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Perturbation Spectra and Renormalization-Group Techniques in Double-Field Inflation and Quantum Gravity Cosmology

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Abstract

We study primordial cosmology with two scalar fields that participate in inflation at the same time, by coupling quantum gravity (i.e., the theory $R+R^2+C^2$ with the fakeon prescription/projection for C^2) to a scalar field with a quadratic potential. We show that there exists a perturbative regime that can be described by an asymptotically de Sitter, cosmic RG flow in two couplings. Since the two scalar degrees of freedom mix in nontrivial ways, the adiabatic and isocurvature perturbations are not RG invariant on superhorizon scales. It is possible to identify the correct perturbations by using RG invariance as a guiding principle. We work out the resulting power spectra of the tensor and scalar perturbations to the NNLL and NLL orders, respectively. An unexpected consequence of RG invariance is that the theory remains predictive. Indeed, the scalar mixing affects only the subleading corrections, so the predictions of quantum gravity with single-field inflation are confirmed to the leading order.

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1 Introduction

Inflation explains the approximate isotropy and homogeneity of the cosmic microwave background radiation by means of a primordial accelerated expansion of the universe [1, 2, 3, 4, 5, 6, 7, 8], which originates the present large-scale structure from the quantum fluctuations [9, 10, 11, 12, 13, 14, 15]. In the most popular approach, the expansion is driven by a scalar field rolling down a potential [16, 17, 18], which in many cases leads to a scalar perturbation spectrum that agrees with observations [19, 20]. There also exists a "geometric" approach, where inflation is driven by the metric itself, as in the Starobinsky $R+R^2$ model [2] and the $f(R)$ theories [18, 21]. In a third approach, instead [22, 23], inflation is described as a cosmic renormalization-group (RG) flow, triggered by the dependence of the background metric (rather than the radiative corrections). The main types of flows can be classified according to the properties of their spectra, without referring to their origins from specific actions or models [24].

The $R+R^2$ theory works well phenomenologically. However, once we include R^2 there is no compelling argument for excluding the square $C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \equiv C^2$ of the Weyl tensor $C_{\mu\nu\rho\sigma}$, which has the same dimension in units of mass. Once both are present, we do not need to add further terms, since the resulting classical action

$$S_{\mathrm{QG}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right) \tag{1.1}$$

is renormalizable by power counting [25]. However, C^2 causes difficulties, if treated as usual, because it gives ghosts, which are responsible for unacceptable physics. Even if the ghosts are assumed to be heavy or short-lived, it is always possible to imagine situations where such assumptions are ineffective and the internal consistency of the theory breaks down. An example is primordial cosmology, where we must properly treat the high-energy limit (through the Bunch-Davies vacuum condition, for example), even if our purpose is just to make predictions about the low-energy limit (which means the superhorizon limit, in this case).

The problem of ghosts is due to an incorrect quantization procedure and disappears [26] once we quantize the associated degrees of freedom as purely virtual particles, or fakeons [27]. The physical predictions change, because the new prescription changes the very definition of Feynman diagram. It does so in the only way that is consistent with the optical theorem and unitarity, hence the theory obtained from (1.1) is both renormalizable and unitary. In particular, the renormalization is unaffected by the prescription[1].

In the end, the theory contains a unique purely virtual particle, with spin 2 and mass m_χ. We denote it by $\chi_{\mu\nu}$. It is removed from the physical spectrum at all energies by "integrating it out" with the fakeon prescription. Similar operations provide a new formulation of Lee-Wick theories [30] and overcome the difficulties of their original formulation.

Cosmology is an arena where we can appreciate how important these operations are in order to get to sensible physics, because, as said, it is not possible to make predictions about the low-energy (superhorizon) limit, if we do not properly take care of the opposite limit. This requires a theory that is consistent (as a perturbative quantum field theory) at all energies.

We can neglect the loop corrections [16], but other challenges appear. For example, the quantization must be performed on a nontrivial background. This can be done by first classicizing quantum gravity [31, 32] and then quantizing the (projected) classical limit by means of the Bunch-Davies quantization condition. The consistency of this procedure on the FLRW background leads to the bound $m_\chi > m_\phi/4$ on the coefficients of the terms R^2 and C^2 [33], which makes the prediction on the tensor-to-scalar ratio r quite sharp ($0.4 \lesssim 1000r \lesssim 3$ with 60 efoldings), at least in the case of single-field inflation.

Although the most popular models of inflation involve a single scalar field, the possibility that more scalars participate in inflation at the same time is not excluded and has attracted considerable attention in the literature [34, 35, 36]. Double-field and multi-field inflation are also studied because of their interesting theoretical aspects. Due to their complexity, they are normally handled by means of numerical methods.

In this paper, we minimally couple the quantum gravity theory defined as explained above to a matter scalar field φ with a quadratic potential. The theory so obtained is renormalizable [37]. We think that this makes it worth of consideration, given that renormalizability is the criterion provided by high-energy physics to overcome the arbitrariness of classical theories. We study a scenario where both scalar fields (φ and the one due to R^2, which we denote by ϕ) participate in inflation at the same time and identify a perturbative regime where we can apply the approach of refs. [22, 23], based on the idea of cosmic RG flow. We recall that this type of flow, originated by the dependence on the background metric, has an asymptotically de Sitter beta function and is such that the spectra of the cosmological perturbations satisfy equations of the Callan-Symanzik type in the superhorizon limit.

We calculate the spectrum of the tensor fluctuations to the next-to-next-to-leading log

[1]The one-loop beta functions can be found in [28, 29].

(NNLL) order, which is given in formula (4.29), and the spectra of the scalar fluctuations to the NLL order, given in formula (5.23). As usual, the spectra of the tensor perturbations are affected by C^2 from the zeroth order, as emphasized by the dependence of the results on m_χ. The spectra of the scalar perturbations are not affected by m_χ to the NLL order included, where they coincide with those of the Starobinsky theory, which is obtained by taking the limit $m_\chi \to \infty$.

Along the way, we meet new challenges. The first difficulty is to identify the right couplings. The quantity $\epsilon = -\dot{H}/H^2$ helps us identify one of them, which we denote by α, but the identification of the second one, which we denote by λ, is not straightforward. The answer is provided by the perturbation spectra, if we require that

a) the beta functions are power series in α and λ and start quadratically;

b) the flow is asymptotically de Sitter in the infinite past, which means that α and λ tend to zero for $t \to -\infty$, where the metric tends to the de Sitter one;

c) the perturbation spectra are power series in α and λ, apart from overall factors;

d) the perturbation spectra are RG invariant on superhorizon scales.

Requirement *a*) means that the beta functions behave like those of an asymptotically free quantum field theory, for example QCD. The possibility of having overall factors with negative powers (or even fractional powers and essential singularities [24]) is due to the fact that the spectra of the scalar perturbations commonly have such a feature.

A second issue is that in double-field inflation the two scalar modes, ϕ and φ, mix in nontrivial ways. This makes the identification of the right physical quantities challenging. Specifically, the adiabatic and isocurvature perturbations $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$, as commonly defined [35, 36], are not RG invariant, which means that they are not conserved on superhorizon scales. Nevertheless, we can use RG invariance to identify conserved combinations $\mathcal{R}_{\text{RG}}^{(1)}$ and $\mathcal{R}_{\text{RG}}^{(2)}$. The right couplings α and λ, the beta functions, and the spectra of $\mathcal{R}_{\text{RG}}^{(1)}$ and $\mathcal{R}_{\text{RG}}^{(2)}$ must be identified altogether, in order to satisfy the requirements *a*-*d*). Once this goal is achieved, the RG properties are exploited to the fullest. The lesson is that cosmic RG invariance is a physical principle to be placed side by side with gauge invariance.

The third issue concerns the predictivity of double-field inflation. Before explaining this point, it is worth to briefly comment on the predictivity of single-field inflation in quantum gravity with purely virtual particles. We have already remarked that, although the typical predictions of primordial cosmology concern the superhorizon limit, it is not enough to work around that limit to make such predictions. This is in contrast to what happens in high-energy physics, where a low-energy effective theory is enough to make predictions about low energies.

There are two main reasons why we must include the high energies to make predictions about the low energies in primordial cosmology. The first reason is that on a nontrivial background we need to provide a quantization condition. This goal is normally achieved by means of the Bunch-Davies vacuum condition, which refers to the high-energy limit of the theory, where the problem can be handled because it reduces to a flat-space one in conformal time. Clearly, it makes sense to impose such a condition only if the theory is consistent at high energies, which is not true when it contains ghosts, because they do not disappear there. Fakeons, instead, disappear everywhere, which leads us to the second reason why it is crucial to include the high energies to make predictions about the low energies. In the low energy regime fakeons disappear for free, because they massive, but in the opposite limit we must impose a condition to ensure that they disappear as well. This condition, which follows from the consistency of the fakeon projection and in particular its classicization [31, 32] on a curved background, is the bound $m_\chi > m_\phi/4$ found in ref. [33] (to which we refer as "ABP bound"). A bonus of the ABP bound is that it makes the physical predictions quite sharp in single-field inflation. Note that in flat space there would not be such issues.

In double-field inflation the ABP bound $m_\chi > m_\phi/4$ remains the same, but in principle the dependence on the second coupling could spoil the relation between the tensor-to-scalar ratio r and the scalar tilt n_{s}. An unexpected result of RG invariance is that, instead, this does not happen, at least to the leading order (in the perturbative regime that we identify in the next section), where the final prediction agrees with the one of the pure quantum gravity theory. This crucial property is satisfied by the RG invariant scalar perturbations $\mathcal{R}_{\mathrm{RG}}^{(1)}$ and $\mathcal{R}_{\mathrm{RG}}^{(2)}$, not by the standard adiabatic and isocurvature perturbations $\mathcal{R}_{\mathrm{adiab}}$ and $\mathcal{R}_{\mathrm{iso}}$. In the end, the predictions of the pure theory found in [33] turn out to be quite robust.

Other troubles with double- and multi-field inflation are known. For example, the curvature perturbations $\mathcal{R}_{\mathrm{adiab}}$ and $\mathcal{R}_{\mathrm{iso}}$ are not conserved in between the horizon exit and the horizon re-entry. This problem must be reconsidered now, given that $\mathcal{R}_{\mathrm{adiab}}$ and $\mathcal{R}_{\mathrm{iso}}$ are not RG invariant and the right physical quantities are $\mathcal{R}_{\mathrm{RG}}^{(1)}$ and $\mathcal{R}_{\mathrm{RG}}^{(2)}$.

We work in the "geometric framework" introduced in ref. [33], which is the one where the higher-derivative terms R^2 and C^2 of the action are kept as such. Apart from the unusual presence of the term C^2, the geometric framework coincides with the familiar Jordan frame. It allows us to treat double-field inflation with just one explicit scalar field.

Alternatively, we could use the "inflaton framework", which is obtained by introducing the inflaton field ϕ explicitly to eliminate R^2. It coincides with the Einstein frame, apart from the presence of C^2. We prefer not to use this approach here, since it complicates the two-scalar potential $\mathcal{V}(\phi, \varphi)$, by mixing a potential of class I with a potential of class II, according to the classification recently introduced in ref. [24].

A third framework is available, where the scalar ϕ and the spin-2 fakeon $\chi_{\mu\nu}$ are introduced explicitly to eliminate both higher-derivative terms R^2 and C^2 [37]. At the practical level, it is not very convenient for single-field inflation [33] and, more generally, cosmology [32].

The interest in a theory like the one we study here is that it is renormalizable, besides being local and unitary, so it can help address the issue of predictivity in quantum gravity when matter is present. As said, the outcome is that the predictions of the pure theory are not affected to the leading order. This is a nontrivial consequence of RG invariance. Simpler models of double-field inflation can be considered, if we relax the renormalizability constraint.

We mention that earlier calculations of the running of spectral indices in various scenarios (without fakeons) can be found in [38] and calculations of subleading corrections are available in [39] and [40] in single-field and multi-field inflation, respectively.

The paper is organized as follows. In section 2 we introduce the theory, derive the beta functions of the cosmic RG flow and recall the RG properties of the spectra. In section 3, we present the action and the fluctuations. In section 4 we compute the spectrum of the tensor perturbations to the NNLL order. In section 5 we study the spectra of the scalar perturbations to the NLL order, working out the RG invariant combinations $\mathcal{R}_{\rm RG}^{(1)}$ and $\mathcal{R}_{\rm RG}^{(2)}$. In section 6 we summarize the results and comment on the physical predictions of the model. Section 7 contains the conclusions, while the appendices collect various reference formulas that are too involved to be displayed in the main sections of the paper.

2 Cosmic RG flow for double-field inflation

In this section we formulate the cosmic RG flow in two couplings, associated with the model of double-field inflation that we study in the paper. We recall that the "cosmic" RG flow is not due to the radiative corrections. It is an alternative approach to inflation itself. In particular, the role of the sliding scale is played by minus the conformal time τ, so RG invariance and conservation are mapped into each other. This is the reason why RG invariance allows us to find the conserved quantities on superhorizon scales. The inflaton framework and the geometric framework mentioned above (which correspond to the usual Einstein frame and Jordan frame, respectively, apart from the presence of the

Weyl-squared term) are viewed as different schemes for the flow. Like in quantum field theory, RG invariance ensures that the physical quantities (such as the spectra) are scheme independent. The main types of cosmic RG flows in single-field inflation have been classified in [24].

The action of the model is

$$S_{\text{QG+scal}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right) + \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}D_\mu\varphi D^\mu\varphi - \mathcal{V}(\varphi) + Q(\varphi)R\right). \tag{2.1}$$

where φ is the extra scalar field. Assuming for simplicity invariance under the symmetry $\varphi \to -\varphi$, renormalizability requires

$$\mathcal{V}(\varphi) = \frac{m^2}{2}\varphi^2 + \frac{f_4}{4!}\varphi^4, \qquad Q(\varphi) = f_2\varphi^2, \tag{2.2}$$

where m, f_2 and f_4 are constants. Throughout the paper, we assume $f_2 = f_4 = 0$, which leaves the theory renormalizable[2]. The choice $f_2 = f_4 = 0$ allows us to reduce the computational effort, which is already considerable, without losing the main properties of double-field inflation.

We look for a background solution where the metric is $g_{\mu\nu} = \mathrm{diag}(1, -a^2, -a^2, -a^2)$ and φ depends only on time, by isotropy and homogeneity. The Friedmann equations and the φ equation are

$$H^2 + \frac{1}{m_\phi^2}\left(6H^2\dot{H} - \dot{H}^2 + 2H\ddot{H}\right) = \frac{\hat{\kappa}^2}{4}\left(\dot{\varphi}^2 + 2\mathcal{V}\right), \qquad \ddot{\varphi} + 3H\dot{\varphi} + \mathcal{V}' = 0, \tag{2.3}$$

where $a(t)$ denotes the scale factor, $H = \dot{a}/a$ is the Hubble parameter and $\hat{\kappa} \equiv \sqrt{16\pi G/3}$. We omit the other Friedmann equation, since it can be obtained (multiplied by H) from the equations just written, by differentiating the first one with respect to time and using the second one to eliminate $\ddot{\varphi}$.

Before proceeding, we recall that, although H is almost constant during inflation, its time dependence is crucial for the spectra of the fluctuations, which are normally expressed

[2]Renormalization does not turn on f_2 and f_4 when they are both absent at the classical level [37]. The reason is that at $f_2 = f_4 = 0$ each external φ leg carries a partial derivative ∂_μ (vertices due to $\sqrt{-g}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$) or a mass m (vertices due to $m^2\varphi^2\sqrt{-g}$). Thanks to this, the superficial degrees of divergence of the Feynman diagrams are lowered enough to forbid the counterterms $R\varphi^2$ and φ^4. Instead, when either f_2 or f_4 is present at the classical level, the other one is generated at the quantum level [41].

by means of the quantity $\epsilon = -\dot{H}/H^2$ (which here becomes one of the couplings). Higher time derivatives of H, on the other hand, parametrize the higher-order corrections to the spectra and can be converted into higher powers of the couplings.

We introduce the couplings α, ς and λ, defined by

$$\alpha = -\frac{\dot{H}}{H^2}, \qquad \varsigma = \frac{\hat{\kappa}\dot{\varphi}}{2H}, \qquad \lambda = \frac{\varsigma^2}{\alpha} = -\frac{\hat{\kappa}^2}{4}\frac{\dot{\varphi}^2}{\dot{H}}, \tag{2.4}$$

together with their beta functions

$$\beta_\alpha = \frac{\mathrm{d}\alpha}{\mathrm{d}\ln|\tau|} = -\frac{\dot{\alpha}}{vH}, \qquad \beta_\varsigma = \frac{\mathrm{d}\varsigma}{\mathrm{d}\ln|\tau|} = -\frac{\dot{\varsigma}}{vH}, \qquad \beta_\lambda = \frac{\mathrm{d}\lambda}{\mathrm{d}\ln|\tau|} = -\frac{\dot{\lambda}}{vH} = \frac{2\varsigma\beta_\varsigma}{\alpha} - \frac{\lambda\beta_\alpha}{\alpha}, \tag{2.5}$$

where $v \equiv -(aH\tau)^{-1}$ and τ denotes the conformal time

$$\tau = -\int_t^{+\infty} \frac{\mathrm{d}t'}{a(t')}. \tag{2.6}$$

We first work in the variables α and ς, which appear to be the natural ones at first. We find that β_α and β_ς are power series in α and ς, but the other functions involved in the calculations, such as H, φ and the perturbation spectra, are singular for $\alpha \to 0$, and not just by overall factors. Yet, it turns out that whenever negative powers of α appear, they are associated with positive powers of ς, which suggests that some combination of ς and α is the correct variable, instead of ς.

It turns out that the correct variables are α and λ. Apart from some appearances of $\sqrt{\lambda}$ in overall factors, the action of the tensor fluctuations and the Mukhanov-Sasaki (MS) action of the scalar fluctuations depends on α and λ perturbatively (both in the limit of infinitely heavy fakeon and at finite m_χ). Second, the beta functions $\beta_\alpha(\alpha, \lambda)$ and $\beta_\lambda(\alpha, \lambda)$ are regular and have the forms

$$\begin{aligned} \beta_\alpha(\alpha,\lambda) &= \alpha^2 \times (\text{power series in } \alpha \text{ and } \lambda) = -2\alpha^2 + \text{higher orders}, \\ \beta_\lambda(\alpha,\lambda) &= \alpha\lambda \times (\text{power series in } \alpha \text{ and } \lambda) = -2\alpha\lambda(1-2\varrho) + \text{higher orders}, \end{aligned} \tag{2.7}$$

where $\varrho = m^2/m_\phi^2$. Third, there is a region in parameter space, which is $\varrho < 1/2$, i.e.,

$$m < \frac{m_\phi}{\sqrt{2}}, \tag{2.8}$$

where the fixed point $\alpha = \lambda = 0$, which is de Sitter space, is asymptotically free. The running couplings tend to it as

$$\alpha(-\tau) \simeq \frac{\bar{\alpha}}{1 + 2\bar{\alpha}\ln|\tau|}, \qquad \lambda(-\tau) \simeq \bar{\lambda}\left(\frac{\alpha(-\tau)}{\bar{\alpha}}\right)^{1-2\varrho}, \tag{2.9}$$

for $\tau \to -\infty$ (which means $t \to -\infty$), where $\bar{\alpha}$ and $\bar{\lambda}$ are integration constants. Finally, the perturbation spectra are power series in α and λ, apart from overall factors. In the rest of the paper we prove these statements and work out the RG invariant perturbation spectra.

As said, we start by viewing v, H, φ and $\mathcal{V}$ as functions of α and ς. Denoting their partial derivatives with respect to the couplings by means of subscripts ($v_\alpha = \partial v/\partial\alpha$, $v_\varsigma = \partial v/\partial\varsigma$, etc.) and converting the time derivatives by means of the identity

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\mathrm{d}}{a\mathrm{d}\tau} = -vH\frac{\mathrm{d}}{\mathrm{d}\ln|\tau|} = -vH\left(\beta_\alpha\frac{\partial}{\partial\alpha} + \beta_\varsigma\frac{\partial}{\partial\varsigma}\right),$$

we find the equations

$$v_\alpha\beta_\alpha + v_\varsigma\beta_\varsigma = 1 - v - \alpha, \qquad \frac{H_\alpha}{H}\beta_\alpha + \frac{H_\varsigma}{H}\beta_\varsigma = \frac{\alpha}{v}, \qquad \varphi_\alpha\beta_\alpha + \varphi_\varsigma\beta_\varsigma = -\frac{2\varsigma}{\hat{\kappa}v}. \tag{2.10}$$

The first equation is obtained by differentiating the very definition of $v = -(aH\tau)^{-1}$, while the second and third equations follow from the definitions (2.4). Using (2.3), the potential $\mathcal{V}$ and its derivative $\mathcal{V}'$ with respect to φ are given by

$$\mathcal{V} = \frac{4H^4}{\hat{\kappa}^2 m_\phi^2}\left[v\beta_\alpha - 3\alpha + \frac{3}{2}\alpha^2 + \frac{m_\phi^2}{2H^2}(1-\varsigma^2)\right], \qquad \mathcal{V}' = \frac{2H^2}{\hat{\kappa}}\left[v\beta_\varsigma - (3-\alpha)\varsigma\right]. \tag{2.11}$$

2.1 Strategy

We first solve (2.10) by assuming generic expansions for the beta functions β_α and β_ς. Then we insert the solutions into (2.11) to determine the coefficients of the expansions.

The purpose is to find a solution in parameter space with the right properties to use perturbative RG methods, that is to say: 1) the beta functions β_α and β_ς are power series in α and ς; 2) de Sitter limit is the fixed point at $\alpha = \varsigma = 0$; and 3) the beta functions behave quadratically around that point, as in asymptotically free quantum field theories. The existence of a regime with such properties emerges a posteriori, once the solution is derived explicitly. We recall that, on the contrary, the regimes mostly studied in double-field inflation so far rely on numerical methods.

In particular, the expansions of β_α and β_ς must start from linear combinations of the monomials α^2, $\alpha\varsigma$ and ς^2. Moreover, it is natural to demand that the case $\varsigma = 0$ returns the usual single-field Starobinsky inflation, triggered by the R^2 term. To this purpose, we can postulate that the expansion of β_ς factorizes an overall factor ς, i.e.

$$\beta_\alpha = \sum_{n=0}^{\infty}\varsigma^n b_n(\alpha), \qquad \beta_\varsigma = \varsigma\sum_{n=1}^{\infty}\varsigma^{n-1}c_n(\alpha),$$

where $b_n(\alpha)$ and $c_n(\alpha)$ are power series in α, with $b_0(\alpha) = \mathcal{O}(\alpha^2)$ and $b_1(\alpha) = \mathcal{O}(\alpha)$, $c_1(\alpha) = \mathcal{O}(\alpha)$. Then the second equation of (2.11) implies $\mathcal{V}' = 0$ at $\varsigma = 0$ and the second equation of (2.4) implies $\dot{\varphi} = 0$: the scalar φ sits at the minimum of the potential $\mathcal{V}$ and does not participate in inflation in this limit.

It is also convenient to expand the solutions for v, h and φ in powers of ς, the coefficients being functions of α:

$$v(\alpha,\varsigma) = \sum_{n=0}^{\infty} \varsigma^n v_n(\alpha), \qquad H(\alpha,\varsigma) = h_0(\alpha)\exp\left(\sum_{n=1}^{\infty} \varsigma^n h_n(\alpha)\right), \qquad \varphi(\alpha,\varsigma) = \sum_{n=1}^{\infty} \varsigma^n \varphi_n(\alpha). \tag{2.12}$$

The zeroth-order functions $v_0(\alpha)$, $h_0(\alpha)$ and $b_0(\alpha)$ satisfy

$$b_0 v_0' + v_0 = 1-\alpha, \qquad v_0 b_0 h_0' = \alpha h_0, \qquad b_0 v_0 = 3\alpha - \frac{3}{2}\alpha^2 - \frac{m_\phi^2}{2h_0^2}.$$

The first two equations come from (2.10) and can be integrated for v_0 and h_0 by quadratures, given b_0. Then, the third equation, which comes from the first equation of (2.11), fixes b_0. The solutions

$$\begin{aligned}
v_0(\alpha) &= 1-\alpha-2\alpha^2-\frac{29}{3}\alpha^3+\mathcal{O}(\alpha^4),\\
h_0(\alpha) &= \frac{m_\phi}{\sqrt{6\alpha}}\left[1-\frac{\alpha}{12}+\frac{19}{288}\alpha^2-\frac{373}{3456}\alpha^3+\mathcal{O}(\alpha^4)\right],\\
b_0(\alpha) &= -\alpha^2\left[2+\frac{5}{3}\alpha+\frac{56}{9}\alpha^2+\frac{742}{27}\alpha^3+\mathcal{O}(\alpha^4)\right],
\end{aligned} \tag{2.13}$$

describe the single-field problem ($\varsigma = 0$) in the geometric framework.

The functions $v_n(\alpha)$, $h_n(\alpha)$ and $\varphi_n(\alpha)$ with $n > 0$ can be worked out iteratively from equations (2.10) by means of quadratures, given the beta functions. For example, the first functions satisfy

$$\begin{aligned}
&b_0 v_1' + (1+c_1)v_1 = -b_1 v_0', && b_0 v_2' + (1+2c_1)v_2 = -b_1 v_1' - b_2 v_0' - c_2 v_1,\\
&b_0 \varphi_1' + c_1\varphi_1 = -\frac{2}{\hat{\kappa} v_0}, && b_0\varphi_2' + 2c_1\varphi_2 = -b_1\varphi_1' - c_2\varphi_1 + \frac{2v_1}{\hat{\kappa} v_0^2},\\
&b_0 h_1' + c_1 h_1 = -\frac{\alpha v_1}{v_0^2} - b_1\frac{h_0'}{h_0}, && b_0 h_2' + 2c_1 h_2 = -b_1 h_1' - b_2\frac{h_0'}{h_0} - c_2 h_1 + \frac{\alpha}{v_0^3}\left(v_1^2 - v_2 v_0\right).
\end{aligned}$$

Assuming that v_j, φ_j and h_j are known for $j \leqslant n$, we first find v_{n+1} from equations like those of the first line and φ_{n+1} from equations like those of the second line. Then we can work out h_{n+1} from equations like those of the third line.

The solutions can be expanded in powers of α, but while $v_n(\alpha)$ are regular, $\varphi_n(\alpha)$ and $h_n(\alpha)$ may contain overall negative powers. More explicitly, they have expressions of the forms

$$\varphi_n(\alpha), h_n(\alpha) = \frac{1}{\alpha^n}(\text{power series in } \alpha), \qquad n > 0.$$

Once v, H are φ are known, $\mathcal{V}$ and $\mathcal{V}'$ are also known, so (2.11) become consistency conditions, which determine the coefficients of the beta functions β_α and β_ς. For example, given the potential $\mathcal{V}(\varphi) = m^2\varphi^2/2$, the first condition coming from the first formula of (2.11) is

$$b_0 v_1 + b_1 v_0 + 4b_0 h_1 v_0 - 6\alpha(2-\alpha)h_1 + \frac{m_\phi^2 h_1}{h_0^2} = 0.$$

The first two conditions coming from the second formula of (2.11) are

$$c_1 v_0 - 3 + \alpha - \frac{m^2\hat{\kappa}\varphi_1}{2h_0^2} = 0, \qquad c_1(v_1 + 2h_1 v_0) + c_2 v_0 - 2(3-\alpha)h_1 - \frac{m^2\hat{\kappa}\varphi_2}{2h_0^2} = 0.$$

2.2 Solution

The instructions just given are enough to work out the solution, which we have done up to the orders $\alpha^{14-p}\varsigma^p$ with $p \leqslant 10$. We report the result in the variables α and λ, even if it is still unclear why we should use them. To the NNLL order, we have

$$\begin{aligned}
\beta_\alpha(\alpha,\lambda) &= -2\alpha^2 - \frac{5}{3}\alpha^3 + 3\alpha^2\lambda\left(\frac{1}{\varrho} - 2\right) - \frac{56}{9}\alpha^4 + \alpha^3\lambda(5 - 8\varrho) - \frac{9}{\varrho}\alpha^2\lambda^2 + \alpha^2\mathcal{O}_3, \\
\beta_\lambda(\alpha,\lambda) &= 2\alpha\lambda(2\varrho - 1) + \frac{\alpha^2\lambda}{3}\left(8\varrho^2 + 2\varrho - 3\right) + 3\alpha\lambda^2\left(4 - \frac{1}{\varrho}\right) + \alpha^2\lambda^2\left(20\varrho - 1 - \frac{2}{\varrho}\right) \\
&\quad + \frac{2\alpha^3\lambda}{9}\left(16\varrho^3 - 28\varrho^2 + 72\varrho - 31\right) + \frac{9\alpha\lambda^3}{\varrho} + \alpha\lambda\mathcal{O}_3,
\end{aligned} \tag{2.14}$$

where $\mathcal{O}_n$ means $\mathcal{O}(\alpha^{n-m}\lambda^m)$, $0 \leqslant m \leqslant n$. The leading log running couplings are (2.9). The NLL running couplings, which are needed for the calculations of the next sections, are derived below.

We see that both positive and negative powers of ϱ appear in the expansions of the beta functions, so we assume that ϱ is neither too small nor too large. Moreover, a large ϱ makes the scalar φ very heavy. In that case, it can be integrated out and effectively decouples, returning the single-field inflation driven by ϕ. Also note that a large ϱ violates the bound (2.8) of asymptotic de Sitter freedom.

As far as the functions v, φ and H are concerned, which are useful to derive the spectra, we collect their lowest orders in formulas (A.1) of the appendix. Observe that

$$\beta_\alpha(\alpha,\lambda)=\alpha^2\mathcal{A}(\alpha,\lambda), \qquad \beta_\lambda(\alpha,\lambda)=\alpha\lambda\mathcal{B}(\alpha,\lambda),$$
$$\mathcal{V}(\varphi(\alpha,\lambda))=\frac{\lambda}{\alpha}\tilde{\mathcal{V}}(\alpha,\lambda), \qquad v(\alpha,\lambda)=1-\alpha+\alpha^2\Delta v(\alpha,\lambda), \tag{2.15}$$

where $\mathcal{A}$, $\mathcal{B}$, $\tilde{\mathcal{V}}$ and Δv are power series in α and λ. In particular, the structures (2.7) are confirmed. We are going to need these properties to show that the Mukhanov-Sasaki action of the scalar perturbations is perturbative in α and λ.

2.3 Running couplings

To the NLL order we have

$$\beta_\alpha(\alpha,\lambda)=-2\alpha^2-\frac{5}{3}\alpha^3+\frac{3}{\varrho}\alpha^2\lambda(1-2\varrho),$$
$$\beta_\lambda(\alpha,\lambda)=-2\alpha\lambda(1-2\varrho)+\frac{\alpha^2\lambda}{3}\left(8\varrho^2+2\varrho-3\right)-\frac{3}{\varrho}\alpha\lambda^2(1-4\varrho). \tag{2.16}$$

We can work out the running couplings to the same order from the ansatz

$$\alpha(-\tau)=\frac{\alpha_k}{z}\left[1+\alpha_k f_1(z)+\lambda_k f_2(z)\right], \qquad \lambda(-\tau)=\lambda_k z^{2\varrho-1}\left[1+\alpha_k f_3(z)+\lambda_k f_4(z)\right], \tag{2.17}$$

where k is a reference momentum scale, $z=1+2\alpha_k\ln(-k\tau)$, α_k and λ_k denote the couplings at $\tau=-1/k$ and $f_i(z)$, $i=1,\ldots 4$ denote unknown functions. Inserting (2.17) into (2.16) and working to the required orders, we obtain

$$\alpha(-\tau)=\frac{\alpha_k}{z}\left[1-\frac{5\alpha_k}{6z}\ln z-\frac{3\lambda_k}{4z\varrho^2}(2\varrho-1)\left(z^{2\varrho}-1\right)\right],$$
$$\lambda(-\tau)=\lambda_k z^{2\varrho-1}\left\{1+(2\varrho-1)\alpha_k\frac{5\ln z+2(z-1)(2\varrho-1)}{6z}\right. \tag{2.18}$$
$$\left.+\frac{3\lambda_k}{4\varrho^2 z(2\varrho-1)}\left[(4\varrho^2+2\varrho-1)(z^{2\varrho}-1)+4\varrho(2\varrho^2-4\varrho+1)(z-1)\right]\right\}.$$

2.4 RG invariance of the perturbation spectra

In the next section we prove that the spectrum $\mathcal{P}_T$ of the tensor fluctuations is RG invariant in the superhorizon limit, where it satisfies an RG evolution equation of the Callan-Symanzik form, with vanishing anomalous dimension. As far as the spectra of the scalar perturbations are concerned, the RG invariant ones can be identified only *a posteriori*, using RG invariance as the guiding principle.

Before proceeding, we recall the meaning of RG invariance. Considering the spectra as functions of α, λ and τ, they are RG invariant if they satisfy

$$\frac{\mathrm{d}\mathcal{P}}{\mathrm{d}\ln|\tau|}=\left(\frac{\partial}{\partial\ln|\tau|}+\beta_{\alpha}(\alpha,\lambda)\frac{\partial}{\partial\alpha}+\beta_{\lambda}(\alpha,\lambda)\frac{\partial}{\partial\lambda}\right)\mathcal{P}=0. \tag{2.19}$$

If we express α and λ as functions of τ, α_k, λ_k, by means of the running couplings, the RG equations imply that the dependence on τ drops out and we remain with

$$\mathcal{P}=\tilde{\mathcal{P}}(\alpha_k,\lambda_k),\qquad \frac{\mathrm{d}\tilde{\mathcal{P}}(\alpha_k,\lambda_k)}{\mathrm{d}\ln k}=-\beta_{\alpha}(\alpha_k,\lambda_k)\frac{\partial\tilde{\mathcal{P}}(\alpha_k,\lambda_k)}{\partial\alpha_k}-\beta_{\lambda}(\alpha_k,\lambda_k)\frac{\partial\tilde{\mathcal{P}}(\alpha_k,\lambda_k)}{\partial\lambda_k}. \tag{2.20}$$

This means that the spectra depend on the momentum k only through α_k and λ_k. Finally, expressing α_k and λ_k as functions of k/k_* and $\alpha_*=\alpha(1/k_*)$, $\lambda_*=\lambda(1/k_*)$, where k_* is the pivot scale, the RG equation reads

$$\left(\frac{\partial}{\partial\ln k}+\beta_{\alpha}(\alpha_*,\lambda_*)\frac{\partial}{\partial\alpha_*}+\beta_{\lambda}(\alpha_*,\lambda_*)\frac{\partial}{\partial\lambda_*}\right)\mathcal{P}(k/k_*,\alpha_*,\lambda_*)=0. \tag{2.21}$$

The RG techniques allow us to calculate RG improved power spectra. This means that, if $\mathcal{P}$ is expanded in powers of α_* and λ_*, the products $\alpha_*\ln(k/k_*)$ and $\lambda_*\ln(k/k_*)$ are considered of order zero and treated exactly.

3 Action and fluctuations

In this section we expand the action and give a preliminary discussion about the physical quantities. Following [33], it is convenient to introduce an auxiliary field Ω and write (2.1) as

$$\begin{aligned}S=&-\frac{1}{16\pi G}\int\mathrm{d}^4x\sqrt{-g}\left[R+\frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}-\frac{1}{6m_\phi^2}(2R-\Omega_0-\Omega)(\Omega_0+\Omega)\right]\\&+\frac{1}{2}\int\mathrm{d}^4x\sqrt{-g}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi-m^2\varphi^2\right).\end{aligned} \tag{3.1}$$

We have also shifted Ω by an arbitrary function Ω_0, to be determined.

The metric is parametrized as

$$\begin{aligned}g_{\mu\nu}=&\,\mathrm{diag}(1,-a^2,-a^2,-a^2)+2\mathrm{diag}(\Phi,a^2\Psi,a^2\Psi,a^2\Psi)-\delta_\mu^0\delta_\nu^i\partial_iB-\delta_\mu^i\delta_\nu^0\partial_iB\\&-2a^2\left(u\delta_\mu^1\delta_\nu^1-u\delta_\mu^2\delta_\nu^2+v\delta_\mu^1\delta_\nu^2+v\delta_\mu^2\delta_\nu^1\right),\end{aligned} \tag{3.2}$$

where $u = u(t, z)$ and $v = v(t, z)$ are the graviton polarizations, chosen, with no loss of generality, so that their space momentum $\mathbf{k}$ is oriented along the z axis after Fourier transform.

We expand the matter scalar field φ as

$$\varphi = \Theta_0 + \Theta.$$

where the background function Θ_0 is the solution $\varphi(\alpha, \lambda)$ calculated in the previous section and Θ is the quantum fluctuation. See [17, 18, 42] for reviews on the parametrizations of the metric fluctuations, their transformations under diffeomorphisms and common conventions.

The fakeon $\chi_{\mu\nu}$ has spin 2, so it has 5 components, which must be projected away as anticipated in the introduction. Two of them make the higher-derivative partners of u and v. Another two make the vector perturbations, which we do not consider since they do not contain physical polarizations (the fakeon projection trivializes them to the quadratic order [33]). The fifth, scalar component of $\chi_{\mu\nu}$ is part of the scalar fluctuations.

The scalar modes are Ψ, Φ, B, Ω and Θ. One such field can be eliminated by means of a gauge choice. Another one appears algebraically and can be integrated out straightforwardly. A third one can be identified with the scalar component of $\chi_{\mu\nu}$ just mentioned. The remaining two are the physical scalar fluctuations, one being provided by the metric and one by the matter field φ.

Specifically, we choose the spatially-flat gauge $\Psi = 0$ and set

$$\Omega_0 = 2R_0 = -6\dot{H} - 12H^2 = 6(\alpha - 2)H^2,$$

where R_0 is the Ricci scalar calculated on the unperturbed metric. Once the action (3.1) is expanded to the quadratic order in the fluctuations u, v, Φ, Ω, Θ and B, we see that the graviton fields u and v decouple from the scalar fields (and from each other). This allows us to treat the tensor fluctuations and the scalar fluctuations separately.

The quadratic Lagrangian $\mathcal{L}_\mathrm{t}$ of the tensor perturbations is derived later, see formula (4.1). Here we borrow the result to analyze it in the superhorizon limit $k/(aH) \to 0$, where it becomes

$$(8\pi G)\frac{\mathcal{L}_\mathrm{t}}{a^3} = \dot{u}^2 \left[1 + (2 - \alpha)H^2 \left(\frac{2}{m_\phi^2} + \frac{1}{m_\chi^2}\right)\right] - \frac{\ddot{u}^2}{m_\chi^2}. \tag{3.3}$$

Clearly, $u =$ constant solves the equations of motion. The arguments of ref. [22, 23] can then be repeated to show that the u two-point function is RG invariant in the superhorizon limit, and so is the spectrum of the tensor perturbations. We are going to use this knowledge to enhance the calculation by one order of magnitude and get to the NNLL order more easily.

As for the scalar fluctuations, the issue is more involved, due to a mixing between the physical degrees of freedom. We can define an adiabatic perturbation $\mathcal{R}_{\text{adiab}}$ and an isocurvature perturbation $\mathcal{R}_{\text{iso}}$ (see appendix D). Both $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ are gauge invariant, by construction, and so is any linear combination

$$\mathcal{R}_{\text{mix}} = C\mathcal{R}_{\text{adiab}} + D\mathcal{R}_{\text{iso}}, \tag{3.4}$$

where C and D are functions of the background. However, $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ are not conserved in the superhorizon limit. In the approach of this paper, this means that they are not RG invariant on superhorizon scales. The questions are: which combinations (3.4) are RG invariant? How to find them?

In single-field inflation, the superhorizon limit of the action shows that $\mathcal{R}_{\text{adiab}}$ is automatically RG invariant [22], so we do not need to multiply by a function C of the background. In double-field inflation it is not simple, in general, to uncover the RG invariant combinations $\mathcal{R}_{\text{RG}}^{(1)}$ and $\mathcal{R}_{\text{RG}}^{(2)}$ from the superhorizon limit of the action. Luckily, the properties of the cosmic RG flow identify $\mathcal{R}_{\text{RG}}^{(1)}$ and $\mathcal{R}_{\text{RG}}^{(2)}$ rather straightforwardly, which motivates us to propose RG invariance as the guiding principle to identify the right physical quantities.

In section 5 we work out the spectra of $\mathcal{R}_{\text{RG}}^{(1)}$ and $\mathcal{R}_{\text{RG}}^{(2)}$ to the NLL order by means of this approach. We upgrade the strategy of [33] by using the RG techniques of [22, 23], which we further extend from the Einstein frame to the Jordan frame and from single-field inflation to double-field inflation. We also compare the spectra with those of $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$.

4 Tensor perturbations

In this section we study the tensor perturbations and work out their spectrum to the NNLL order. It is easy to show that the quadratic action of the tensor fluctuations u and v is unaffected by the scalar field φ and formally coincides with the one of the single-field case found in [33]. After Fourier transforming the space coordinates to the momentum $\mathbf{k}$, we obtain

$$(8\pi G)\frac{\mathcal{L}_{\text{t}}}{a^3} = \left(\dot{u}^2 - \frac{k^2u^2}{a^2}\right)\left[1 + 2(2-\alpha)\frac{H^2}{m_\phi^2} + \frac{k^2}{a^2m_\chi^2}\right] + \frac{\dot{u}^2}{m_\chi^2}\left[(2-\alpha)H^2 + \frac{k^2}{a^2}\right] - \frac{\ddot{u}^2}{m_\chi^2}, \tag{4.1}$$

plus an identical contribution for v, where $k = |\mathbf{k}|$. What changes is that now H and a (and consequently the spectra) depend on both couplings α and λ. In (4.1) $\dot{u}^2$, u^2 and $\ddot{u}^2$ stand for $\dot{u}_{\mathbf{k}}\dot{u}_{-\mathbf{k}}$, $u_{\mathbf{k}}u_{-\mathbf{k}}$ and $\ddot{u}_{\mathbf{k}}\ddot{u}_{-\mathbf{k}}$, respectively, where $u_{\mathbf{k}}$ is the Fourier transform of u with respect to the space coordinates. We drop the subscripts $\mathbf{k}$ and $-\mathbf{k}$ when no confusion is expected to arise.

We first eliminate the higher derivatives of $\mathcal{L}_{\mathrm{t}}$ by adding an auxiliary field U and considering the extended Lagrangian

$$\mathcal{L}_{\mathrm{t}}' = \mathcal{L}_{\mathrm{t}} + \frac{a^3}{8\pi G m_\chi^2}\left(m_\chi^2 S - \ddot{u} - f\dot{u} - gu\right)^2, \tag{4.2}$$

where f and g are functions to be determined. The theories described by the Lagrangians $\mathcal{L}_{\mathrm{t}}$ and $\mathcal{L}_{\mathrm{t}}'$ are equivalent, because S appears algebraically. Indeed, when we solve the S field equation and insert the solution back into $\mathcal{L}_{\mathrm{t}}'$, we retrieve $\mathcal{L}_{\mathrm{t}}$. If we keep S, instead, and integrate by parts to remove $\ddot{u}$ from the term proportional to $\ddot{u}S$, $\mathcal{L}_{\mathrm{t}}'$ contains two fields (u and S), but no higher derivatives.

4.1 De Sitter diagonalization

We can diagonalize $\mathcal{L}_{\mathrm{t}}'$ in the de Sitter limit $\alpha = \lambda = 0$ by choosing

$$f = (3 - 2\alpha)H, \qquad g = \frac{m_\chi^2}{2} + \frac{k^2}{a^2} - \left(2\alpha + \frac{\alpha}{\xi} - \alpha^2 + v\beta_\alpha\right)H^2,$$

and making the field redefinitions

$$\begin{aligned} u &= m_\phi\sqrt{\zeta\pi G}\frac{U+V}{aH}\left[1 + \zeta(\xi-1)\frac{\alpha}{2} + \zeta^2(\xi-1)^2\frac{\alpha^2}{8}\right], \\ S &= -\frac{4}{m_\phi}\sqrt{\frac{\pi G}{\zeta}}\frac{H}{a}\left[V + \zeta(\xi-1)\frac{\alpha}{2}U + \zeta^2(\xi-1)^2\frac{\alpha^2}{8}V\right], \end{aligned} \tag{4.3}$$

where U and V are new fields and $\xi = m_\phi^2/m_\chi^2$, $\zeta = 2/(2+\xi)$. Switching to conformal time, we find the action

$$S_{\mathrm{t}} = \int \mathrm{d}\tau a\mathcal{L}_{\mathrm{t}}' = \int \mathrm{d}\tau \left(\mathcal{L}_U + \mathcal{L}_V + \mathcal{L}_{UV}\right), \tag{4.4}$$

where we have separated the U sector $\mathcal{L}_U$, from the V sector $\mathcal{L}_V$ and the mixing sector $\mathcal{L}_{UV}$.

Since the results to the NNLL order are involved, we just report the intermediate steps in concise forms, leaving the explicit expressions to the appendix. We find

$$\mathcal{L}_U = \frac{1}{2}U'^2 + q_2 U^2 + \alpha \mathcal{O}_3, \qquad \mathcal{L}_V = -\frac{1}{2}V'^2 + q_4 V^2 + \alpha \mathcal{O}_1, \qquad \mathcal{L}_{UV} = q_5 UV + q_6 UV' + \alpha \mathcal{O}_3, \tag{4.5}$$

where the coefficients q_2, q_4, q_5 and q_6 are given in formula (B.1). The primes on U and V denote derivatives with respect to τ.

4.2 Fakeon projection

The fakeon projection requires to "integrate out V", by first solving the V field equation with the fakeon prescription and then inserting the solution $V(U)$ back into the action. The form of $\mathcal{L}_{UV}$ makes it clear that $V(U)$ is $\mathcal{O}_2$. When $V(U)$ is inserted into (4.4), both $\mathcal{L}_V$ and $\mathcal{L}_{UV}$ give contributions that are $\mathcal{O}_4$. Since our plan is to work out the spectra to the NNLL order, it is enough to keep the orders $\mathcal{O}_3$, so the projected Lagrangian is just $\mathcal{L}_U$. Higher-order corrections are more complicated, since S_t contains nonlocalities from $\mathcal{O}_4$ onwards.

The projection $V(U)$ makes sense when no tachyonic behaviors are present, i.e., the ABP bound of ref. [33] is fulfilled. Now we show that the bound is independent of α and λ and coincides with the one of the single-field case, which is $m_\chi > m_\phi/4$.

The bound is obtained by ensuring that the fakeon projection makes sense not only in the superhorizon limit $k/(aH) \to 0$, but also in the opposite limit $k/(aH) \to \infty$. As recalled in the introduction, a crucial feature of primordial cosmology is that in order to make predictions about the superhorizon limit, it is not enough to study that limit, but it is necessary to interpolate from the infinite past.

This is also a key ingredient of the fakeon projection, because we must be sure that $\chi_{\mu\nu}$ is a purely virtual particle at every energy. Now, fakeons are better understood in flat space, having been introduced for the theory of scattering in perturbative quantum field theory, but here we are expanding around a nontrivial background. Luckily, in both limits $k/(aH) \to 0$ and $k/(aH) \to \infty$ the problem reduces to a flat-space one, in suitable variables, where the fakeon projection is also understood [31, 32].

Let us consider the V field equation

$$V'' + k^2 V + \frac{4V}{\xi\tau^2} = \mathcal{O}_2. \tag{4.6}$$

This form is useful to study the limit $|k\tau| \gg 1$, where $V'' + k^2 V \sim \mathcal{O}_2$ and the projection reduces to the flat-space one in conformal time. The fakeon Green function is given by

[31, 32]

$$\left.\frac{1}{\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}+k^2}\right|_{\mathrm{f}} = \frac{1}{2k}\sin\left(k|\tau-\tau'|\right), \tag{4.7}$$

where the subscript "f" means "fakeon prescription".

Introducing the "inflaton cosmological time"

$$\bar{t} = -\frac{\ln(-H_0\tau)}{H_0},$$

where H_0 is a constant, $\bar{a} = \mathrm{e}^{H_0\bar{t}}$ and $W(\bar{t}) = \bar{a}^{1/2}H_0^2 V$, we can write (4.6) in the equivalent form

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\bar{t}^2} + \frac{k^2}{\bar{a}^2}W + \frac{H_0^2}{4\xi}\left(16-\xi\right)W = \mathcal{O}_2, \tag{4.8}$$

which is efficient to study the superhorizon limit $k/(\bar{a}H_0) \rightarrow 0$.

The fakeon Green function $\hat{G}_{\mathrm{f}}(\bar{t},\bar{t}')$ for equation (4.8) is uniquely determined by requiring that it matches (4.7) for $k/(\bar{a}H_0) \gg 1$. The result is [33]

$$\hat{G}_{\mathrm{f}}(\bar{t},\bar{t}') = \frac{i\pi\mathrm{sgn}(\bar{t}-\bar{t}')}{4H_0\sinh\left(n_\chi\pi\right)}\left[J_{in_\chi}(\check{k})J_{-in_\chi}(\check{k}') - J_{in_\chi}(\check{k}')J_{-in_\chi}(\check{k})\right], \qquad n_\chi = \sqrt{\frac{4m_\chi^2}{m_\phi^2}-\frac{1}{4}},$$

where J_n denotes the Bessel function of the first kind, $\check{k} = k\mathrm{e}^{-H_0\bar{t}}/H_0$, $\check{k}' = k\mathrm{e}^{-H_0\bar{t}'}/H_0$, and $\mathrm{sgn}(\bar{t})$ is the sign function. Thus, the fakeon Green function in the superhorizon limit becomes [33]

$$\left.\frac{1}{\frac{\mathrm{d}^2}{\mathrm{d}\bar{t}^2}+H_0^2 n_\chi^2}\right|_{\mathrm{f}} = \frac{1}{2H_0 n_\chi}\sin\left(H_0 n_\chi|\bar{t}-\bar{t}'|\right). \tag{4.9}$$

This function exhibits no tachyonic behaviors only for

$$m_\chi > \frac{m_\phi}{4}. \tag{4.10}$$

When this bound is violated, we have a hyperbolic sine in (4.9), which means that the theory is in a different phase and the large scale structure of the universe as we know it cannot form. This scenario is beyond the scope of this paper and needs to be explored separately.

Note that we have not switched from τ to t, which is the cosmological time of the geometric approach. If we do it, we do not obtain a harmonic oscillator with constant frequency. Then we need a further reparametrization to reduce to a problem of that type. An example of such a reparametrization is precisely the one that switches from t to $\bar{t}$. Any other one leads to the same conclusion (4.10), since a theorem proved in [33] ensures that once the squared mass is constant in some paramerization, any further reparametrization that leaves it constant preserves its sign. For this reason, it is enough to derive the ABP bound in the variable $\bar{t}$.

Having established when the projection makes sense, the projected field $V(U)$ can be obtained relatively easily, because we just need it in the superhorizon limit $k/(aH) \to 0$ [3]. We can derive $V(U)$ for $k/(aH) \to 0$ by means of the ansatz

$$V(U) = s_1(\alpha, \lambda)U + s_2(\alpha, \lambda)\tau U', \tag{4.11}$$

where s_1, s_2 are functions to be determined. We insert this expression into the V field equation, truncated to the order we are interested in, which is

$$V'' + 2q_4 V + (q_5 - q_6')U - q_6 U' = \alpha \mathcal{O}_3. \tag{4.12}$$

Then we use the U field equation $U'' = 2q_2 U + \alpha \mathcal{O}_3$ to eliminate U''. Finally, we determine s_1, s_2 by killing the terms proportional to U and U' in (4.12). The result is given in formulas (B.2) of the appendix.

4.3 Projected action and RG invariance

As said, the projected Lagrangian to the order we need is just $\mathcal{L}_U$. We write the action in the form

$$S_{\mathrm{t}}^{\mathrm{proj}} = \frac{1}{2}\int \mathrm{d}\tau \left[U'^2 - h_{\mathrm{t}} k^2 U^2 + (2+\sigma_{\mathrm{t}})\frac{U^2}{\tau^2} + \mathcal{O}_4\right], \tag{4.13}$$

where

$$\begin{aligned} h_{\mathrm{t}} &= 1 - 2\xi\zeta\alpha^2 - \frac{3}{\varrho}\xi\zeta\alpha^2\lambda(2\varrho - 1) + \frac{1}{6}\xi\zeta^2\alpha^3(14 - 11\xi) + \mathcal{O}_4, \\ \sigma_{\mathrm{t}} &= 9\zeta\alpha^2\left(1 + \frac{3}{2}\lambda\right) + \frac{3\zeta^2\alpha^3}{2}(32 + 37\xi + 6\xi^2) + \mathcal{O}_4. \end{aligned}$$

Defining $\eta = -k\tau$ and $w(\eta) = U(\tau)\sqrt{k}$, (4.13) becomes

$$S_{\mathrm{t}}^{\mathrm{proj}} = \frac{1}{2}\int \mathrm{d}\eta \left[w'^2 - h_{\mathrm{t}} w^2 + (2+\sigma_{\mathrm{t}})\frac{w^2}{\eta^2}\right], \tag{4.14}$$

where the prime on w stands for the derivative with respect to η and we have dropped the orders that do not contribute to our calculation. The w equation of motion is

$$w'' + h_{\mathrm{t}} w - \frac{2+\sigma_{\mathrm{t}}}{\eta^2} w = 0. \tag{4.15}$$

[3] The large $k/(aH)$ behavior is required only for the Bunch-Davies vacuum condition on the solution of the projected U equation, which, as remarked above, is unaffected by $V(U)$ to the order we are considering.

If we want the tensor spectrum to the NNLL order, we actually need the $\mathcal{O}_4$ contributions to $\sigma_{\rm t}$. We can infer them indirectly, by means of RG invariance and the method of [23]. What makes this possible is the knowledge, from section 3, that the tensor mode u is RG invariant in the superhorizon limit[4].

We parametrize the $\mathcal{O}_4$ contributions to $\sigma_{\rm t}$ as

$$\zeta^2\alpha^2(c_1\alpha^2 + c_2\alpha\lambda + c_3\lambda^2) \tag{4.16}$$

and determine the constants c_i as follows. First, we decompose $\eta w(\eta)$ as the sum

$$\eta w = Q(\ln\eta) + Y(\eta). \tag{4.17}$$

of a power series $Q(\ln\eta)$ in $\ln\eta$ plus a power series $Y(\eta)$ in η and $\ln\eta$, such that $Y(\eta) \to 0$ term-by-term for $\eta \to 0$. Inserting (4.17) into (4.15), we find

$$Q'' - 3Q' - \sigma_{\rm t}Q = \sigma_{\rm t}Y + 2\eta Y' - \eta^2 Y'' - h_{\rm t}\eta^2(Y + Q), \tag{4.18}$$

where the primes denote the derivatives with respect to the arguments of the functions. Both sides of equation (4.18) must be separately zero, because the left-hand side is an expansion in powers of $\ln\eta$, while the right-hand side is at least an overall factor η times an expansion in powers of $\ln\eta$. Thus, the Q equation reads

$$Q'' - 3Q' = \left(\frac{\mathrm{d}}{\mathrm{d}\ln\eta} - 3\right)\frac{\mathrm{d}Q}{\mathrm{d}\ln\eta} = \sigma_{\rm t}Q.$$

Actually, this equation can be further reduced, since the contributions proportional to η^3, allowed by the operator in parenthesis, do not belong to Q, by construction. The workaround is to invert that operator perturbatively, which gives [22]

$$DQ = -\frac{1}{3}\frac{1}{1-\frac{D}{3}}(\sigma_{\rm t}Q), \tag{4.19}$$

where

$$D \equiv \frac{\mathrm{d}}{\mathrm{d}\ln\eta} = \beta_\alpha\frac{\partial}{\partial\alpha} + \beta_\lambda\frac{\partial}{\partial\lambda}. \tag{4.20}$$

It is understood that $\left(1-\frac{D}{3}\right)^{-1}$ on the right-hand side of (4.19) must be expanded as a power series in D. Indeed, D is $\mathcal{O}_1$, since the beta functions β_α and β_λ are $\mathcal{O}_2$ to the leading order.

[4] We cannot proceed like this for the scalar perturbations, because the RG invariant combinations $\mathcal{R}_{\rm RG}^{(1)}$ and $\mathcal{R}_{\rm RG}^{(2)}$ are not known *a priori*.

Viewing $Q(\ln \eta)$ as a function $\tilde{Q}$ of α and λ, we can write

$$D\tilde{Q} = -\frac{\sigma_{\rm t}\tilde{Q}}{3} - \frac{1}{9}D(\sigma_{\rm t}\tilde{Q}) - \frac{1}{27}D^2(\sigma_{\rm t}\tilde{Q}), \tag{4.21}$$

to the order we need. We express the solution as

$$\tilde{Q}_{\rm t}(\alpha, \lambda; \alpha_k, \lambda_k) = \frac{J_{\rm t}(\alpha, \lambda)}{J_{\rm t}(\alpha_k, \lambda_k)}\tilde{Q}_{\rm t}(\alpha_k, \lambda_k), \tag{4.22}$$

where $J_{\rm t}$ and $\tilde{Q}_{\rm t}$ are functions to be determined. We calculate the constants c_i of (4.16) from the RG invariance of u in the superhorizon limit, which ensures that u cannot depend on α and λ in that limit.

Specifically, formula (4.3) gives

$$u = \frac{m_\phi}{aH}\sqrt{\zeta\pi G}\,[U + s_1(\alpha, \lambda)U + s_2(\alpha, \lambda)\tau U']\left[1 + \zeta(\xi - 1)\frac{\alpha}{2} + \zeta^2(\xi - 1)^2\frac{\alpha^2}{8}\right]. \tag{4.23}$$

Replacing $U = w/\sqrt{k}$ with $\tilde{Q}/(\eta\sqrt{k})$, we obtain that the expression

$$[J_{\rm t} + s_1(\alpha, \lambda)J_{\rm t} + s_2(\alpha, \lambda)(DJ_{\rm t} - J_{\rm t})]\, v\left[1 + \zeta(\xi - 1)\frac{\alpha}{2} + \zeta^2(\xi - 1)^2\frac{\alpha^2}{8}\right] \tag{4.24}$$

must be a numerical constant to the NNLL order. Solving (4.21) for $J_{\rm t}(\alpha, \lambda)$ and inserting the solution into (4.24), we find (B.3) and (B.4), the numerical constant being equal to unity.

Note that the expression (4.22) does not match the one of [23] for λ, $\lambda_k \to 0$, because the calculations of [23] were done in the inflaton framework, while we are working in the geometric framework here. Only the physical quantities (spectra, tilts, running coefficients, etc.) need to match, as they do (see below).

4.4 Solution of the projected equation of motion

It remains to determine the constant $\tilde{Q}_{\rm t}(\alpha_k, \lambda_k)$ of formula (4.22). To achieve this goal, we need to solve the equation of motion (4.15) of the projected action (4.13). To impose the correct Bunch-Davies condition, we need to make a change of variables and eliminate the mass renormalization $h_{\rm t}$. Following [22], we rewrite the action (4.13) as

$$\tilde{S}_{\rm t}^{\rm prj} = \frac{1}{2}\int {\rm d}\tilde{\eta}\left(\tilde{w}'^2 - \tilde{w}^2 + \frac{2\tilde{w}^2}{\tilde{\eta}^2} + \tilde{\sigma}_{\rm t}\frac{\tilde{w}^2}{\tilde{\eta}^2}\right), \tag{4.25}$$

where the new variable $\tilde{\eta}(\eta)$ is defined as the solution of the differential equation $\tilde{\eta}'(\eta) = \sqrt{h_{\rm t}(\eta)}$ with the initial condition $\tilde{\eta}(0) = 0$, while

$$\tilde{w}(\tilde{\eta}(\eta)) = h_{\rm t}(\eta)^{1/4} w(\eta), \qquad \tilde{\sigma}_{\rm t} = \frac{\tilde{\eta}^2(\sigma_{\rm t}+2)}{\eta^2 h_{\rm t}} + \frac{\tilde{\eta}^2}{16 h_{\rm t}^3}\left(4h_{\rm t}h_{\rm t}'' - 5h_{\rm t}'^2\right) - 2. \tag{4.26}$$

The Bunch-Davies vacuum condition for (4.25) is the usual one,

$$\tilde{w}(\tilde{\eta}) \simeq \frac{\mathrm{e}^{i\tilde{\eta}}}{\sqrt{2}} \qquad \text{for } \tilde{\eta} \to \infty. \tag{4.27}$$

Using (4.26) and the running couplings (2.18), we expand $\tilde{\sigma}_{\rm t}$ in powers of α_k and λ_k, then expand the $\tilde{w}$ equation of motion by writing

$$\tilde{w} = \tilde{w}_0 + \alpha_k^2 \tilde{w}_2 + \alpha_k^3 \tilde{w}_3 + \alpha_k^2 \lambda_k \tilde{w}_{21} + \cdots \tag{4.28}$$

The solution is given by

$$\begin{aligned} \tilde{w}_2(\tilde{\eta}) &= \zeta W_2(\tilde{\eta}), \qquad \tilde{w}_{21}(\tilde{\eta}) = \frac{3\zeta}{2} W_2(\tilde{\eta}), \\ \tilde{w}_3(\tilde{\eta}) &= \zeta W_3(\tilde{\eta}) - \frac{\zeta}{6}\left[4(8+3\xi) - \zeta(32+37\xi+6\xi^2)\right] W_2(\tilde{\eta}), \end{aligned}$$

the functions $W_i(\tilde{\eta})$ being defined in formula (E.1). Since each $W_i(\tilde{\eta})$, $i > 0$, tends to zero for $\tilde{\eta} \to \infty$, (4.28) satisfies (4.27).

Next, the first formula of (4.26) gives $w(\eta)$, where from we can extract $Q(\ln \eta)$ by means of (4.17) and the asymptotic behaviors (E.2). At $\ln \eta = 0$ we obtain the desired spectral normalization $\tilde{Q}_{\rm t}(\alpha_k, \lambda_k) = Q(0)$, reported in formula (B.5). Finally, the η dependence of $Q(\ln \eta)$ provides a check of (4.22) up to the RG improvement.

4.5 Perturbation spectrum

Inserting (B.5) into (4.22) and using (4.17), we obtain $w(\eta)$ in the superhorizon limit, hence $U(\tau) = w(\eta)/\sqrt{k}$ in the same limit. Then (4.23) gives u and (E.3) gives the spectrum. The result is

$$\begin{aligned} \mathcal{P}_T(k) = \frac{4m_\phi^2 \zeta G}{\pi} \Bigg[& 1 - 3\zeta\alpha_k\left(1 + 2\alpha_k\gamma_M + 4\gamma_M^2\alpha_k^2 - \frac{\pi^2\alpha_k^2}{3} - \frac{\alpha_k^2}{3}\right) + \frac{\zeta^2\alpha_k^2}{8}(94+11\xi) \\ & + 3\gamma_M\zeta^2\alpha_k^3(14+\xi) - \frac{\zeta^3\alpha_k^3}{12}(614+191\xi+23\xi^2) - \frac{9}{4\varrho}\zeta\alpha_k\lambda_k \\ & + \frac{3}{4}\left(\frac{20+\xi}{\varrho} + 3(1-2\gamma_M)(2+\xi)\right)\zeta^2\alpha_k^2\lambda_k + \mathcal{O}_4\Bigg]. \end{aligned} \tag{4.29}$$

At $\lambda_k = 0$ it agrees with the one of [23], as expected, but with one caveat: the two couplings α_k used here and there do not coincide, since the single-field calculation of [23] was performed in the inflaton framework (Einstein frame), while the present calculation is performed in the geometric framework (Jordan frame). The conversion between the two (check for example the appendix of [33]) is

$$\alpha_k^{\text{infl}} = \alpha_k - \frac{\alpha_k^3}{3} + \mathcal{O}(\alpha_k^4). \tag{4.30}$$

Once the conversion is taken into account, we have a perfect match with the result of [23] for $\lambda_k = 0$.

We see that $\mathcal{P}_T(k)$ has a regular expansion in powers of α_k and λ_k, but also a regular expansion in powers of α_k and $\varsigma_k = \sqrt{\alpha_k \lambda_k}$. The spectra of the scalar perturbations will only have a regular expansion in powers of α_k and λ_k.

One virtue of the RG approach is that it returns manifestly RG invariant spectra. As in the dimensional transmutation familiar from quantum field theory, the sliding scale (which is minus the conformal time τ) has disappeared from $\mathcal{P}_T$, replaced by $1/k$ inside the running couplings α_k and λ_k. The relation (4.30) can be viewed as a scheme change, the two schemes being the Einstein frame and the Jordan frame. The match between (4.29) and the result of [23] at $\lambda_k = 0$ provides an explicit confirmation that the physical quantities, such as the spectra, are scheme independent.

The Starobinsky limit is obtained by letting m_χ tend to infinity, where the theory becomes $R + R^2$ plus the extra scalar field φ. There we have

$$\begin{aligned}\mathcal{P}_T^{m_\chi=\infty}(k) = \frac{4m_\phi^2 G}{\pi}\Bigg[&1 - 3\alpha_k + \frac{\alpha_k^2}{4}(47 - 24\gamma_M) - \frac{9\alpha_k\lambda_k}{4\varrho} \\ &- \left(\frac{301}{6} + 12\gamma_M^2 - 42\gamma_M - \pi^2\right)\alpha_k^3 + \frac{3}{2}\left(\frac{10}{\varrho} + 3 - 6\gamma_M\right)\alpha_k^2\lambda_k + \mathcal{O}_4\Bigg].\end{aligned} \tag{4.31}$$

5 Scalar perturbations

In this section we study the spectra of the scalar perturbations, which we compute to the NLL order. We show that, as in the single-field case, they are unaffected by the fakeon $\chi_{\mu\nu}$, since the first corrections that depend on m_χ are the NNLL ones. This means that the results of this section coincide with those of the theory $R + R^2$ plus the extra scalar field φ.

Nevertheless, to prove the statements just made, we still need to start from the complete $R + R^2 + C^2$ theory, perform the fakeon projection and verify that the results are actually

m_χ independent to the NLL order. We do so by expanding in powers of $\xi = m_\phi^2/m_\chi^2$.

The four scalar fields Ω, B, Φ, Θ appearing in the action play the following roles: Φ is an auxiliary field and can be integrated out straightforwardly, B (plus certain corrections) is the scalar fakeon, while Ω and Θ (plus corrections) encode the physical perturbations U_1 and U_2. The nontrivial mixing between U_1 and U_2 complicates the identification of the RG invariant combinations. The difficulty can be overcome if we use RG invariance as a guiding principle. Since most formulas we are going to meet in the intermediate steps are lengthy, we cannot report them here, but just give the main details to work out the final result.

We expand the Lagrangian (3.1) by means of (3.2) (with $u = v = 0$) to the quadratic order in the fluctuations. Then we make the field redefinitions $\Omega, B, \Phi, \Theta \rightarrow U_1, E, \tilde{\Phi}, U_2$, where

$$\Omega = 8m_\phi\sqrt{3\pi G}\frac{H}{a}U_1, \qquad B = 2\sqrt{3\pi G}\frac{a}{k^2}E, \qquad \Phi = 8m_\phi\sqrt{3\pi G}\frac{\tilde{\Phi}}{aH}, \qquad \Theta = \frac{U_2}{a}. \tag{5.1}$$

Next, we simplify the Lagrangian by using the equations (2.3) and (2.11) of the background metric and the background scalar φ. At the end, we switch to conformal time. Using the properties (2.15), it is easy to show that the quadratic Lagrangian we obtain is analytic in α and λ, apart from the linear terms in U_2 and U_2', which are multiplied by an extra factor $\sqrt{\lambda}$. In particular, the de Sitter limit $\alpha, \lambda \rightarrow 0$ is regular and reads

$$\begin{aligned}\mathcal{L}_{\mathrm{dS}} = {}& \frac{1}{2}U_2'^2 - \frac{k^2}{2}U_2^2 + \frac{U_2^2}{\tau^2} - \frac{E'^2}{2m_\chi^2} + \frac{2U_1E'}{\tau m_\phi} - \frac{6EU_1}{\tau^2 m_\phi} - \frac{2}{\tau^2}U_1^2 \\ & + \frac{4\tilde{\Phi}}{\tau^2}\left(12U_1 + 12\frac{E}{m_\phi} + 6\tau U_1' + k^2\tau^3\frac{m_\phi}{m_\chi^2}E' - 2k^2\tau^2 U_1\right) - \frac{8\tilde{\Phi}^2}{\tau^2}\left(36 + \frac{m_\phi^2}{m_\chi^2}k^4\tau^4\right).\end{aligned}$$

In this limit the extra scalar U_2 decouples and the quantum gravity sector can be treated as in the single-scalar case. The negative sign in front of the E kinetic term E'^2 shows that E (plus corrections) is the fakeon.

5.1 ABP bound

If we integrate out the auxiliary field $\tilde{\Phi}$, we obtain, on superhorizon scales $k/(aH) \rightarrow 0$ in the de Sitter limit,

$$\mathcal{L}_{\mathrm{dS}}^{k\rightarrow 0} = \frac{1}{2}U_1'^2 + \frac{U_1^2}{\tau^2} + \frac{1}{2}U_2'^2 + \frac{U_2^2}{\tau^2} - \frac{1}{2m_\chi^2}\left(E'^2 - 4\frac{m_\chi^2}{m_\phi^2}\frac{E^2}{\tau^2}\right).$$

The E-dependent part of the action reads, in cosmological time,

$$\int \mathrm{d}\tau \mathcal{L}_{C\mathrm{dS}}^{k\to 0} \simeq -\frac{1}{2m_\chi^2}\int a\mathrm{d}t \left[\dot{E}^2 - \frac{2m_\chi^2}{3\alpha}E^2\right],$$

up to subleading corrections, where we have used $v = -1/(aH\tau) = 1 + \mathcal{O}_1$ and $H = m_\phi/\sqrt{6\alpha}(1+\mathcal{O}_1)$. As in the previous section, we can read the ABP bound [33] by defining a new cosmological time $t' = t'(t)$ to reduce the problem to one with constant coefficients, up to corrections of higher orders. For example, we can choose $\mathrm{d}t/\mathrm{d}t' = \sqrt{\alpha}$ and $\tilde{E}(t') = \alpha^{-1/4}E(t)\sqrt{a}$, which gives

$$\int \mathrm{d}\tau \mathcal{L}_{C\mathrm{dS}}^{k\to 0} \simeq -\frac{1}{2m_\chi^2}\int \mathrm{d}t' \left[\left(\frac{\mathrm{d}\tilde{E}}{\mathrm{d}t'}\right)^2 + \left(m_\phi^2 - 16m_\chi^2\right)\frac{\tilde{E}^2}{24}\right].$$

The squared mass of the fakeon is positive if and only if (4.10) holds, so the ABP bound of the scalar perturbations coincides with the one of the tensor perturbations. Again, any reparametrization that gives a constant squared mass leads to the same ABP bound. These properties suggest that the ABP bound is a universal property of the theory.

At this point, we can proceed to project the fakeon out.

5.2 Fakeon projection

When we restore the dependence on α and λ, we move away from the de Sitter point. To study this situation, we still eliminate the auxiliary field $\tilde{\Phi}$ and then expand in powers of ξ to perform the fakeon projection, which expresses E in terms of U_1, U_2 and their derivatives. We have done this explicitly to the order ξ^3 included. In the small ξ expansion, the projection is straightforward, since the E field equation has the form

$$\frac{4\hat{E}}{\xi\tau^2} - \frac{k^2}{3}\hat{E} + \hat{E}'' = \mathcal{O}_1 + \mathcal{O}(\xi), \tag{5.2}$$

having defined

$$E = \sqrt{\frac{3}{\pi G}}\hat{E} + \frac{m_\phi k^2\tau^2}{6}U_1,$$

so the solution can be worked out algebraically by iteration.

Inserting the solution back into the Lagrangian, we obtain the projected Lagrangian, which has the form

$$\mathcal{L}_{\mathrm{MS}} = \frac{1}{2}\sum_{i=1}^{2}\left[U_i'^2 + \left(\frac{2}{\tau^2} - k^2\right)U_i^2\right] + \mathcal{O}_1 + \mathcal{O}(\xi). \tag{5.3}$$

The corrections can contain higher derivatives of U_1, multiplied by higher powers of ξ. They can be eliminated by means of a change of variables

$$U_1 = u_1\left(1-\frac{\alpha}{2}+\frac{\alpha^2}{2}+\frac{9}{8\varrho}\alpha\lambda\right)-\alpha\sqrt{\frac{3\lambda}{2}}u_2+\mathcal{O}_{5/2}+\mathcal{O}(\xi), \qquad U_2 = u_2, \tag{5.4}$$

which turns (5.3) into the standard form

$$\begin{aligned}\mathcal{L}_{\mathrm{MS}} = {} & \frac{1}{2}\sum_{i=1}^{2}\left[u_i'^2+\left(\frac{2}{\tau^2}-k^2\right)u_i^2\right]+\frac{\alpha}{\tau}\sqrt{6\lambda}u_1'u_2+\frac{\alpha}{\tau^2}\left[(3+10\alpha)u_1^2-\frac{9}{2\varrho}\lambda u_1^2\right] \\ & +\frac{\alpha u_2}{\tau^2}\left[\frac{3}{2}(1-2\varrho)(1+4\alpha)u_2+\frac{11}{2}\varrho\alpha u_2+4\sqrt{6\lambda}u_1-\frac{9}{2}\lambda u_2\right]+\mathcal{O}_{5/2}.\end{aligned} \tag{5.5}$$

Note that this expression is exact in ξ, which proves our statement: the scalar spectra are unaffected by the fakeon to the NLL order.

5.3 Mukhanov-Sasaki equations

Having two mixing scalars, we need to work with vectors and matrices. Let us define

$$w(\eta)=\sqrt{k}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \qquad \Xi = \alpha\sqrt{\frac{3\lambda}{2}}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}+\begin{pmatrix} \mathcal{O}_3 & \mathcal{O}_{5/2} \\ \mathcal{O}_{5/2} & \mathcal{O}_3 \end{pmatrix},$$

$$\Sigma = \alpha\begin{pmatrix} 6+22\alpha-\frac{9\lambda}{\varrho} & \sqrt{\frac{3\lambda}{2}}\left(9+(3-2\varrho)\alpha\right) \\ \sqrt{\frac{3\lambda}{2}}\left(9+(3-2\varrho)\alpha\right) & 3(1-2\varrho)+(12-13\varrho)\alpha-9\lambda \end{pmatrix}+\begin{pmatrix} \mathcal{O}_3 & \mathcal{O}_{5/2} \\ \mathcal{O}_{5/2} & \mathcal{O}_3 \end{pmatrix}. \tag{5.6}$$

The Mukhanov-Sasaki action is

$$S_{\mathrm{MS}} = \int \mathrm{d}\eta \mathcal{L}_{\mathrm{MS}} = \frac{1}{2}\int \mathrm{d}\eta \left[w'^{\mathrm{T}}w'-w^{\mathrm{T}}w+\frac{2}{\eta^2}w^{\mathrm{T}}w+\frac{1}{\eta^2}w^{\mathrm{T}}\Sigma w+\frac{2}{\eta}w^{\mathrm{T}}\Xi w'\right], \tag{5.7}$$

where the superscript T means transpose. The MS equations are

$$w''+w-\frac{2}{\eta^2}w=\frac{1}{\eta^2}\left(\Sigma-\Xi+D\Xi\right)w+\frac{2}{\eta}\Xi w'. \tag{5.8}$$

5.4 RG invariance

To study RG invariance we write

$$\eta w(\eta) = Q(\ln\eta)+Y(\eta), \tag{5.9}$$

where $Q(\ln\eta)$ is a vector with entries that are power series in $\ln\eta$, while $Y(\eta)$ is a vector with entries that are power series in η and $\ln\eta$ and such that $Y(\eta)\rightarrow 0$ term-by-term for $\eta\rightarrow 0$. Proceeding as in [22] and above formula (4.19) (but paying attention to the fact that now we have to deal with matrices and vectors), it is easy to derive the Q equations

$$DQ=-\frac{1}{3}\frac{1}{1-\frac{D}{3}}\left[(\Sigma-3\Xi-D\Xi)\,Q+2D(\Xi Q)\right].$$

As usual, the right-hand side must be expanded in powers of D. Since the beta functions are $\mathcal{O}_2$ to the lowest order, D is $\mathcal{O}_1$, while Σ is also $\mathcal{O}_1$ and Ξ is $\mathcal{O}_{3/2}$. To the NLL order we can truncate the equation to

$$D\tilde{Q}=-\frac{1}{3}\tilde{\Sigma}\tilde{Q}, \tag{5.10}$$

where

$$\tilde{Q}=\left(1+\frac{\Sigma}{9}+\frac{\Xi}{3}\right)Q,\qquad \tilde{\Sigma}=(\Sigma-3\Xi-D\Xi)\left(1+\frac{\Sigma}{9}+\frac{\Xi}{3}\right)^{-1}.$$

The solution is

$$\tilde{Q}(\ln\eta)=K(\eta)\tilde{Q}_0, \tag{5.11}$$

where $\tilde{Q}_0$ is a constant vector and the kernel K is given by the ordered exponential

$$\begin{aligned}K(\eta)&=T\exp\left(-\frac{1}{3}\int_0^{\ln\eta}\mathrm{d}\ln\eta'\,\tilde{\Sigma}(\eta')\right)\\&\equiv 1-\frac{1}{3}\int_0^{\ln\eta}\mathrm{d}\ln\eta'\,\tilde{\Sigma}(\eta')+\frac{1}{9}\int_0^{\ln\eta}\mathrm{d}\ln\eta'\,\tilde{\Sigma}(\eta')\int_0^{\ln\eta'}\mathrm{d}\ln\eta''\,\tilde{\Sigma}(\eta'')+\cdots.\end{aligned} \tag{5.12}$$

It is always possible to write $\tilde{Q}(\ln\eta)$ as the product $\mathcal{K}(\alpha,\lambda)X_0$ of a matrix $\mathcal{K}$ that depends only on α and λ, times an arbitrary constant vector X_0. To prove this statement, we use the running couplings (2.18) to, say, express λ as a function

$$\lambda=f_\lambda(\alpha,\lambda_0) \tag{5.13}$$

of α and an arbitrary constant λ_0. The function f_λ is the solution of the first order differential equation

$$\frac{\mathrm{d}f_\lambda}{\mathrm{d}\alpha}=\frac{\beta_\lambda(\alpha,f_\lambda)}{\beta_\alpha(\alpha,f_\lambda)}.$$

obtained by inserting (5.13) into $\beta_\lambda(\alpha,\lambda)=\mathrm{d}\lambda/\mathrm{d}\ln\eta$, and λ_0 is the initial condition. Then, we can write the differential operator D appearing in (5.10) as

$$D=\frac{\mathrm{d}}{\mathrm{d}\ln\eta}=\beta_\alpha\frac{\partial}{\partial\alpha}+\beta_\lambda\frac{\partial}{\partial\lambda}=\beta_\alpha(\alpha,f_\lambda)\frac{\mathrm{d}}{\mathrm{d}\alpha},$$

where the total derivative $\mathrm{d}/\mathrm{d}\alpha$ also acts on the α dependence due to f_λ.

Since the initial conditions for the system of differential equations (5.10) are collected into a constant vector X_0 on the right of everything, the solution can be written as the product $\tilde{\mathcal{K}}(\alpha,\lambda_0)X_0$ of a matrix $\tilde{\mathcal{K}}(\alpha,\lambda_0)$ depending on α and λ_0 times X_0. Inverting (5.13), λ_0 can be expressed as a function of α and λ. This allows us to write $\tilde{\mathcal{K}}(\alpha,\lambda_0)$ as a matrix $\mathcal{K}(\alpha,\lambda)$ whose entries are functions of α and λ, as we wished to prove.

At the practical level, we evaluate the solution $\tilde{Q}$ as follows. First, we use the running couplings of formulas (2.18) to expand Σ and Ξ in powers of α_k and λ_k. This makes the dependence of $\ln\eta$ explicit and allows us to perform the integrals of (5.12). From (5.11) we learn that $\tilde{Q}(\ln\eta) = K(\eta)\tilde{Q}_0 = \mathcal{K}(\alpha,\lambda)X_0$, which implies $\tilde{Q}_0 = \mathcal{K}(\alpha_k,\lambda_k)X_0$ at $\ln\eta = 0$. Since X_0 is arbitrary, we can rewrite the result of (5.12) in the factorized form

$$K(\eta) = \mathcal{K}(\alpha,\lambda)\mathcal{K}^{-1}(\alpha_k,\lambda_k).$$

We find

$$\mathcal{K}(\alpha,\lambda) = \begin{pmatrix} \alpha\left(1+\frac{13}{6}\alpha\right)+\mathcal{O}_3 & -\frac{1}{\varrho}\sqrt{\frac{3\lambda}{2}}\,(\lambda+\mathcal{O}_2) \\ \sqrt{\frac{3\lambda}{2}}\,(\alpha+\mathcal{O}_2) & \lambda\left[1+\frac{19-16\varrho}{12}\alpha-\frac{3(3-2\varrho)}{4\varrho(1-2\varrho)}\lambda\right]+\mathcal{O}_3 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} \end{pmatrix}.$$

The further factorization appearing here is introduced to better organize the orders we are neglecting.

We conclude that the solutions of the Mukhanov-Sasaki equations (5.8) read, in the superhorizon limit,

$$w = \sqrt{k}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \simeq \frac{1}{\eta}\left(1+\frac{\Sigma}{9}+\frac{\Xi}{3}\right)^{-1}\mathcal{K}(\alpha,\lambda)\mathcal{K}^{-1}(\alpha_k,\lambda_k)\tilde{Q}_0. \tag{5.14}$$

At this point, what remains to do is compute $\tilde{Q}_0$.

5.5 Solution of the Mukhanov-Sasaki equations

To find $\tilde{Q}_0$, we go back to equations (5.8) and study them beyond the superhorizon limit. We need to impose the Bunch-Davies vacuum condition for large momenta $k/(aH) \gg 1$. Equations (5.8) contain a term proportional to w'/η, which turns out to be incompatible with such a condition. We remedy by switching to new variables ϖ through the field redefinition

$$w = \varpi + \Xi\varpi\ln\eta. \tag{5.15}$$

The action becomes

$$S_{\rm MS}=\int \mathrm{d}\eta \mathcal{L}_{\rm MS}=\frac{1}{2}\int \mathrm{d}\eta \left[\varpi_{-\mathbf{k}}^{\prime \rm T}\varpi_{\mathbf{k}}^{\prime}-\varpi_{-\mathbf{k}}^{\rm T}\varpi_{\mathbf{k}}+\frac{1}{\eta^{2}}\varpi_{-\mathbf{k}}^{\rm T}(2+\Sigma)\varpi_{\mathbf{k}}\right], \tag{5.16}$$

plus higher orders, having restored the subscripts $\mathbf{k}$ and $-\mathbf{k}$. The momenta are

$$\Pi_{-\mathbf{k}}=-\frac{\delta\mathcal{L}_{\rm MS}}{\delta\varpi_{-\mathbf{k}}^{\prime \rm T}}=-\varpi_{\mathbf{k}}^{\prime}, \tag{5.17}$$

the minus sign being due to the fact that the orientation of η (which is equal to $-k\tau$) is opposite to the orientation of time. We quantize the system by setting

$$[\hat{\Pi}_{\mathbf{k}},\hat{\varpi}_{\mathbf{k}'}^{\rm T}]=-i\mathbb{I}(2\pi)^{3}\delta^{(3)}(\mathbf{k}-\mathbf{k}'),\qquad [\hat{\varpi}_{\mathbf{k}},\hat{\varpi}_{\mathbf{k}'}^{\rm T}]=[\hat{\Pi}_{\mathbf{k}},\hat{\Pi}_{\mathbf{k}'}^{\rm T}]=0, \tag{5.18}$$

where $\hat{\Pi}_{\mathbf{k}}$ and $\hat{\varpi}_{\mathbf{k}}$ are the operators associated with $\Pi_{\mathbf{k}}$ and $\varpi_{\mathbf{k}}$ and $\mathbb{I}$ is the 2×2 identity matrix. Then we write

$$\hat{\varpi}_{\mathbf{k}}=P(\eta)\hat{A}_{\mathbf{k}}+P^{*}(\eta)\hat{A}_{-\mathbf{k}}^{\dagger},\qquad \hat{\Pi}_{\mathbf{k}}=-P'(\eta)\hat{A}_{-\mathbf{k}}-P'^{*}(\eta)\hat{A}_{\mathbf{k}}^{\dagger},$$

where $P(\eta)$ is an η dependent two-by-two matrix, to be determined, while

$$\hat{A}_{\mathbf{k}}=\begin{pmatrix}\hat{a}_{1\mathbf{k}}\\ \hat{a}_{2\mathbf{k}}\end{pmatrix},\qquad \hat{A}_{\mathbf{k}}^{\dagger}=\begin{pmatrix}\hat{a}_{1\mathbf{k}}^{\dagger}\\ \hat{a}_{2\mathbf{k}}^{\dagger}\end{pmatrix},$$

are vectors of (η-independent) annihilation and creation operators $\hat{a}_{i\mathbf{k}}$, $\hat{a}_{i\mathbf{k}}^{\dagger}$, satisfying

$$[\hat{A}_{\mathbf{k}},\hat{A}_{\mathbf{k}'}^{\dagger\rm T}]=\mathbb{I}(2\pi)^{3}\delta^{(3)}(\mathbf{k}-\mathbf{k}'),\qquad [\hat{A}_{\mathbf{k}},\hat{A}_{\mathbf{k}'}^{\rm T}]=[\hat{A}_{\mathbf{k}}^{\dagger},\hat{A}_{\mathbf{k}'}^{\dagger\rm T}]=0. \tag{5.19}$$

Then, (5.18) and (5.19) give

$$P'P^{*\rm T}-P^{*\prime}P^{\rm T}=i\mathbb{I}.$$

We proceed by expanding

$$P=P_{0}+\alpha_{k}P_{1}+\alpha_{k}\sqrt{\lambda_{k}}P_{2}+\alpha_{k}^{2}P_{3}+\alpha_{k}\lambda_{k}P_{4}+\mathcal{O}_{5/2}. \tag{5.20}$$

where the P_i are matrices of functions of η. We expand (the operatorial versions of) the MS equations derived from (5.16) and derive the equations solved by each P_i. The solutions are reported in formula (C.1) of appendix C to the orders we need. It is easy to check that the Bunch-Davies conditions

$$P\simeq\frac{\mathrm{e}^{i\eta}}{\sqrt{2}}\mathbb{I}\qquad \text{for } \eta\gg 1$$

are satisfied.

In the superhorizon limit we use the asymptotic behaviors (E.2) and find the operatorial version of formula (5.14) with $\hat{w}_{\mathbf{k}}$ on the left-hand side and

$$\tilde{Q}_0 = \frac{i}{\sqrt{2}} \left[\mathbb{I} + \frac{\alpha_k}{3} (7 - 3\tilde{\gamma}_M) \begin{pmatrix} 2 & 0 \\ 0 & 1 - 2\varrho \end{pmatrix} + \begin{pmatrix} \mathcal{O}_2 & \mathcal{O}_{3/2} \\ \mathcal{O}_{3/2} & \mathcal{O}_2 \end{pmatrix} \right] (\hat{A}_{\mathbf{k}} - \hat{A}^{\dagger}_{-\mathbf{k}}) \qquad (5.21)$$

on the right-hand side.

5.6 Perturbation spectra

The matrix $\mathcal{R}_{\text{RG}}$ of RG invariant perturbations is obtained by multiplying both sides of equation (5.14) by η times an α, λ-dependent matrix, to remove any dependence on τ in the superhorizon limit. This leaves us with an expression that just depends on α_k and λ_k, which reads

$$\mathcal{R}_{\text{RG}} = \frac{C}{k^{3/2}} \mathcal{K}^{-1}(\alpha, \lambda) \left(1 + \frac{\Sigma}{9} + \frac{\Xi}{3} \right) \eta \hat{w}_{\mathbf{k}} \simeq \frac{C}{k^{3/2}} \mathcal{K}^{-1}(\alpha_k, \lambda_k) \tilde{Q}_0, \qquad (5.22)$$

where C is a constant matrix. The factor $k^{-3/2}$ is determined by the behavior of the curvature perturbations $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ of formulas (D.3) in the de Sitter limit.

The matrix C cannot be fixed by RG invariance, but we can arrange it so that one entry of the matrix $\mathcal{R}_{\text{RG}}$ matches the curvature perturbation $\mathcal{R}_{\text{adiab}}$ in the de Sitter limit. The behaviors of the other entries of $\mathcal{R}_{\text{RG}}$ do not allow us to match $\mathcal{R}_{\text{iso}}$.

Specifically, we choose C proportional to the identity matrix and determine the overall constant so that the 1-1 entry of the matrix $\mathcal{R}_{\text{RG}}$ matches $\mathcal{R}_{\text{adiab}}$ for $\alpha_k, \lambda_k \simeq 0$. Using (5.21), we find

$$\mathcal{R}_{\text{RG}} = \sqrt{\frac{\pi G}{6}} \frac{i m_\phi}{k^{3/2}} T \left[\begin{pmatrix} \frac{1}{\alpha_k} \mathcal{R}_{\text{RG}}^{(11)} + \mathcal{O}_1 & \frac{1}{\alpha_k \varrho} \sqrt{\frac{3\lambda_k}{2}} + \mathcal{O}_{1/2} \\ -\sqrt{\frac{3}{2\lambda_k}} + \mathcal{O}_{1/2} & \frac{1}{\lambda_k} \mathcal{R}_{\text{RG}}^{(22)} + \mathcal{O}_1 \end{pmatrix} \right] (\hat{A}_{\mathbf{k}} - \hat{A}^{\dagger}_{-\mathbf{k}}),$$

where

$$T = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{\lambda_k} \end{pmatrix}, \qquad \mathcal{R}_{\text{RG}}^{(11)} = 1 + \frac{\alpha_k}{2} (5 - 4\tilde{\gamma}_M) - \frac{3\lambda_k}{2\varrho},$$

$$\mathcal{R}_{\text{RG}}^{(22)} = 1 + \frac{\alpha_k}{12} (9 - 40\varrho) - \tilde{\gamma}_M \alpha_k (1 - 2\varrho) + \frac{3(1 + 2\varrho)\lambda_k}{4\varrho(1 - 2\varrho)}.$$

Finally, the matrix $\mathcal{P}$ of perturbation spectra is defined from

$$\left\langle \mathcal{R}_{\text{RG}\mathbf{k}}(\tau) \mathcal{R}_{\text{RG}\mathbf{k}'}^{\text{T}}(\tau) \right\rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}.$$

The result is

$$\mathcal{P}=\frac{m_\phi^2 G}{12\pi}\left(\begin{array}{cc}\tilde{\mathcal{P}}_{11} & \tilde{\mathcal{P}}_{12}\\ \tilde{\mathcal{P}}_{21} & \tilde{\mathcal{P}}_{22}\end{array}\right), \tag{5.23}$$

where

$$\tilde{\mathcal{P}}_{11}=\frac{1}{\alpha_k^2}\left[1+\alpha_k(5-4\gamma_M)+\frac{3\lambda_k(1-2\varrho)}{2\varrho^2}\right]+\mathcal{O}_0,\qquad \tilde{\mathcal{P}}_{12}=\tilde{\mathcal{P}}_{21}=\sqrt{\frac{3}{2}}\frac{1-\varrho}{\varrho\alpha_k}+\mathcal{O}_0,$$
$$\tilde{\mathcal{P}}_{22}=\frac{1}{\lambda_k}\left\{1+(9-40\varrho)\frac{\alpha_k}{6}-2\alpha_k\gamma_M(1-2\varrho)+\frac{3}{2}\lambda_k\left(1+\frac{1}{\varrho}+\frac{4}{1-2\varrho}\right)\right\}+\mathcal{O}_1. \tag{5.24}$$

Again, the spectra are manifestly RG invariant. In passing, the results prove that α and $\varsigma=\sqrt{\alpha\lambda}$ are not the right couplings, since if we used them, we would have negative powers of α_k in the expansion, and not just as overall factors. This shows that it is not possible to identify the right couplings just from the equations of the background metric and the background field φ.

We conclude by showing that the standard curvature perturbations $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ of formulas (D.3) are not the right physical quantities. They can be worked out from equations (D.1), (D.2), then (5.1) (which express them in terms of U_1 and U_2), then (5.4) (which relate U_1 and U_2 to u_1 and u_2), and finally (5.14) and (5.21) (which give $w=\sqrt{k}\,(u_1,\,u_2)$). We find, to the NLL order,

$$\mathcal{R}_{\text{adiab}}=\frac{im_\phi}{k^{3/2}\alpha_k}\sqrt{\frac{\pi G}{6}}\left(1+\frac{\alpha_k}{2}(5-4\tilde{\gamma}_M)-\frac{3}{2}\lambda_k+6\ell(1-\varrho)\lambda_k+\mathcal{O}_2\;,\right.$$
$$\left.\sqrt{\frac{3\lambda_k}{2}}(1-4\ell(1-\varrho))+\mathcal{O}_{3/2}\right)(\hat{A}_{\mathbf{k}}-\hat{A}^\dagger_{-\mathbf{k}}),$$
$$\mathcal{R}_{\text{iso}}=-\frac{im_\phi}{k^{3/2}\alpha_k}\sqrt{\frac{\pi G}{6}}(1+\ell+2\ell\varrho)\left(-\sqrt{\frac{3\lambda_k}{2}}+\mathcal{O}_{3/2},1+\mathcal{O}_1\right)(\hat{A}_{\mathbf{k}}-\hat{A}^\dagger_{-\mathbf{k}}), \tag{5.25}$$

up to corrections that contain higher powers $\alpha_k\ln\eta\equiv\ell$. We recall that ℓ must be considered of order unity when we make a log expansion, while it is of order one when we make an ordinary expansion in powers of α_k and λ_k. The terms proportional to ℓ of formulas (5.25) are reported to show that ℓ does not disappear. This proves that $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ are not RG invariant (i.e., they are not η independent) in the superhorizon limit, differently from the combinations $\mathcal{R}_{\text{RG}}$ identified above. For this reason, they are not the correct physical perturbations. The violation of RG invariance starts from the subleading corrections in $\mathcal{R}_{\text{adiab}}$, but also affects the leading contributions to $\mathcal{R}_{\text{iso}}$.

6 Predictions

In this section we comment on the predictivity of the double-field model. Formula (2.9) gives, to the leading order,

$$\alpha_k \sim \frac{\alpha_*}{1+2\alpha_* \ln(k_*/k)}, \qquad \lambda_k \sim \frac{\bar{\lambda}}{\alpha_*^{1-2\varrho}}\alpha_k^{1-2\varrho}, \tag{6.1}$$

where k_* is some pivot scale and α_* is the pivot coupling. We see that when α_k is small, λ_k is generically larger, assuming that the constant $\bar{\lambda}/\alpha_*^{1-2\varrho}$ is of order unity. Then, formulas (5.24) imply that the leading contribution to the scalar spectrum is given by the 1-1 entry of the matrix $\mathcal{P}$, while the rest can be treated as a correction. The spectra to the leading order are

$$\mathcal{P}_T \simeq \frac{4m_\phi^2 \zeta G}{\pi}(1-3\zeta\alpha_k), \qquad \mathcal{P}_{11} \simeq \frac{m_\phi^2 G}{12\pi\alpha_k^2},$$

while the tilts and the tensor-to-scalar ratio r read

$$n_{\mathrm{t}} \simeq -6\zeta\alpha_k^2, \qquad n_{\mathrm{s}}-1 \simeq -4\alpha_k, \qquad r=\frac{\mathcal{P}_T}{\mathcal{P}_{11}} \simeq \frac{96\alpha_k^2 m_\chi^2}{2m_\chi^2+m_\phi^2} \simeq \frac{6(n_{\mathrm{s}}-1)^2 m_\chi^2}{2m_\chi^2+m_\phi^2}. \tag{6.2}$$

Recalling that, by the ABP bound, m_χ must lie in the interval $m_\phi/4 < m_\chi < \infty$, we conclude that r lies the interval

$$\frac{(n_{\mathrm{s}}-1)^2}{3} \lesssim r \lesssim 3(n_{\mathrm{s}}-1)^2, \tag{6.3}$$

at least in the perturbative regime that we have identified in this paper. This result coincides with the one of pure quantum gravity found in [33]. Moreover, in [24] it was shown that the same prediction holds when, in single-field inflation, the renormalizability requirement is relaxed and the Starobinsky potential (which is equivalent to the R^2 term in (1.1)) is replaced by any potential of class I (as per the classification of [24], which means a power series $\mathcal{V}(\mathrm{e}^{-c\phi})$ in $\mathrm{e}^{-c\phi}$, where c is a constant, ϕ is the inflaton and $\mathcal{V}(0)\neq 0$). These observations suggest that the prediction (6.3) is a robust prediction of quantum gravity.

The second scalar affects the subleading corrections. If we define $n_{\mathrm{s}}-1$ from $\mathcal{P}_{11}$, we find

$$\begin{aligned} n_{\mathrm{s}}-1 &= -\beta_\alpha(\alpha_k,\lambda_k)\frac{\partial \ln \mathcal{P}_{11}}{\partial \alpha_k} - \beta_\lambda(\alpha_k,\lambda_k)\frac{\partial \ln \mathcal{P}_{11}}{\partial \lambda_k} \\ &\simeq -4\alpha_k + \frac{4}{3}\alpha_k^2(5-6\gamma_M) + \frac{3(1-2\varrho)}{\varrho^2}\alpha_k\lambda_k + \mathcal{O}_3. \end{aligned}$$

Depending on the value of $\varrho = m^2/m_\phi^2$, the correction proportional to $\alpha_k\lambda_k$ can be larger than the one proportional to α_k^2.

If the constant $\bar{\lambda}/\alpha_*^{1-2\varrho}$ is large enough the analysis just made does not hold and it is possible to have a dominant contribution from φ to the scalar spetrum.

We point out that the conclusions just reached are nontrivial consequences of RG invariance. To some extent, they are also unexpected. To illustrate this point, let us assume for the moment that $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ are the correct scalar perturbations, as one would naively do. Then formulas (5.25) show that to the leading order they both behave like $1/\alpha_k$. However, $\mathcal{R}_{\text{iso}}$ has an extra factor $1+\ell+2\ell\varrho$. If we take $\mathcal{R}_{\text{iso}}$ in the tensor-to-scalar ratio r, we obtain

$$\mathcal{P}_T \simeq \text{as above}, \qquad \mathcal{P}_{\text{iso}} \simeq \frac{m_\phi^2 G}{12\pi\alpha_k^2}(1+\ell+2\ell\varrho)^2, \qquad n_{\text{s}}-1=\frac{\mathrm{d}\ln\mathcal{P}_{\text{iso}}}{\mathrm{d}\ln k}\simeq -\frac{2(1-2\varrho)\alpha_k}{1+\ell+2\ell\varrho}, \tag{6.4}$$

hence

$$r \simeq \frac{48\zeta\alpha_k^2}{(1+\ell+2\ell\varrho)^2} \simeq \frac{24m_\chi^2(n_{\text{s}}-1)^2}{(1-2\varrho)^2(2m_\chi^2+m_\phi^2)}. \tag{6.5}$$

With respect to (6.2), r is multiplied by an extra factor $4/(1-2\varrho)^2$, which can be as large as we wish if ϱ is close to its bound (2.8). We conclude that $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$ do not provide a meaningful prediction for r with the data available at present.

7 Conclusions

We have shown that there are situations where we can import RG techniques from high-energy physics into double-field inflation and study it as an asymptotically de Sitter RG flow in two couplings. This means that there exists a perturbative region where the cosmic RG flow resembles the one of an asymptotically free quantum field theory.

The tensor perturbations are RG invariant in the superhorizon limit, but the usual adiabatic and isocurvature perturbations are not, due to the nontrivial mixing between the scalar fields. Nonetheless, RG invariance allows us to work out the correct curvature perturbations algorithmically.

We worked out the power spectra of the tensor perturbations to the NNLL order and the ones of the scalar perturbations to the NLL order. An unexpected consequence of RG invariance is that, under mild assumptions, the theory remains predictive and the predictions to the leading order confirm those of pure quantum gravity. This does not occur if we use the adiabatic and isocurvature perturbations as commonly defined. The results suggest that, very much like gauge invariance, cosmic RG invariance is a guiding principle to identify the right physical quantities.

Referring to the classification of potentials and cosmic RG flows done in ref. [24], the model studied here describes a combination of a flow of class I (due to the Starobinsky potential) with a flow of class II (due to the powerlike φ potential). The asymmetry between the two scalar perturbations is due to the renormalizability requirement, because there is no way to combine two flows of class I (or two flows of class II) into a renormalizable theory. To further appreciate the features of double- and multi-field inflation in simpler setups, it may be interesting to renounce renormalizability and study symmetric configurations.

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Appendices: reference formulas

In these appendices we collect involved formulas that are referred to in the paper.

A Cosmic RG flow

We start with some formulas used in the formulation of the cosmic RG flow of section 2, which are the first contributions to the functions v, φ and H of expressions (2.12):

$$\begin{aligned}
v(\alpha,\lambda) &= 1-\alpha+\alpha^2\left[-2-\frac{29\alpha}{3}-\frac{638}{9}\alpha^4+3\lambda\left(\frac{1}{\varrho}-2\right)\right. \\
&\quad \left. +\alpha\lambda\left(\frac{30}{\varrho}-67+16\varrho\right)-\frac{9\lambda^2}{\varrho}+\alpha^2\mathcal{O}_3\right], \\
\frac{\hat{\kappa}\sqrt{\alpha}}{\sqrt{\lambda}}\varphi(\alpha,\lambda) &= -\frac{1}{\varrho}+\frac{2\alpha}{3}-\frac{\alpha}{6\varrho}+\frac{3\lambda}{2\varrho^2}-\frac{\alpha^2}{3}\left(2-\frac{4\varrho}{3}-\frac{5}{12\varrho}\right)+\frac{\alpha\lambda}{\varrho}-\frac{9\lambda^2}{4\varrho^3} \\
&\quad +\alpha^2\lambda\left(\frac{8}{3}-\frac{11}{6\varrho}+\frac{1}{8\varrho^2}\right)+\frac{3\alpha\lambda^2}{8\varrho^3}(1-4\varrho) \\
&\quad +\frac{\alpha^3}{216}\left(304-\frac{57}{\varrho}-336\varrho+128\varrho^2\right)+\frac{27\lambda^3}{8\varrho^4}+\mathcal{O}_4, \qquad \text{(A.1)} \\
\frac{\sqrt{6\alpha}}{m_\phi}H(\alpha,\lambda) &= 1-\frac{\alpha}{12}-\frac{3\lambda}{4\varrho}+\frac{19\alpha^2}{288}-\frac{\alpha\lambda}{2}\left(1-\frac{3}{8\varrho}\right)+\frac{27\lambda^2}{32\varrho^2}-\frac{373\alpha^3}{3456} \\
&\quad +\alpha^2\lambda\left(\frac{11}{8}-\frac{53}{128\varrho}-\varrho\right)+\frac{3\alpha\lambda^2}{8\varrho}\left(1-\frac{15}{16\varrho}\right)-\frac{135\lambda^3}{128\varrho^3}+\mathcal{O}_4.
\end{aligned}$$

B Tensor spectrum

Here we collect formulas about the spectrum of the tensor perturbations studied in section 4. We start with the coefficients of the de-Sitter diagonalized Lagrangians of formulas (4.4) and (4.5), which are

$$q_2 = \frac{1}{\tau^2} - \frac{k^2}{2} + \left(\frac{9}{2\tau^2} + \xi k^2\right)\zeta\alpha^2 + \left[\frac{3\,(32+37\xi+6\xi^2)}{\tau^2} - \frac{k^2\xi}{3}(14-11\xi)\right]\frac{\zeta^2}{4}\alpha^3$$
$$+\left[\frac{27}{2\tau^2} + 3k^2\xi\left(2-\frac{1}{\varrho}\right)\right]\frac{\zeta}{2}\alpha^2\lambda, \qquad q_4 = \frac{2}{\xi\tau^2} + \frac{k^2}{2} + \frac{6\alpha}{\xi\tau^2},$$

$$\begin{aligned} q_5 &= -\frac{(2-37\xi+14\xi^2)\,\zeta\alpha^2}{4\xi\tau^2} + 2k^2\xi\zeta\alpha^2 + \frac{9\alpha\lambda}{2\varrho\xi\tau^2} \\ &-\frac{(19-239\xi-182\xi^2+42\xi^3)\,\zeta^2\alpha^3}{12\xi\tau^2} - \frac{1}{6}k^2\,(14-11\xi)\,\xi\zeta^2\alpha^3 \\ &+\frac{3\,(2(3+\varrho)+2\,(2\varrho+3)\,\xi+3\,(1-2\varrho)\,\xi^2)\,\zeta\alpha^2\lambda}{2\varrho\xi\tau^2} + 3k^2\xi\zeta\left(2-\frac{1}{\varrho}\right)\alpha^2\lambda, \\ q_6 &= -\frac{\zeta\alpha^2(1-\xi)}{3\tau}\left[6+5\alpha+9\left(2-\frac{1}{\varrho}\right)\lambda\right]. \end{aligned} \tag{B.1}$$

Fakeon projection. The coefficients of the fakeon projection (4.11) read

$$\begin{aligned} s_1(\alpha,\lambda) &= \frac{\zeta\alpha}{16}\left[\zeta\alpha\left(2-29\xi+6\xi^2\right) - \frac{18\lambda}{\varrho}\right] + \frac{\zeta^3\alpha^3}{48}(1+132\xi-303\xi^2+62\xi^3) \\ &+\frac{3\zeta^2\alpha^2\lambda}{8\varrho}(3-5\xi-\xi^2-\varrho(2+3\xi-2\xi^2)) + \alpha\mathcal{O}_3, \\ s_2(\alpha,\lambda) &= -\frac{\xi(1-\xi)}{2}\zeta^2\alpha^2 + \frac{\xi\zeta^3\alpha^3}{24}(32-130\xi+35\xi^2) - \frac{3\xi\zeta^2\alpha^2\lambda}{4\varrho}(2+\xi-\varrho(1+2\xi)) + \alpha\mathcal{O}_3. \end{aligned} \tag{B.2}$$

RG invariance. The solution of equation (4.21) is (4.22) with

$$\begin{aligned} J_{\mathrm{t}}(\alpha,\lambda) &= 1 + \frac{3\zeta\alpha}{2} + \frac{56+61\xi+12\xi^2}{16}\zeta^2\alpha^2 + \frac{9}{8\varrho}\zeta\alpha\lambda \\ &+\left(1544+2147\xi+1052\xi^2+162\xi^3\right)\frac{\zeta^3\alpha^3}{96} \\ &-\frac{3}{16\varrho}\left[13+14\xi+6\xi^2-6\varrho(6+7\xi+2\xi^2)\right]\zeta^2\alpha^2\lambda + \mathcal{O}_4. \end{aligned} \tag{B.3}$$

The constants c_i that parametrize the $\mathcal{O}_4$ contributions (4.16) to $\sigma_{\rm t}$ read

$$
\begin{aligned}
c_1 &= 277 + \frac{3109}{8}\xi + \frac{345}{4}\xi^2 + 81\zeta, \qquad c_3 = \frac{81}{4\zeta\varrho}, \\
c_2 &= -\frac{9}{8\varrho}\left[112 + 107\xi + 30\xi^2 - \varrho(262 + 293\xi + 72\xi^2) + 8\varrho^2(8 + 10\xi + 3\xi^2)\right]. \qquad \text{(B.4)}
\end{aligned}
$$

Normalization of the spectrum. The constant $\tilde{Q}_{\rm t}(\alpha_k, \lambda_k)$ of (4.22) is

$$
\begin{aligned}
\tilde{Q}_{\rm t}(\alpha_k, \lambda_k) = \frac{i}{\sqrt{2}} \Bigg[& 1 + \frac{3\zeta\alpha_k^2}{2}(4 + \xi - 2\tilde{\gamma}_M) - \zeta\alpha_k^3\pi^2 - 6\zeta\alpha_k^3\tilde{\gamma}_M^2 + \frac{3}{2}\tilde{\gamma}_M\zeta^2\alpha_k^3(8 + \xi) \\
& + \frac{9}{8}\xi\zeta^2\alpha_k^3(10 + 3\xi) + \frac{9}{4}\zeta\alpha_k^2\lambda_k\left(4 - 2\tilde{\gamma}_M + 2\xi - \frac{\xi}{\varrho}\right) + \mathcal{O}_4\Bigg]. \qquad \text{(B.5)}
\end{aligned}
$$

C Scalar spectrum

The solutions of the MS equations derived from (5.16) with the expansion (5.20) are given by

$$
P_0 = W_0\mathbb{I}, \quad P_1 = \frac{W_2}{3}\begin{pmatrix} 2 & 0 \\ 0 & 1 - 2\varrho \end{pmatrix}, \quad P_2 = \sqrt{\frac{3}{2}}W_2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad P_4 = -\frac{W_2}{\varrho}\begin{pmatrix} 1 & 0 \\ 0 & \varrho \end{pmatrix},
$$

$$
P_3 = \frac{W_4}{4}\begin{pmatrix} 4 & 0 \\ 0 & (1 - 2\varrho)^2 \end{pmatrix} + \frac{1}{6}\left[\frac{1}{2}(1 - 4\varrho^2)W_3 + \frac{5 + 18\varrho - 12\varrho^2}{3}W_2\right]\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \qquad \text{(C.1)}
$$

D Adiabatic and isocurvature perturbations

The adiabatic and isocurvature perturbations can be defined by switching to the inflaton framework through a Weyl transformation combined with a diffeomorphism (see the appendix of [33] for details). The latter amounts to defining a new cosmological time $\bar{t}$, to preserve the structure of the parametrization (3.2) of the metric. We have

$$
g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \mathrm{e}^{2\sigma}\bar{g}_{\mu\nu}\mathrm{d}\bar{x}^\mu\mathrm{d}\bar{x}^\nu, \qquad \mathrm{e}^{-2\sigma} = 1 - \frac{\Omega_0 + \Omega}{3m_\phi^2}, \qquad \frac{\mathrm{d}\bar{t}}{\mathrm{d}t} = \mathrm{e}^{-\sigma_0}, \qquad \text{(D.1)}
$$

where the bars are used to denote quantities in the inflaton framework. The space coordinates are not affected by the transformation.

From the transformed action (3.1), we find the scalar fields $\phi^i = (\sigma, \varphi) = (\sigma_0 + \delta\sigma, \Theta_0 + \Theta) \equiv \phi_0^i + \delta\phi^i$ and the matrix $G_{ij}(\phi_0) = \mathrm{diag}(4/\hat{\kappa}^2, \mathrm{e}^{2\sigma_0})$, defined so that the scalar kinetic terms of the new action read

$$\frac{1}{2}\int \mathrm{d}^4\bar{x}\sqrt{-\bar{g}}G_{ij}(\phi)\bar{g}^{\mu\nu}\bar{\partial}_\mu\phi^i\bar{\partial}_\nu\phi^j.$$

The formulas of [33] give

$$\bar{\Psi}=\delta\sigma=\frac{\Omega}{6m_\phi^2}\mathrm{e}^{2\sigma_0}. \tag{D.2}$$

The standard definitions of adiabatic and isocurvature perturbations are [35]

$$\mathcal{R}_{\text{adiab}}=\bar{\Psi}+\bar{H}\frac{G_{ij}(\phi_0)(\partial_{\bar{t}}\phi_0^i)\delta\phi^j}{G_{kl}(\phi_0)(\partial_{\bar{t}}\phi_0^k)(\partial_{\bar{t}}\phi_0^l)},\qquad \mathcal{R}_{\text{iso}}=\frac{G_{ij}(\phi_0)\psi_0^i(\delta\phi^j\bar{H}+\bar{\Psi}\partial_{\bar{t}}\phi_0^j)}{\sqrt{G_{kl}(\phi_0)(\partial_{\bar{t}}\phi_0^k)(\partial_{\bar{t}}\phi_0^l)}}, \tag{D.3}$$

where ψ_0^i are defined so that $G_{ij}(\phi_0)\psi_0^i\psi_0^j=1$ and $G_{ij}(\phi_0)\psi_0^i(\partial_{\bar{t}}\phi_0^j)=0$.

The explicit expressions of $\mathcal{R}_{\text{adiab}}$ and $\mathcal{R}_{\text{iso}}$, which we do not report here, can be worked out from the formulas just given. We have used them to work out (5.25) and the predictions (6.4) and (6.5) of section 6.

E Other formulas

The functions

$$\begin{aligned}
W_0 &= \frac{i(1-i\eta)}{\eta\sqrt{2}}\mathrm{e}^{i\eta},\qquad W_2=\frac{6W_0}{1-i\eta}-3\left(i\pi-\mathrm{Ei}(2i\eta)\right)W_0^*,\\
W_3 &= \left[6(\ln\eta+\tilde{\gamma}_M)^2+24i\eta F_{2,2,2}^{1,1,1}(2i\eta)+\pi^2\right]W_0^*+\frac{24W_0}{1-i\eta}-4(\ln\eta+1)W_2,\\
W_4 &= -\frac{16W_0}{1+\eta^2}+\frac{2(13+i\eta)W_2}{9(1+i\eta)}+\frac{W_3}{3}+4G_{2,3}^{3,1}\left(-2i\eta\,\middle|\,{}^{0,1}_{0,0,0}\right)W_0,
\end{aligned} \tag{E.1}$$

introduced in [23] appear frequently and can be used to express the solutions of the Mukhanov-Sasaki equations to the lowest orders. Ei denotes the exponential-integral function, $F_{b_1,\cdots,b_q}^{a_1,\cdots,a_p}(z)$ the generalized hypergeometric function ${}_pF_q(\{a_1,\cdots,a_p\},\{b_1,\cdots,b_q\};z)$ and $G_{p,q}^{m,n}$ the Meijer G function.

The asymptotic behaviors in the superhorizon limit $\eta\simeq 0$ are

$$\begin{aligned}
&\eta W_2\simeq\frac{3i}{\sqrt{2}}\left(2-\tilde{\gamma}_M-\ln\eta\right),\qquad \eta W_4\simeq -8i\sqrt{2}+\frac{26}{9}\eta W_2+\frac{\eta W_3}{3}+i\sqrt{2}(\ln\eta+\tilde{\gamma}_M)^2+\frac{i\pi^2}{\sqrt{2}},\\
&\eta W_3\simeq -3i\sqrt{2}(\ln\eta+\tilde{\gamma}_M)^2-\frac{i\pi^2}{\sqrt{2}}+12i\sqrt{2}-4(\ln\eta+1)\eta W_2.
\end{aligned} \tag{E.2}$$

We quantize (4.14) as usual, by introducing the operator

$$\hat{w}_{\mathbf{k}}(\eta) = w_{\mathbf{k}}(\eta)\hat{a}_{\mathbf{k}} + w^*_{-\mathbf{k}}(\eta)\hat{a}^\dagger_{-\mathbf{k}},$$

where $\hat{a}^\dagger_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}$ are creation and annihilation operators satisfying $[\hat{a}_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k} - \mathbf{k}')$, $[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = [\hat{a}^\dagger_{\mathbf{k}}, \hat{a}^\dagger_{\mathbf{k}'}] = 0$, so that

$$\langle w_{\mathbf{k}}(\eta)w_{\mathbf{k}'}(\eta)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}')|w_{\mathbf{k}}|^2.$$

Summing over the graviton polarizations u and v, the power spectrum $\mathcal{P}_T$ of the tensor perturbations is defined by the two-point function

$$\langle \hat{u}_{\mathbf{k}}(\tau)\hat{u}_{\mathbf{k}'}(\tau)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}')\frac{\pi^2}{8k^3}\mathcal{P}_T, \qquad \mathcal{P}_T = \frac{8k^3}{\pi^2}|u_{\mathbf{k}}|^2, \tag{E.3}$$

For convenience, we recall some notations frequently used in the paper, which are

$$\varrho = \frac{m^2}{m_\phi^2}, \qquad \xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad \tilde{\gamma}_M = \gamma_M - \frac{i\pi}{2}, \qquad \gamma_M = \gamma_E + \ln 2,$$

γ_E being the Euler-Mascheroni constant.

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