Predictions in Inflationary Cosmology from Quantum Gravity with Purely Virtual Quanta

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OUTLINE

- Brief overview
- Purely virtual quanta in cosmology
- Purely virtual quanta in quantum field theory

Overview of the theory

- Purely virtual quantum (fakeon):

  a degree of freedom that mediates interactions and circulate inside loops but cannot appear as external state in physical processes.
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Quantum gravity with fakeons

- Degrees of freedom of the theory:

massless spin-2 $h_{\mu\nu}$,	massive scalar ϕ,	massive spin-2 $\chi_{\mu\nu}$.
(graviton)	(inflaton)	(fakeon)

- Parameters:

$$M_{\rm Pl}, \qquad \Lambda_C, \qquad m_\phi, \qquad m_\chi.$$

- Physical content in cosmology:

  scalar perturbations, tensor perturbations, no vectors.

PURELY VIRTUAL QUANTA IN COSMOLOGY

Results: Amplitudes and spectral indices

D. Anselmi, E. Bianchi and MP, arXiv:2005.10293

- Leading order

$A_{\mathcal{R}}$	A_T	r	$n_{\mathcal{R}} - 1$	n_T
$\frac{m_\phi^2 N^2}{3\pi M_{\rm Pl}^2}$	$\frac{4m_\phi^2}{\pi M_{\rm Pl}^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$\frac{12}{N^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$	$-\frac{2}{N}$	$-\frac{3}{2N^2} \frac{2m_\chi^2}{m_\phi^2+2m_\chi^2}$

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- Higher order corrections

$$A_{\mathcal{R}} = \frac{GN^2m_\phi^2}{3\pi}\left(1-\frac{\ln N}{6N}+\mathcal{O}\left(\frac{1}{N}\right)\right).$$

$$A_T = \frac{8G}{\pi}\frac{m_\chi^2 m_\phi^2}{m_\phi^2+2m_\chi^2}\left(1-\frac{3m_\chi^2}{N(m_\phi^2+2m_\chi^2)}\left(1+\frac{\ln N}{12N}\right)+\mathcal{O}\left(\frac{1}{N^2}\right)\right).$$

With $N=60$, the first correction to A_T is between 0.3% and 2.5%.

Results: ratio

- From Planck 2018, $n_{\mathcal{R}} = 0.9649 \pm 0.0042$ at 68% CL.
- From the predictions, $n_{\mathcal{R}} = 1 - 2/N$.
- From the bound $m_\chi > m_\phi/4$,

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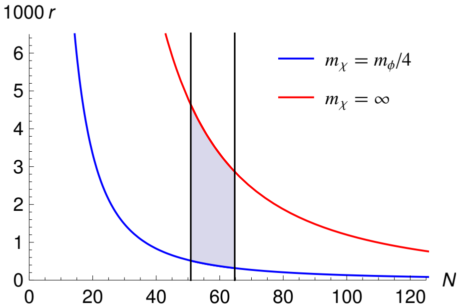

$N = 60 \qquad 0.4 \lesssim 1000r \lesssim 3, \qquad -0.4 \lesssim 1000 n_T \lesssim -0.05.$

Action and properties

$$S_{\mathrm{QG}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int\sqrt{-g}\left[R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right]$$

Properties:

- Unitarity
  [D. Anselmi and MP, PRD 96 (2017) 045009, D. Anselmi, JHEP 02 (2018) 141].
- Renormalizability (once Λ_{CC} is reinstated).
- Violation of microcausality [D. Anselmi and MP, JHEP 11 (2018) 21].
- No violation of macrocausality
  [D. Anselmi and A. Marino, Class. Quantum Grav. 37 (2020) 095003].

Frameworks

Classical background: FLRW $\quad g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2).$
De Sitter expansion

$$\varepsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = 2\varepsilon - \frac{\dot{\varepsilon}}{2H\varepsilon}.$$

- Geometric framework

$$S_{\mathrm{geom}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int\sqrt{-g}\left[R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right]$$

m_ϕ and H are unrelated

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  - Expand in ε with H^2/m_χ^2 fixed;
  - Expand in ε with m_χ^2/m_ϕ^2 fixed.

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- Inflaton framework

$$S_{\mathrm{infl}}(g) = -\frac{M_{\mathrm{Pl}}^2}{16\pi}\int \sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + S_\phi(g,\phi),$$

$$S_\phi(g,\phi) = \frac{1}{2}\int \sqrt{-g}\left(\nabla_\mu\phi\nabla^\mu\phi - 2V(\phi)\right), \qquad V(\phi) = \frac{m_\phi^2}{2\hat{\kappa}^2}\left(1 - \mathrm{e}^{\hat{\kappa}\phi}\right)^2$$

Action for cosmological perturbations

$$S(u) = \frac{1}{2}\int \mathrm{d}t\ a(t)^3 \left[f(t)\dot{u}^2 - h(t)\ddot{u}^2 - g(t)u^2\right], \qquad u \equiv u_{\mathbf{k}}(t)$$

$$\Downarrow$$

$$S'(U,V) = \frac{1}{2}\int \mathrm{d}t\ Z\left(\dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma UV\right).$$

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Procedure:

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iii) Quantize the new action with the standard methods.

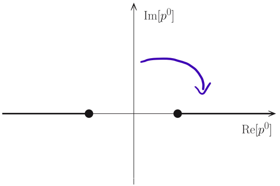
- The physical variable is still $u = F(U,V)$.
- After the procedure, the two-point function is

$$\langle uu\rangle = \langle F(U,V)F(U,V)\rangle, \qquad \text{with} \qquad V = V(U).$$

Fakeon prescription and fakeon Green function

- The fakeon prescription comes from high-energy physics and deals with scattering amplitudes.

$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$



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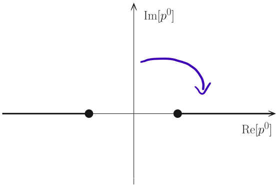
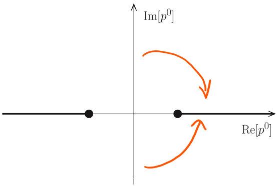


$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{A}_+(p) + \mathcal{A}_-(p)\right].$$



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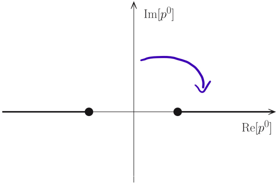

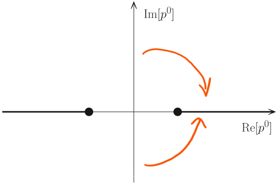

- Classical level: fakeon Green function.

$$G_{\rm f} = \frac{1}{2}\left(G_{\rm adv} + G_{\rm ret}\right).$$

Example:

$$\ddot{V} + \omega^2 V = \left(\frac{{\rm d}^2}{{\rm d}t^2} + \omega^2\right) V = F(t) \qquad \Rightarrow \qquad V(t) = (G_{\rm f} * F)(t).$$

This is enough for tree level correlation functions in cosmology.

Fakeon Green function in FLRW spacetime

- In flat spacetime

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}t^2} + A^2\right) G_{\mathrm{f}}(t, t') = \delta(t - t'), \qquad A = \text{const.}$$

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$$G_\mathrm{f}(t,t')=\frac{i\pi\,\mathrm{sgn}(t-t')}{4H\sinh(n_\chi\pi)}\left[J_{in_\chi}(\check{k})J_{-in_\chi}(\check{k}')-J_{in_\chi}(\check{k}')J_{-in_\chi}(\check{k})\right],\qquad \check{k}^{(\prime)}=\frac{k}{a(t^{(\prime)})H}.$$

Projected action

$$S'(U,V) = \frac{1}{2}\int \mathrm{d}t\ Z\left(\dot{U}^2 - \omega^2 U^2 - \dot{V}^2 + \Omega^2 V^2 + 2\sigma UV\right).$$

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$$V(U) = -G_{\mathrm{f}} * (\sigma U), \qquad S^{\mathrm{prj}}(U) = S'(U, V(U)).$$

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$$S^{\mathrm{prj}}_w = \frac{1}{2}\int \mathrm{d}\tau \left[w'^2 - \bar{k}^2 w^2 + \frac{w^2}{\tau^2}\left(\nu_{\mathrm{t}}^2 - \frac{1}{4}\right)\right], \qquad \bar{k} = k\Big(1 + \mathcal{O}(\varepsilon)\Big).$$

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- The nonlocalities enter in the correlation functions since the physical variable is still

$$u = U + \alpha V(U), \qquad U = U(w).$$

Consistency condition

The fakeon eom's can always be turned into the form

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Imposing a no-tachyon condition on $m(t)^2$ in the flat-space limit

$$m(t)^2\big|_{k/(aH)\to 0} > 0. \qquad \Rightarrow \qquad n_\chi \in \mathbb{R}.$$

It can be seen also from the Green function

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All perturbations give the same bound

$$\boxed{m_\chi > \frac{m_\phi}{4}.}$$

Predictions in inflationary cosmology

- Amplitudes and spectral indices in de Sitter and quasi de Sitter.

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The bound restricts the possible values for the tensor-to-scalar ratio r

$$\frac{1}{9} \lesssim \frac{N^2}{12} r \lesssim 1.$$

$$N = 60 \qquad 0.4 \lesssim 1000r \lesssim 3,$$

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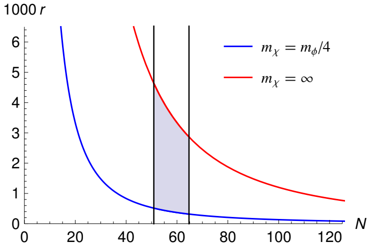

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PURELY VIRTUAL QUANTA IN QUANTUM FIELD THEORY

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Using the Feynman prescription

Einstein gravity: unitary but nonrenormalizable theory

$$S_{\rm HE} = -\frac{1}{2\kappa^2}\int \sqrt{-g}R, \quad \kappa^2 = 8\pi G.$$

$$\Gamma^{(\rm ct)}_{\rm HE} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

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Stelle gravity: renormalizable but not unitary theory

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[\gamma R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + 2\Lambda_{\mathrm{C}}\big].$$

$$\Gamma^{(\mathrm{ct})}_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \sqrt{-g}\big[a_\gamma R + a_\alpha R_{\mu\nu}R^{\mu\nu} + a_\beta R^2 + 2a_{\mathrm{C}}\big].$$

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- Pseudo-unitarity equation (cutting equations)

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in general

$$H = \mathrm{diag}(\ldots, 1, \ldots, 1, -1, \ldots, -1, \ldots).$$

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$$\mathrm{H} = \mathrm{diag}(\ldots, 1, \ldots, 1, -1, \ldots, -1, \ldots).$$

- To have unitarity

$$\mathrm{H} = \mathbb{1} \qquad \text{or} \qquad \mathrm{H}|_{\mathrm{V}} = \mathbb{1} \quad + \quad \textbf{consistent projection}$$

Unitarity

$$SS^\dagger = 1, \qquad S = 1 + iT.$$

- Unitarity equation (optical theorem)

$$2\mathrm{Im}T = TT^\dagger.$$

- Pseudo-unitarity equation (cutting equations)

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- Fakeon prescription sets

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Prescription for the propagator $\frac{\pm 1}{k^2 - m^2}$

Prescription for the amplitude $\mathcal{A}(p)$

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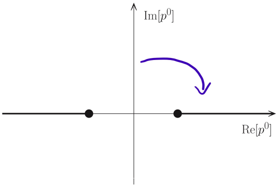
Standard particle : $\quad \dfrac{1}{k^2 - m^2 + i\epsilon}.$

Ghost : $\quad \dfrac{-1}{k^2 - m^2 + i\epsilon}.$

Prescription for the amplitude $\mathcal{A}(p)$



$$\mathcal{A}_+(p) = \mathcal{A}(p + i\epsilon)$$

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Fake particle (fakeon):

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+ integration domain deformations

(D. Anselmi and MP, JHEP 1706 (2017) 066.)

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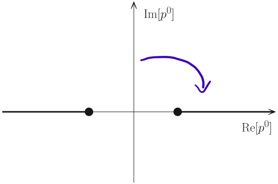

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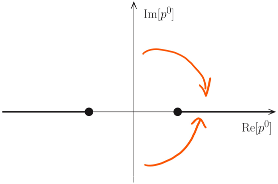

$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{A}_+(p) + \mathcal{A}_-(p)\right].$$

A theory of particles and fakeons is unitary.

D. Anselmi and MP, PRD 96 (2017) 045009. D. Anselmi, JHEP 02 (2018) 141.

$$D(k^2, m^2, \mathcal{E}) = \frac{1}{(k^2 - m^2)^2 + \mathcal{E}^4}.$$

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Minkowski models

$$i\mathcal{M}(p) = c\int_{\mathbb{R}} \frac{\mathrm{d}k^0}{2\pi} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 k}{(2\pi)^3} D(p-k, m_1, \mathcal{E}) D(k, m_2, \mathcal{E}).$$

Not unitary, nonlocal and non-Hermitean divergences (Aglietti and Anselmi).

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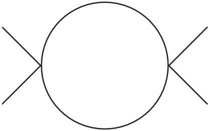
The fakeon propagator is not the Cauchy principal value.

$$\lim_{\mathcal{E}\to 0} \frac{k^2 - m^2}{(k^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P}\frac{1}{k^2 - m^2} = \lim_{\epsilon \to 0^+} \frac{1}{2}\left(\frac{1}{k^2 - m^2 + i\epsilon} + \frac{1}{k^2 - m^2 - i\epsilon}\right).$$

A very simple example

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{\lambda}{4!}\varphi^4.$$

One-loop bubble diagram



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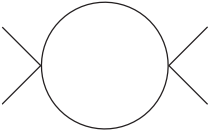

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After renormalizing the UV divergence

Feynman prescription

$$\mathcal{A}_+(p) = \frac{1}{2(4\pi)^2}\ln\frac{-p^2 - i\epsilon}{\mu^2}.$$

Fakeon presciption

$$\mathcal{A}_{\mathrm{AV}}(p) = \frac{1}{4(4\pi)^2}\ln\frac{(p^2)^2}{\mu^4}.$$

note: the fakeon prescription does not spoil renormalizability

The theory of quantum gravity and fakeons

$$S_{\mathrm{QG}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{1}{6m_\phi^2}R^2\right].$$

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- iv) Sum back and project onto the subspace where only particles are external states.

Absorptive part of graviton self energy at $\Lambda_C = 0$

D. Anselmi and MP, JHEP 11 (2018) 021.

Equivalent action:
auxiliary fields ϕ, $\chi_{\mu\nu}$ + Weyl transformation + field redefinitions.

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$$S(g, \phi, \chi, \Phi) = S_{\mathrm{EH}}(g) + S_\chi(g, \chi) + S_\phi(g + 2\chi, \phi) + S_m(ge^{\kappa\phi} + 2\chi e^{\kappa\phi}, \Phi).$$

S_{EH} = Einstein-Hilbert, S_χ = (−)Pauli-Fierz + interactions, S_m = Standard Model,

$$S_\phi(g, \phi) = \frac{3\zeta}{4} \int \sqrt{-g} \left[\nabla_\mu \phi \nabla^\mu \phi - \frac{m_\phi^2}{\kappa^2} (1 - e^{\kappa\phi})^2 \right].$$

Violation of microcausality

Resumming the self energies the corrected χ propagator at the peak m_χ is

$$\langle \chi_{\mu\nu}(p)\chi_{\rho\sigma}(-p)\rangle_{s\sim\bar{m}_\chi^2} = -\frac{i\kappa^2}{\zeta}\frac{Z_\chi}{s-\bar{m}_\chi^2+i\bar{m}_\chi\Gamma_\chi}\Pi^{(2)}_{\mu\nu\rho\sigma}(p,s),$$

$$\Gamma_\chi = -\frac{m_\chi^3}{M_{\text{Pl}}^2}C, \qquad C = \frac{N_s+6N_f+12N_v}{120}, \qquad \Gamma_\chi < 0.$$

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Duration $\sim 1/|\Gamma_\chi|$

If $m_\chi \sim 10^{12}$GeV then $1/|\Gamma_\chi| \sim 4\cdot 10^{-20}$s

For time intervals of the order $1/|\Gamma_\chi|$ past, present and future, as well as cause and effect lose meaning.

Conclusions

New predictions from quantum gravity with purely virtual quanta

- Purely virtual quanta provide a local, unitary and renormalizable theory of quantum gravity.
- The theory is essentially unique.
- Computational power (Feynman diagrams).
- Predictive.
- Falsifiable.
  - Amplitudes and spectral indices of scalar and tensor perturbations
  - Once new cosmological data will be available, m_ϕ and m_χ will be fixed and other predictions will be stringent tests of the theory.