterterms (6.4) are trivial. However, this condition is incompatible with (6.1).

We conclude that Lorentz violating gauge theories with strictly renormalizable gauge couplings are problematic at $n > 1$.

7 Proca theories

We conclude considering Proca versions of our theories and study the UV behaviors of their propagators. Instead of being gauge-fixed, now the lagrangian (3.7) is equipped with a mass term

$$\mathcal{L}_m = \frac{m^2}{2} \left(\hat{A}^2 + \bar{A} \tilde{\zeta} \left(\bar{\upsilon} \right) \bar{A} \right)$$

for the gauge fields, where $\tilde{\zeta}$ is a polynomial of degree $n - 1$. At the free-field level, acting with a derivative on the field equations we get the identity

$$\hat{\partial} \cdot \hat{A} + \tilde{\zeta} \left(\bar{\upsilon} \right) \bar{\partial} \cdot \bar{A} = 0,$$

which kills one degree of freedom.

Expressing the propagator of $\mathcal{L}_Q + \mathcal{L}_m$ in the form (3.15) we find

$$u = \frac{1}{D(1, \eta) + m^2}, \qquad v = \frac{1}{D(\tilde{\eta}, \tau) + m^2 \tilde{\zeta}}, \qquad r = \frac{\eta}{m^2 \left(\eta D(1, \tilde{\zeta}) + m^2 \tilde{\zeta} \right)},$$
$$s = \frac{ur}{\eta} \left(\eta D(1, \eta) + m^2 \tilde{\zeta} \right), \qquad t = \frac{vr}{\eta} \left(\eta D(\tilde{\eta}, \tau) + m^2 \tilde{\tau} \right).$$

While u and v have regular behaviors, we have

$$r, s, t \sim \frac{1}{m^2 \hat{k}^2}, \quad \text{for } \hat{k}^2 \to \infty, \qquad r, s, t \sim \frac{1}{m^2 \tilde{\zeta} \bar{k}^2}, \quad \text{for } \bar{k}^2 \to \infty.$$

We see that the large-momentum behaviors of u and v agree with weighted power counting, but those of r, s and t dot not. We conclude that the Lorentz violating Proca theories are not renormalizable.

8 Conclusions

In this paper we have constructed Lorentz violating gauge theories that can be renormalized by weighted power counting. The theories contain higher space derivatives, but are arranged so that no counterterms with higher time derivatives are generated by renormalization. The absence of spurious subdivergences privileges the models where spacetime is split into space and time. We have focused on the simplest class of models, leaving the general classification of renormalizable theories to a separate paper.

If Lorentz invariance is violated at high energies there remains to explain why it should be recovered at low energies, since generically renormalization makes the couplings run independently and there is no apparent reason why the parameters of the low-energy theory should belong to the Lorentz invariant surface. It is of course possible to restore Lorentz invariance at low energies by means of a fine tuning, which is easier to justify when the Lorentz invariant surface is RG stable.

Acknowledgments

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Appendix A: Classification of the quadratic terms

In this appendix we derive the form (3.7) of the quadratic lagrangian $\mathcal{L}_Q$. It contains all terms of weights $\leq$đ, constructed with two field strengths and possibly covariant derivatives. Clearly there exists a single such term with two $\hat{F}$'s, that is $\hat{F}^2_{\mu\nu}$. Consider now the terms constructed with two $\bar{F}$'s and possibly derivatives $\bar{D}$. We start from

$$F_{\bar{\mu}\bar{\nu}} D_{\bar{\lambda}} \cdots D_{\bar{\tau}} F_{\bar{\rho}\bar{\sigma}} \tag{A.1}$$

and study all possible contractions. Up to addition of vertices, we can freely commute the covariant derivatives, since $[D_{\bar{\alpha}}, D_{\bar{\beta}}] = gF_{\bar{\alpha}\bar{\beta}}$. From now on every formula of this appendix is meant up to vertices and total derivatives. Clearly, contracting four derivatives of (A.1) with the indices $\bar{\mu},\bar{\nu},\bar{\rho},\bar{\sigma}$ we obtain a vertex. Contracting two derivatives with field-strength indices we obtain

$$\mathcal{I}_p \equiv F_{\bar{\mu}\bar{\nu}}(\bar{D}^2)^p D_{\bar{\rho}} D_{\bar{\mu}} F_{\bar{\rho}\bar{\nu}}.$$

Using the Bianchi identity on $D_{\bar{\mu}}F_{\bar{\rho}\bar{\sigma}}$ we easily get

$$\mathcal{I}_p = \frac{1}{2} F_{\bar{\mu}\bar{\nu}}(\bar{D}^2)^{p+1} F_{\bar{\mu}\bar{\nu}}.$$

This is the unique independent contraction of (A.1). By weighted power counting, p can be at most $2n-3$. The terms of this type are those corresponding to the function τ of (3.7).

Next, consider terms with two $\bar{F}$'s and derivatives $\bar{D}$ and $\hat{D}$. By $O(1,\hat{d}-1)$ invariance these terms can contain two $\hat{D}$'s, which must be contracted among themselves, or no $\hat{D}$, which is the case already considered. Arguing as before, we get new terms of the form

$$\mathcal{I}'_p = \frac{1}{2} F_{\bar{\mu}\bar{\nu}} \hat{D}^2 (\bar{D}^2)^p F_{\bar{\mu}\bar{\nu}}, \qquad p \leq n-2,$$

which correspond to the function ξ in (3.7).

The terms with one $\bar{F}$ and one $\tilde{F}$ are

$$\mathcal{I}''_p = F_{\hat{\mu}\bar{\nu}} D_{\hat{\mu}} (\bar{D}^2)^p D_{\bar{\sigma}} F_{\bar{\nu}\bar{\sigma}}.$$

Using the Bianchi identities we get $\mathcal{I}''_p = -\mathcal{I}'_p$, therefore nothing new. The terms with two $\tilde{F}$'s are

$$\mathcal{I}'''_p = F_{\hat{\mu}\bar{\nu}} D_{\bar{\lambda}_1} \cdots D_{\bar{\lambda}_{2p+2}} F_{\hat{\mu}\bar{\sigma}}.$$

No $\hat{D}$ derivatives can have place here, by weighted power counting. Contracting two derivatives with the field-strength indices $\bar{\nu}$ and $\bar{\sigma}$ we get the unique new contraction

$$\mathcal{I}'''_p - \mathcal{I}''_p = F_{\hat{\mu}\bar{\nu}} (\bar{D}^2)^{p+1} F_{\hat{\mu}\bar{\nu}}, \qquad p \leq n-2, \tag{A.2}$$

which gives the terms corresponding to η in (3.7). Finally, it is easy to see that no term with one $\hat{F}$ and one $\tilde{F}$, or one $\hat{F}$ and one $\bar{F}$, are allowed by weighted power counting.

Appendix B: Absence of spurious subdivergences

In this appendix we study the spurious subdivergences. We first need to classify the lagrangian terms that contain derivatives $\hat{\partial}$. We know that no term can contain more than two $\hat{\partial}$'s. Moreover, vertices with two $\hat{\partial}$'s cannot contain fermions and ghosts, because their weights would exceed đ. For the same reason, they cannot contain more than one scalar and do not depend on $\hat{A}$. Moreover, vertices with one $\hat{\partial}$ cannot contain fermions. Summarizing, $\hat{\partial}$-dependent vertices have necessarily the forms

$$X_1 \equiv \hat{\partial} f_1(\hat{A}, \bar{A}, \varphi, \bar{C}, C, \bar{\partial}), \qquad X_2 \equiv f_2(\bar{A}, \varphi, \bar{\partial})(\hat{\partial}\bar{A})(\hat{\partial}\bar{A}), \qquad X_2' \equiv f_2'(\bar{A}, \varphi, \bar{\partial})(\hat{\partial}^2\bar{A}), \quad \text{(B.1)}$$

where the $\bar{\partial}$-derivatives are allowed to act anywhere, as well as the $\hat{\partial}$-derivative in X_1. The functions f_1, f_2 and f_2' are constrained by locality, weighted power counting and BRST invariance, and their structure depends on n. However, their form is not relevant for the proof that follows.

The quadratic terms that do not fall in the classes (B.1) are

$$(\hat{\partial}\hat{A})^2, \qquad \bar{C}\hat{\partial}^2 C, \qquad \varphi\hat{\partial}^2\varphi, \qquad \bar{\psi}\hat{\partial}\psi. \quad \text{(B.2)}$$

Every other lagrangian term is $\hat{\partial}$-independent.

Our purpose is to derive sufficient conditions to ensure that all integrals are free of subdivergences, once counterterms for proper divergent subdiagrams are included. In particular, spurious subdivergences must be absent, because they do not correspond to any subdiagram, so there exist no counterterms that can subtract them. We work in the Feynman gauge (3.17)-(3.18) and assume that the spacetime dimension is even, together with $\hat{d} = 1$, n =odd. We use the dimensional regularization and proceed inductively in the loop order. The proof is considerably involved, and we have to split it in three steps.

First step: structure of integrals Consider a generic N-loop integral

$$\int \frac{\mathrm{d}\hat{k}_1}{(2\pi)^{\hat{d}}} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}_1}{(2\pi)^{\bar{d}}} \cdots \int \frac{\mathrm{d}\hat{k}_N}{(2\pi)^{\hat{d}}} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}_N}{(2\pi)^{\bar{d}}}, \quad \text{(B.3)}$$

with loop momenta $(k_1, \ldots, k_N)$. We have to prove that all subintegrals, in all parametrizations $(k_1', \ldots, k_N')$ of the momenta, are free of subdivergences. By the inductive assumption, all subintegrals

$$\prod_{j=1}^{M} \int \frac{\mathrm{d}\hat{k}_j'}{(2\pi)^{\hat{d}}} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}_j'}{(2\pi)^{\bar{d}}}, \quad \text{(B.4)}$$

where $M < N$, are subtracted, if divergent, by appropriate counterterms. We need to consider integrals where some hatted integrations are missing and the corresponding barred integrations are present, and/or viceversa. Start from subintegrals $\bar{\mathcal{I}}$ containing (B.4) and one integral

$$\int \frac{\mathrm{d}^{\bar{d}}\bar{k}_a'}{(2\pi)^{\bar{d}}} \quad \text{(B.5)}$$

for some a, but no integration over $\hat{k}_a'$. The form of propagators (3.18) and (3.16) ensures that differentiating $\bar{\mathcal{I}}$ a sufficient number of times with respect to $\hat{k}_a'$ the subintegral $\bar{\mathcal{I}}$ becomes overall convergent. Thus, its overall (spurious) $\bar{\mathcal{I}}$-subdivergence is polynomial in $\hat{k}'_a$. However, in the complete integral (B.3) $\bar{\mathcal{I}}$ must eventually be integrated over $\hat{k}'_a$. This operation kills the spurious subdivergence, because in dimensional regularization the integral of a polynomial vanishes.

Next, consider subintegrals $\bar{\mathcal{I}}$ containing (B.4) and a product

$$\prod_a \int \frac{\mathrm{d}^{\bar{d}} \bar{k}'_a}{(2\pi)^{\bar{d}}},$$

but no integration over the corresponding $\hat{k}'_a$s. Then there exists a combination of derivatives

$$\prod_a \frac{\partial^{n_a}}{\partial \hat{k}'^{\,n_a}_a},$$

for suitable n_as, that cures not only the overall divergence of $\bar{\mathcal{I}}$, but also its subdivergences (e.g. those of type (B.5)). Thus, the spurious subdivergences are a combination of contributions, each of which is local in at least one $\hat{k}'_a$: again, such spurious subdivergences are killed by the integrals over the $\hat{k}'_a$s.

We see that we do not need to worry about subintegrals $\bar{\mathcal{I}}$ containing an excess of barred integrations. On the other hand, we do need to worry about subintegrals $\hat{\mathcal{I}}$ containing an excess of hatted integrations or excesses of both types. We start from the subintegrals containing only hatted integrations.

Second step: $\hat{k}$-subintegrals Consider the $\hat{k}$-subintegral of a diagram G with L loops, v_1 vertices of type X_1, v_2 vertices of type X_2 and X'_2, Δv vertices of other types, I_B internal bosonic legs (including ghosts) and I_F internal fermionic legs. We have seen in section 3 that every bosonic propagator behaves at least like $1/\hat{k}^2$, for $\hat{k}$ large, while the fermionic propagator behaves like $1/\hat{k}$. The $\hat{k}$-subintegral behaves like

$$\int \mathrm{d}^{L\hat{d}}\hat{k}\ \frac{\hat{k}^{v_1+2v_2}}{(\hat{k}^2)^{I_B}\hat{k}^{I_F}}, \qquad \hat{\omega}(G) = L\hat{d} + v_1 + 2v_2 - 2I_B - I_F, \tag{B.6}$$

$\hat{\omega}(G)$ denoting its degree of divergence. Using $L = 1 + I_B + I_F - v_1 - v_2 - \Delta v$, we can write

$$\hat{\omega}(G) = L(\hat{d} - 2) + 2 + \Delta\hat{\omega}(G), \qquad \Delta\hat{\omega}(G) = I_F - v_1 - 2\Delta v.$$

We know that no vertices with four or more fermionic legs are allowed in $1/\alpha$ theories. Because of this fact, the fermionic internal lines must end at different vertices of the set Δv. Therefore, we have $I_F \leq \Delta v$ and $\Delta\hat{\omega}(G) \leq 0$. Then the integral (B.6) is certainly convergent for $\hat{d} = 1$ and $L > 2$. Instead, for $\hat{d} > 1$ there exist divergent diagrams with arbitrarily many loops.

From now on we assume $\hat{d} = 1$. We must consider the one- and two-loop diagrams more explicitly. Observe that no logarithmic divergence exists at one loop in odd (i.e. $\hat{d}$) dimensions.

More precisely,

$$\int \mathrm{d}^{\hat{d}}\hat{k}\ \frac{\hat{k}_{\hat{\mu}_1}\cdots\hat{k}_{\hat{\mu}_{2m-1}}}{(\hat{k}^2)^m}$$

is UV convergent by symmetric integration. Here we have kept $\hat{d}$ generic to emphasize that it is continued to complex values. Moreover, power-like divergences are absent using the dimensional-regularization technique.

We remain only with two-loop diagrams. Setting $\hat{d} = 1$ and $L = 2$ we find $\hat{\omega}(G) = I_F - v_1 - 2\Delta v \leq -v_1 - \Delta v \leq 0$. The potential divergence is logarithmic and can only occur for $I_F = v_1 = \Delta v = 0$. Let us leave the quadratic terms (B.2) aside for a moment. Divergent diagrams can contain only vertices of types X_2 and X_2'. Moreover, the $\hat{\partial}$'s of X_2 and X_2' must act on internal legs and the external momenta can be set to zero. The internal legs can only be of type $\bar{A}\bar{A}$, plus possibly one internal φ-leg. At $L = 2$ we have $v_2 + 1$ internal legs, so the diagram has the form of Fig. 1, (a) or (b). However, diagram (a) is the product of two one-loop diagrams, so it does not diverge. Consider now the vertex F of diagram (b). It can be an X_2 or an X_2'. The two $\hat{k}$'s shown in the figure are those belonging to F. If F is an X_2, the vertex and the two propagators attached to it make in total $\hat{k}\cdot\hat{k}/(\hat{k}^2)^2 = 1/\hat{k}^2$. The same conclusion holds if F is an X_2'. Therefore, the divergent part of diagram (b) coincides (apart from external factors) with the one of the modified diagram where F is suppressed. Similarly, we can suppress the vertices A, B, C, D, E and G, and reduce to the diagram of Fig. 1, (b′), which has the form

$$\int \mathrm{d}^{\hat{d}}\hat{k}\ \mathrm{d}^{\hat{d}}\hat{p}\ \frac{P_4(\hat{k},\hat{p})}{\hat{k}^2\hat{p}^2(\hat{k}+\hat{p})^2}, \tag{B.7}$$

where $P_4(\hat{k},\hat{p})$ is a scalar degree-4 polynomial in $\hat{k}$ and $\hat{p}$. Clearly, $P_4(\hat{k},\hat{p})$ can also be written as a degree-2 polynomial P_2 in $\hat{k}^2$, $\hat{p}^2$ and $(\hat{k}+\hat{p})^2$. We see that P_2 is a linear combination of terms, each of which simplifies at least one denominator of (B.7), leaving a sum of integrals of the form

$$\int \mathrm{d}^{\hat{d}}\hat{k}\ \mathrm{d}^{\hat{d}}\hat{p}\ \frac{a\hat{k}^2 + b\hat{p}^2 + c\hat{k}\cdot\hat{p}}{\hat{k}^2\hat{p}^2}$$

(possibly after a $\hat{k}$- or $\hat{p}$-translation), where a, b and c are constants. The first two contributions are zero in dimensional regularization, while the third contribution factorizes into the product of two one-loop integrals, which cannot have logarithmic divergences for the reasons explained before.

The two-leg vertices (B.2) leave $\hat{\omega}(G)$ unchanged. On the other hand, we have seen that potentially divergent diagrams have only $\bar{A}$- or φ-internal legs, so only the scalar term of (B.2) can be used. However, it simplifies a scalar propagator, so we end up again with (B.7).

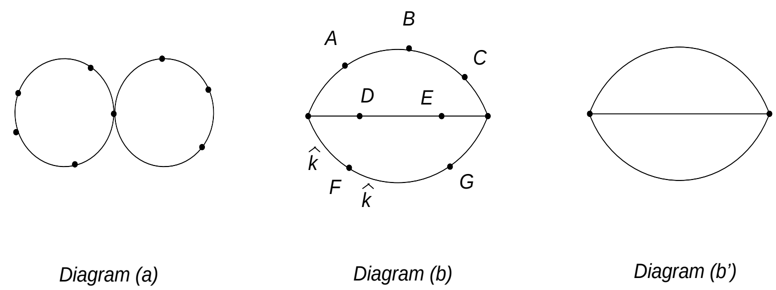

Figure 1: Analysis of the spurious subdivergences

Third step: mixed subintegrals Now we consider subintegrals of the form

$$\prod_{i=1}^{L} \int \frac{\mathrm{d}\hat{k}'_i}{(2\pi)^{\hat{d}}} \left[\prod_{j=L+1}^{L+M} \int \frac{\mathrm{d}\hat{k}'_j}{(2\pi)^{\hat{d}}} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}'_j}{(2\pi)^{\bar{d}}} \right], \tag{B.8}$$

which are "incomplete" in L barred directions. The complete subintegrals in square brakets can be regarded as products of (nonlocal, but one-particle irreducible) "subvertices". Let $\tilde{v}_r$ be the number of subvertices of type r, with $\tilde{n}_{\hat{A}r}$, $\tilde{n}_{\bar{A}r}$, $\tilde{n}_{Cr}$, $\tilde{n}_{fr}$, $\tilde{n}_{sr}$ external legs of types $\hat{A}$, $\bar{A}$, ghost, fermion and scalar, respectively. Since subvertices are at least one-loop, each leg has a factor g attached to it (see (3.24)). Thus, the weight $\tilde{\delta}_r$ of the subvertex of type r satisfies the bound

$$\tilde{\delta}_r \leq đ - \tilde{n}_{\hat{A}r} - \frac{\tilde{n}_{\bar{A}r}}{n} - \tilde{n}_{Cr} - \frac{3}{2}\tilde{n}_{fr} - \tilde{n}_{sr}. \tag{B.9}$$

Now we repeat the argument leading to (B.6) for diagrams that may contain such subvertices. We have

$$\hat{\omega}(G) = L + v_1 + 2v_2 - 2I_B - I_F + \sum_r \tilde{v}_r \tilde{\delta}_r. \tag{B.10}$$

Moreover, the topological identity $L - I + V = 1$ gives

$$L = 1 + I_B + I_F - v_1 - v_2 - \Delta v - \sum_r \tilde{v}_r. \tag{B.11}$$

We can write $\Delta v = \Delta v_B + \Delta v_F$, to distinguish the Δv_B vertices containing no fermionic legs from the Δv_F vertices containing two fermionic legs. Counting the fermionic legs of our subdiagram we have

$$2I_F + E_F = 2\Delta v_F + \sum_r \tilde{v}_r \tilde{n}_{fr}, \tag{B.12}$$

where E_F denotes the number of external fermionic legs. Combining (B.10), (B.11) and (B.12) we get

$$\hat{\omega}(G) = 2 - L - v_1 - 2\Delta v_B - \Delta v_F - \frac{E_F}{2} + \sum_r \tilde{v}_r \left(\tilde{\delta}_r - 2 + \frac{1}{2}\tilde{n}_{fr} \right). \tag{B.13}$$

We know that in the realm of the usual power counting, odd-dimensional integrals do not have logarithmic divergences. In the realm of the weighted power counting, such a property generalizes as follows: if $\hat{d} = 1$, $d =$even and $n =$odd, then odd-dimensional (weighted) integrals do not have logarithmic divergences. The proof is simple and left to the reader. Thus, the case $L = 1$ is excluded. Sufficient conditions to have $\hat{\omega}(G) \leq 0$ are then

$$\tilde{\delta}_r - 2 + \frac{1}{2}\tilde{n}_{fr} < 0 \qquad \text{for every } r. \tag{B.14}$$

Indeed, if (B.14) hold (B.13) gives $\hat{\omega}(G) < 0$ unless subvertices are absent, which is the case considered previously.

Finally, the most general mixed subintegrals have the form

$$\prod_{i=1}^{L} \int \frac{\mathrm{d}\hat{k}'_i}{(2\pi)^{\hat{d}}} \left[\prod_{j=L+1}^{L+M} \int \frac{\mathrm{d}\hat{k}'_j}{(2\pi)^{\hat{d}}} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}'_j}{(2\pi)^{\bar{d}}} \prod_{m=L+M+1}^{L+M+P} \int \frac{\mathrm{d}^{\bar{d}}\bar{k}'_m}{(2\pi)^{\bar{d}}} \right].$$

They can be treated as above, considering the integrals between square brakets as subvertices. Now formula (B.13) has an extra $-P$ on the right-hand side, since P hatted intergations are missing. The situation, therefore, can only improve. The only caveat is that now L can also be one (if P is odd). Even in that case, however, $2 - L - P \leq 0$, since $P \geq 1$.

Restrictions Using (B.9), sufficient conditions for (B.14) are

$$\text{đ} - \tilde{n}_{\hat{A}} - \frac{\tilde{n}_{\bar{A}r}}{n} - \tilde{n}_{Cr} - \tilde{n}_{fr} - \tilde{n}_{sr} < 2.$$

The worst case is $\tilde{n}_{\bar{A}r} = 2$, $\tilde{n}_{\hat{A}} = \tilde{n}_{Cr} = \tilde{n}_{fr} = \tilde{n}_{sr} = 0$, which gives

$$\text{đ} < 2 + \frac{2}{n}.$$

This condition is always satisfied in four dimensions (for $n > 1$). Summarizing, we have been able to prove the absence of spurious subdivergences under the sufficient conditions (3.22).

Some final remarks are in order. The conclusions of this appendix do not apply to the case $n = 1$, because then τ, η and ζ are constant and ξ vanishes, so the propagators (3.15) are regular. Thus for $n = 1$ all types of Lorentz breakings are allowed, which is well-known. Since (3.22) are sufficient, but not necessary, conditions, we cannot exclude all models that violate them. In specific cases other types of cancellations can take place, because of symmetries or peculiar types of expansions or resummations (e.g. large N). Generically speaking, even some theories with $\hat{d} > 1$, $n > 1$ might work, although we are unable to give explicit examples of that kind right now.

References

[1] V.A. Kostelecky, Ed., *Proceedings of the Fourth Meeting on CPT and Lorentz Symmetry*, World Scientific, Singapore, 2008.

[2] V.A. Kostelecky and S. Samuel, Spontaneous breaking of Lorentz symmetry in string theory, Phys. Rev. D 39 (1989) 683;

S.M. Carroll, G.B. Field and R. Jackiw, Limits on a Lorentz and Parity Violating Modification of Electrodynamics, Phys. Rev. D 41 (1990) 1231;

G. Amelino-Camelia, J.R. Ellis, N.E. Mavromatos, D.V. Nanopoulos, S. Sarkar, Tests of quantum gravity from observations of gamma-ray bursts, Nature 393 (1998) 763 and arXiv:astro-ph/9712103.

[3] D. Colladay and V.A. Kostelecký, Lorentz-violating extension of the Standard Model, Phys. Rev. D58 (1998) 116002 and arXiv:hep-ph/9809521.

[4] R. Jackiw and V.A. Kostelecky, Radiatively induced Lorentz and CPT violation in electrodynamics, Phys. Rev. Lett. 82 (1999) 3572 and arXiv:hep-ph/9901358;

O. Bertolami and C.S. Carvalho, Proposed astrophysical test of Lorentz invariance, Phys. Rev. D 61 (2000) 103002 and arXiv:gr-qc/9912117;

N.R. Bruno, G. Amelino-Camelia and J. Kowalski-Glikman, Deformed boost transformations that saturate at the Planck scale, Phys. Lett. B 522 (2001) 133 and arXiv:hep-th/0107039;

A.A. Andrianov, P. Giacconi and R. Soldati, Lorentz and CPT violations from Chern-Simons modifications of QED, JHEP 0202 (2002) 030 and arXiv:hep-th/0110279;

V.A. Kostelecky, Gravity, Lorentz violation, and the standard model, Phys. Rev. D 69 (2004) 105009 and arXiv:hep-th/0312310;

J. Gamboa, J. Lopez-Sarrion and A.P. Polychronakos, Ultraviolet modified photons and anisotropies in the cosmic microwave background radiation, Phys. Lett. B 634 (2006) 471 and arXiv:hep-ph/0510113;

T. Jacobson, S. Liberati and D. Mattingly, Lorentz violation at high energies: concepts, phenomena and astrophysical constraints, Annals Phys. 321 (2006) 150 and arXiv:astro-ph/0505267.

[5] V.A. Kostelecky and N. Russell, *Data tables for Lorentz and CTP violation*, in [1], p. 308 and arXiv:0801.0287 [hep-ph].

[6] D. Anselmi and M. Halat, Renormalization of Lorentz violating theories, Phys. Rev. D 76 (2007) 125011 and arXiv:0707.2480 [hep-th].

[7] D. Anselmi, Weighted scale invariant quantum field theories, JHEP 02 (2008) 051 and arXiv:0801.1216 [hep-th].

[8] D. Anselmi, Weighted power counting and Lorentz violating gauge theories. II: Classification, arXiv:0808.3474 [hep-th].

[9] H.B. Nielsen and M. Ninomiya, Beta function in a noncovariant Yang-Mills theory, Nucl.Phys. B 141 (1978) 153;

S. Chadha and H.B. Nielsen, Lorentz invariance as a low-energy phenomenon, Nucl. Phys. B 217 (1983) 125.

[10] R.M. Hornreich, M. Luban and S. Shtrikman, Critical behavior at the onset of $\overrightarrow{k}$-space instability on the λ line, Phys. Rev. Lett. 35 (1975) 1678;

See also, M.M. Leite, Renormalization-group picture of the Lifshitz critical behavior, Phys. Rev. B 67 (2003) 104415;

M.A. Shpot, Yu.M. Pis'mak and H.W.Diehl, Large-n expansion for m-axial Lifshitz points, J. Phys. Condens. Matter 17 (2005) S1947 and arXiv:cond-mat/0412405.

[11] D. Anselmi, Deformed dimensional regularization for odd (and even) dimensional theories, IJMPA 20 (2005) 1389 and hep-th/0404053.

[12] C. Becchi, A. Rouet and R. Stora, The Abelian Higgs-Kibble model. Unitarity of the S Operator, Phys. Lett. B 52 (1974) 344;

C. Becchi, A. Rouet and R. Stora, Renormalization of Gauge Theories, Annals Phys. 98 (1976) 287;

I.V. Tyutin, Gauge invariance in field theory and statistical physics in operator formalism, LEBEDEV-75-39, 1975 and arXiv:0812.0580 [hep-th].

[13] G. 't Hooft, Confinement and topology in non-abelian gauge theories, Acta Physica Austriaca, Suppl. XXII (1980) 531;

T. Appelquist and U. Heinz, Three-dimensional O(N) theories at large distances, Phys. Rev. D 24 (1981) 2169;

R. Jackiw and S. Templeton, How super-renormalizable interactions cure their infrared divergences, Phys. Rev. D 23 (1981) 2291;

T. Appelquist and R.D. Pisarski, High-remperature Yang-Mills theories and three-dimensional quantum chromodynamics, Phys. Rev. D 23 (1981) 2305.

[14] F. Bloch and A. Nordsieck, Note on the radiation field of the electron, Phys. Rev. D 52 (1937) 54.

[15] I.A. Batalin and G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, Phys. Rev. D 28 (1983) 2567.

[16] See for example, C. Itzykson and J.B. Zuber, *Quantum field theory*, Mcgraw-hill, New York, 1980, p. 599-603;

S. Weinberg, *The quantum theory of fields*, Vol. 2, Cambridge University Press, Cambridge, 2000, p. 80-91.

[17] S.D. Joglekar and B.W. Lee, General theory of renormalization of gauge invariant operators, Annals Phys. 97 (1976) 160.

[18] G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in the antifield formalism. I. General theorems, Commun. Math. Phys. 174 (1995) 57 and arXiv:hep-th/9405109.

II. Application to Yang-Mills theory, ibid. p. 116 and arXiv:hep-th/9405194.

[19] B. DeWitt, Quantum theory of gravity. 2. The manifestly covariant theory, Phys. Rev. 162 (1967) 1195;

Quantum theory of gravity. 3. Applications of the covariant theory, ibid. p. 1239;

L.F. Abbott, The background field method beyond one loop, Nucl. Phys. B 185 (1981) 189.

[20] D. Colladay and P. McDonald, One-Loop renormalization of pure Yang-Mills with Lorentz violation, Phys. Rev. D 75 (2007) 105002 and hep-ph/0609084.