Quantum gravity from fakeons

Damiano Anselmi

- The idea of fake particle ("fakeon") leads to the right theory of quantum gravity
- Physical implications and predictions
- Is the Higgs boson a fakeon?
- Classical limit : new and unexpected features
- FLRW solution

The problem of quantum gravity is to make it renormalizable and unitary at the same time

$$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

Higher derivatives lead to renormalizability, but violate unitarity, unless ...

Unitarity: $$S^\dagger S = 1$$

$S = 1 + iT$

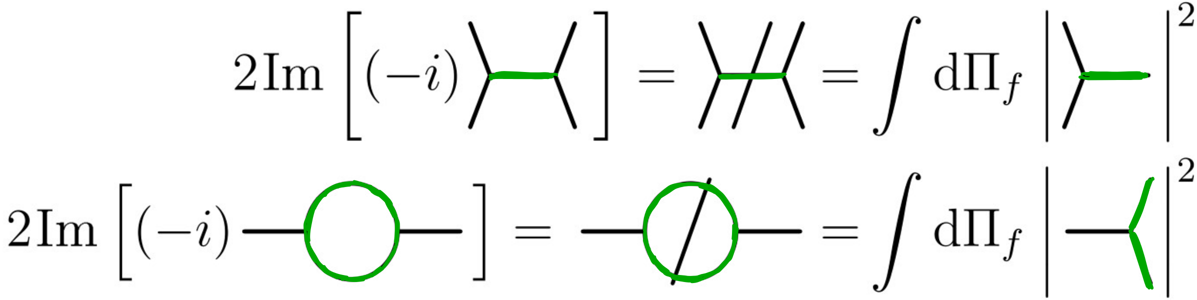
$$2\,\mathrm{Im}T = T^\dagger T$$ Optical theorem

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \langle b|T^\dagger\underbrace{|n\rangle\langle n|}_{\pm 1}T|a\rangle, \qquad |a\rangle, |b\rangle \in W.$$

In general, W may contain unphysical states in higher-derivative theories (those with $\sigma_n = 1$)

Feynman prescription $\longrightarrow$ pseudounitarity equation

Diagrammatic version of the optical theorem :



cutting equations

Propagator :

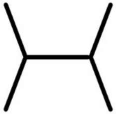
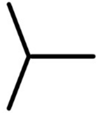
$$G(p,m) = \frac{1}{p^2 - m^2}$$

A prescription is needed

The Feynman prescription gives

$$G_+(p,m,\epsilon) = \frac{1}{p^2 - m^2 + i\epsilon}$$

The optical theorem is ok:

2Im $\Big[(-i)$  $\Big]$ $=$  $= \int d\Pi_f \Big|$  $\Big|^2 \geq 0$

Indeed:

$$\mathrm{Im}\left[-\frac{1}{p^2-m^2+i\epsilon}\right] = \pi\delta(p^2-m^2) \quad \geq 0$$

Ghost: opposite residue $-\dfrac{1}{p^2-m^2+i\epsilon}$

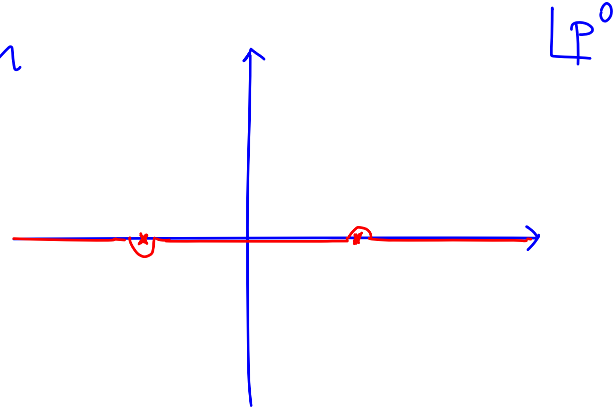
The optical theorem is violated

$-\dfrac{1}{p^2-m^2-i\epsilon}$ would be ok

Note that the prescription is crucial

Correct answer: the fakeon

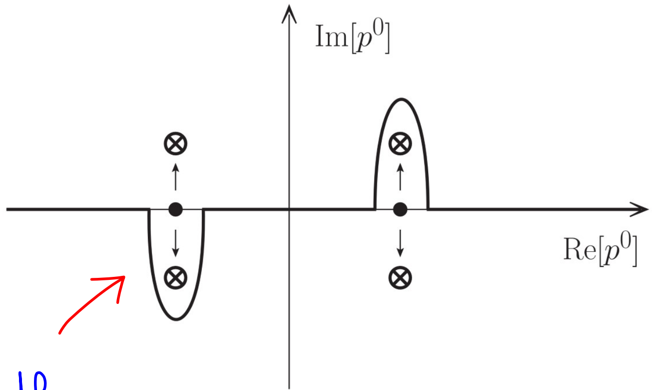
$$G(p,m) = \frac{1}{p^2 - m^2}$$



Write $\pm \dfrac{p^2 - m^2}{(p^2 - m^2)^2}$ and split the poles into pairs

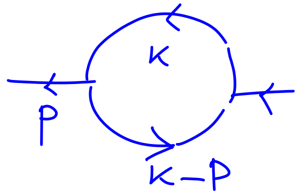
$$\mathbb{G}_\pm(p,m,\mathcal{E}^2) = \pm \frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4}$$

infinitesimal width



Lee-Wick integration path

Example: bubble diagram



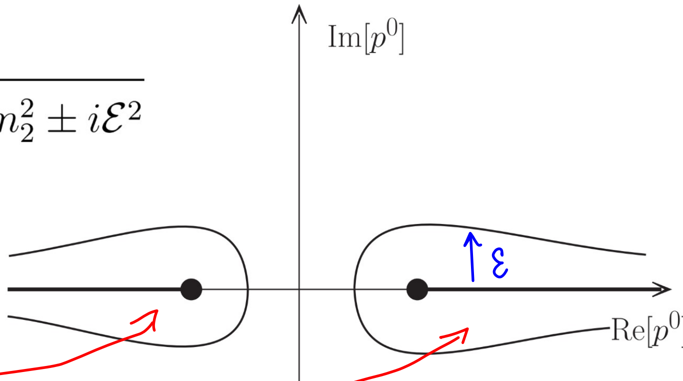
$$i\mathcal{M} \propto \int \frac{dk^0}{2\pi} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \mathbb{G}_\pm(p-k, m_1, \mathcal{E}^2)\mathbb{G}_\pm(k, m_2, \mathcal{E}^2)$$

LW

$$= \int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p, \mathbf{k})$$

$w(p, \vec{k})$ is singular for

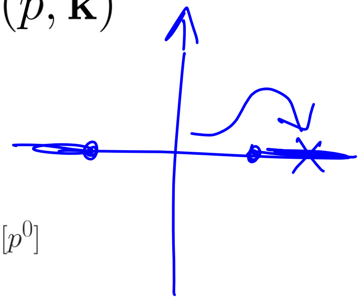
$$|p^0| = \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_1^2 \pm i\mathcal{E}^2} + \sqrt{\mathbf{k}^2 + m_2^2 \pm i\mathcal{E}^2}$$



Here Lorentz invariance & analiticity are violated

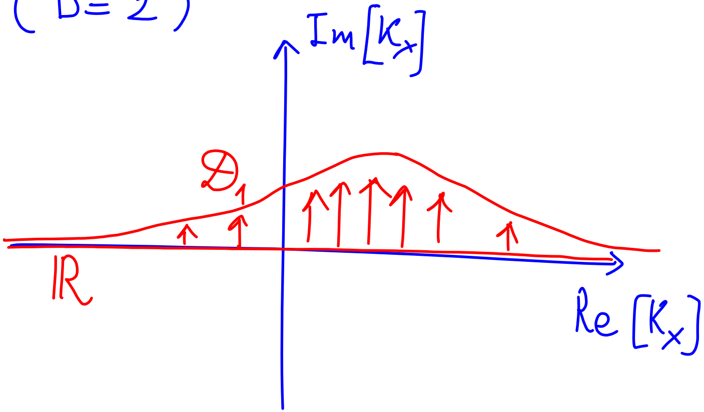
The integration domain on the loop space momentum must be deformed

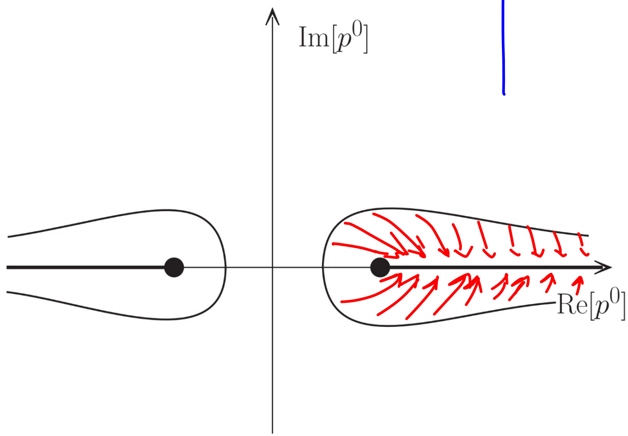
$$\int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p,\mathbf{k}) \longrightarrow \int_{\mathcal{D}_3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p,\mathbf{k})$$



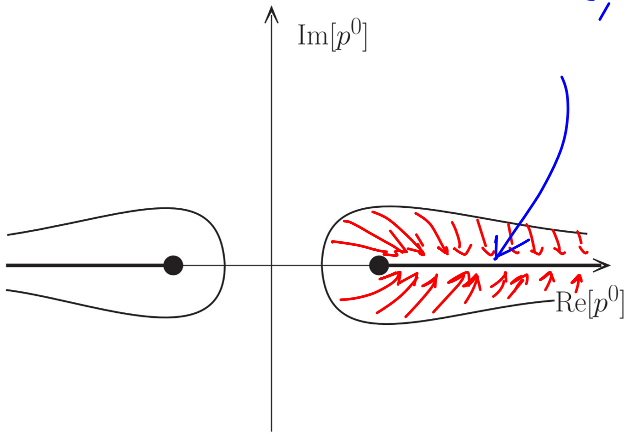
Deformation: $\mathbb{R} \to \mathcal{D}_1$

$(D=2)$





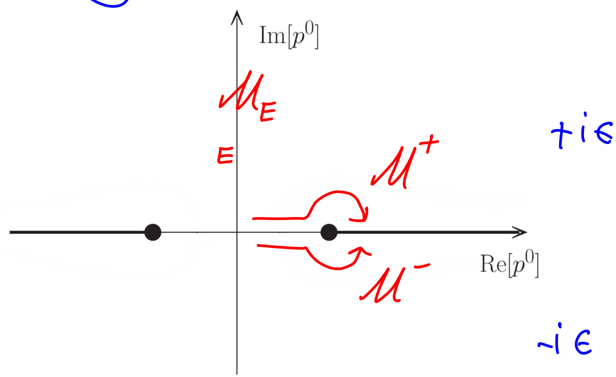
calculate INSIDE, then deform the domain (and, possibly, take $\varepsilon \to 0$)



Lorentz invariance and analyticity are recovered in the limit

Result: average continuation

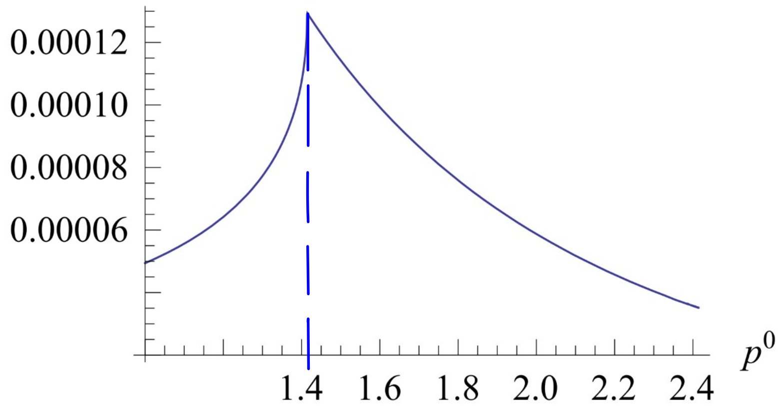
$$\frac{1}{2}\left(\mathcal{M}^+ + \mathcal{M}^-\right)$$



Result :

Re [(-i)–○–] fakeon threshold

Im [(-i)–○–] = 0



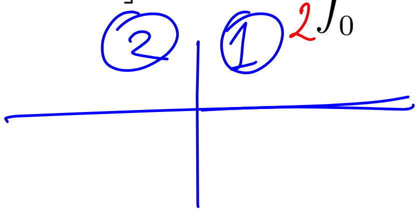
Feynman :

$$\int_0^1 \mathrm{d}x \ln\left[-p^2x(1-x)+m^2-i\epsilon\right]$$

$\ln z$

Fakeon :

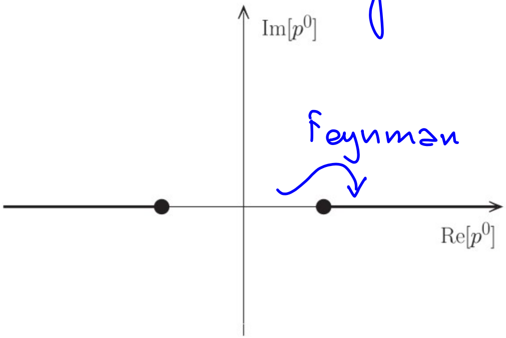
$$\frac{1}{2}\int_0^1 \mathrm{d}x \ln\left[\left(-p^2x(1-x)+m^2\right)^2\right]$$

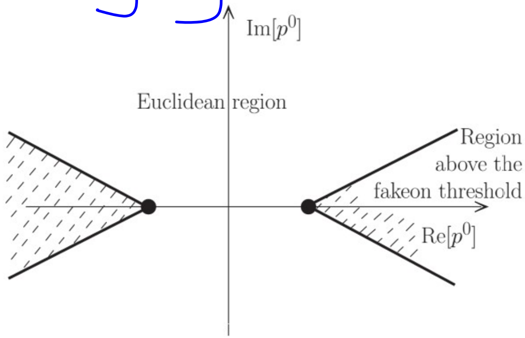


$\frac{1}{2} \ln z^2$

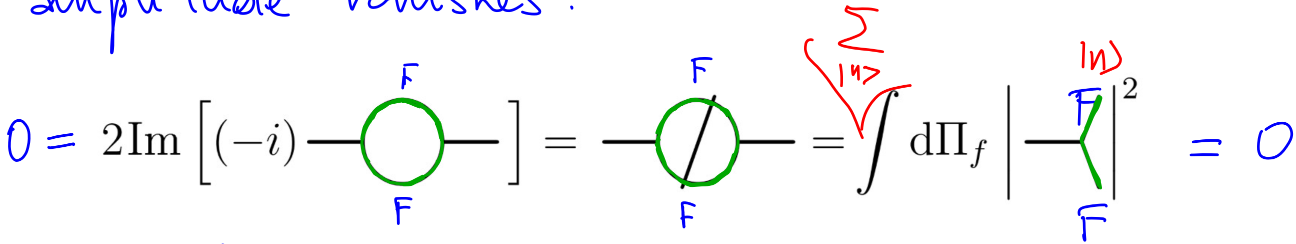
D. A. & M. Piva, J. High Energy Phys. 06 (2017) 066 and arXiv: 1703.04584 [hep-th]

Regionwise analyticity





Since the prescription is symmetric w.r.t. the real axis, the imaginary part of the amplitude vanishes:



⇒ the fakeon F MUST be projected away :

From

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \langle b|T^\dagger \overbrace{|n\rangle\langle n|}^{\substack{1,0\\ \cancel{\pm 1}}} T|a\rangle, \qquad |a\rangle, |b\rangle \in W.$$

to

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in V} \langle b|T^\dagger|n\rangle\langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in V$$

with $V \subset W$, $|F\rangle \in W$, $|F\rangle \notin V$

To all orders:

D. Anselmi (2018), Fakeons & Lee-Wick models, JHEP

Quantum gravity

Consider the (renormalizable) higher-derivative action

$$S_{\mathrm{QG}}=-\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C+\zeta R+\alpha\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)-\frac{\xi}{6}R^2\right]+S_{\mathfrak{m}}(g,\Phi)$$

$\zeta,\alpha,\xi>0$

Standard Model

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

Eliminate the higher derivatives by means of extra fields.

$$\mathcal{S}_{\rm QG}(g,\phi,\chi,\Phi)=S_{\rm H}(g)+S_{\chi}(g,\chi)+S_{\phi}(\tilde{g},\phi)+S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

where $\tilde{g}_{\mu\nu}=g_{\mu\nu}+2\chi_{\mu\nu}$

$$S_{\rm H}(g)=-\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R,\qquad S_{\phi}(g,\phi)=\frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_{\mu}\phi\nabla^{\mu}\phi-\frac{m_{\phi}^2}{\kappa^2}\left(1-\mathrm{e}^{\kappa\phi}\right)^2\right]$$

$$S_{\chi}(g,\chi)=S_{\rm H}(\tilde{g})-S_{\rm H}(g)-2\int\chi_{\mu\nu}\frac{\delta S_{\rm H}(\tilde{g})}{\delta g_{\mu\nu}}+\frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu}-\chi^2)\big|_{g\rightarrow\tilde{g}}\,.$$

D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renormalization.com and arXiv:1806.03605 [hep-th].

Now, the $\chi_{\mu\nu}$ action has the wrong overall sign:

$$S_\chi(g,\chi) = -\frac{\zeta}{\kappa^2} S_{\mathrm{PF}}(g,\chi,m_\chi^2) - \frac{\zeta}{2\kappa^2}\int \sqrt{-g} R^{\mu\nu}(\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho) + S_\chi^{(>2)}(g,\chi)$$

where S_{PF} is the covariantized Pauli-Fierz action

This means that $\chi_{\mu\nu}$ MUST be quantized as a fakeon. This way, we have both renormalizability and unitarity.

$$\pm \frac{1}{p^2 - m^2}$$

Instead, ϕ can be quantized either as a fakeon or as a physical particle

Graviton multiplet: $\{h_{\mu\nu}, \phi, \chi_{\mu\nu}\}$

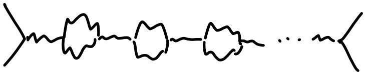
$$m_\chi = \frac{\zeta}{\alpha}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$$

fluctuation of the metric

massive scalar

spin-2 fakeon of mass m_χ



Fakeon width:

$$\Gamma_\chi = -\alpha_\chi C m_\chi \qquad C = \frac{N_s + 6N_f + 12N_v}{120}$$

$$\alpha_\chi = \left(\frac{m_\chi}{M_{Pl}}\right)^2$$

$\Gamma_\chi < 0$: causality is violated by $\chi_{\mu\nu}$

D. A. and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energy Phys. 05 (2018) 027 and arxiv:1803.0777 [hep-th]

JHEP 11 (2018) 21

D. A. and M. Piva, Quantum gravity, fakeons and microcausality, arxiv:1806.03605 [hep-th]

$$\frac{i}{E - m + i\frac{\Gamma}{2}} \longrightarrow \underbrace{\mathrm{sgn}(t)\,\theta(\Gamma t)\,\exp\left(-imt - \frac{\Gamma t}{2}\right)}_{\equiv\, G_{BW}(t)}$$

$$\varphi(x) = \int d^4y \; G_{BW}(x-y)\, J(y)$$

$\Gamma < 0$:

$$\varphi(t) = -\int_t^\infty dt' \, e^{-im(t-t') - \frac{\Gamma}{2}(t-t')} J(t')$$

	Fermions			Bosons	
Quarks	u	c	t	γ	H
	d	s	b	$W^{\pm}$	g
Leptons	e	μ	τ	Z^{0}	ϕ
	ν_e	ν_μ	ν_τ	g	χ

QG triplet

ϕ: fakeon ?

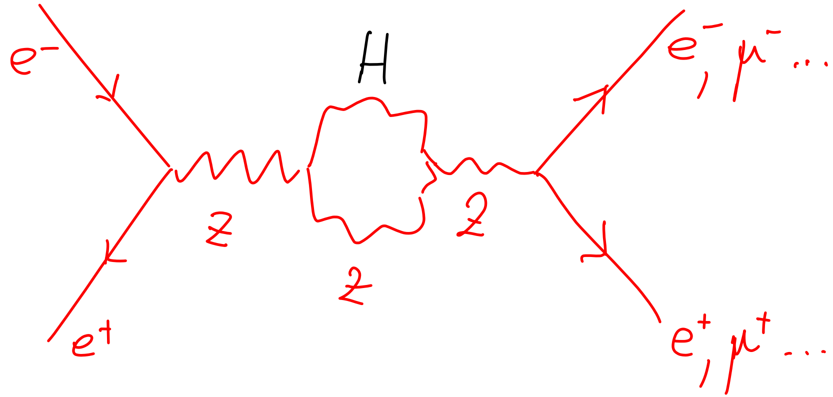
χ: fakeon

The fields of the standard model and quantum gravity could explain everything we know

The fakeon prescription can be applied universally

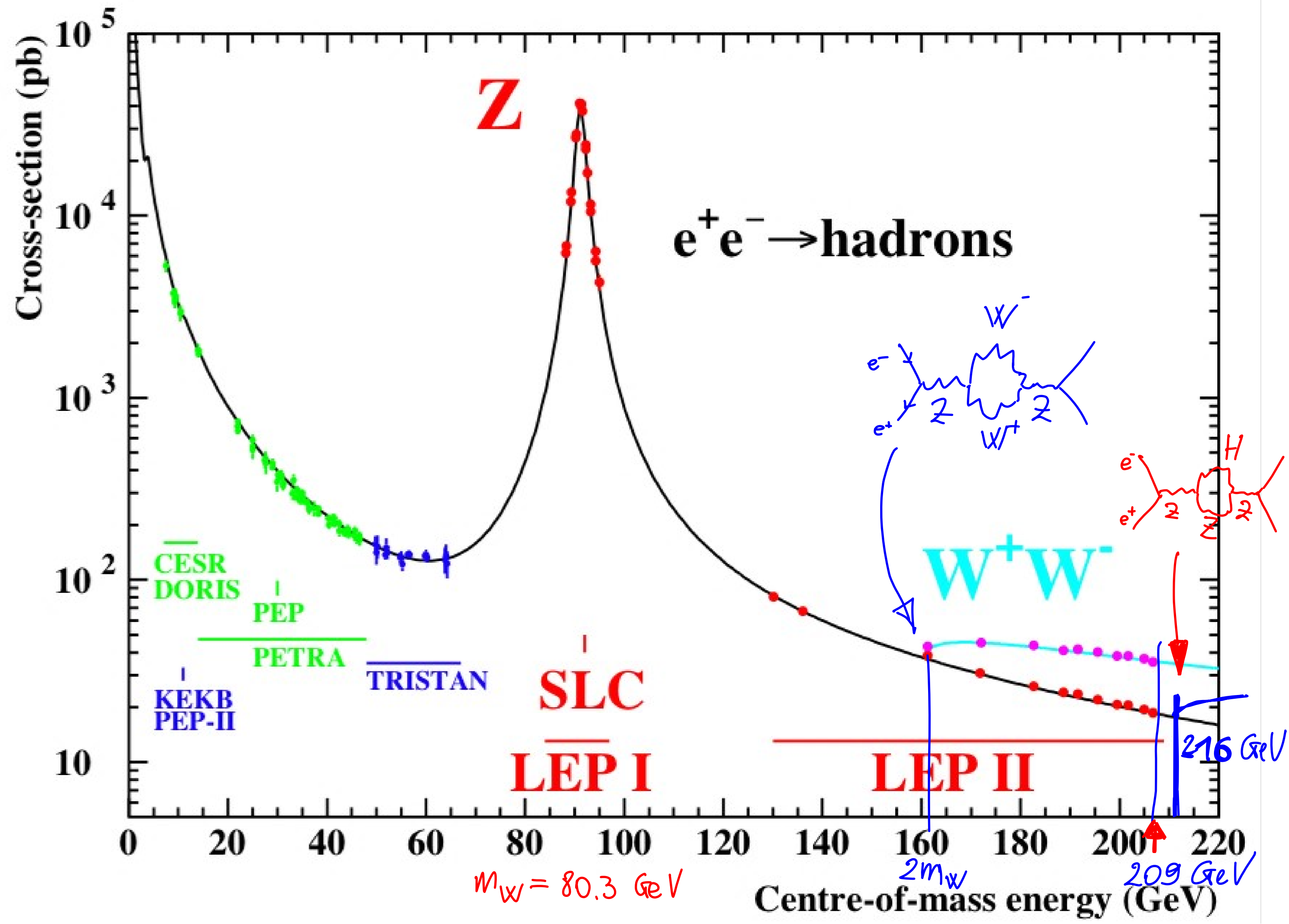
We should talk about "quantum field theory of particles and fakeons".

Is H a fakeon? Maybe...



$$m_H + m_Z = 216 \text{ GeV}$$

D. A., On the nature of the Higgs boson, MPLA 33 (2019) 1950123
arXiv: 1811.02600 [hep-ph]



Cross-section (pb)
10^5
10^4
10^3
10^2
10
Z
$e^+e^- \rightarrow$ hadrons
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
W^+W^-
0
20
40
60
80
100
120
140
160
180
200
220
Centre-of-mass energy (GeV)
e^-
e^+
Z
W^-
W^+
Z
H
e^-
e^+
Z
Z
Z
$2m_W$
$m_W = 80.3$ GeV
209 GeV
216 GeV

PROJECTION

$$Z_{\mathrm{pr}}(J) = \int [\mathrm{d}\varphi \mathrm{d}\chi] \exp\left(iS(\varphi, \chi) + i \int J\varphi \right) = \exp\left(iW_{\mathrm{pr}}(J)\right)$$

NO SOURCE J_χ FOR χ

Projection = integrating out the fakeons $\boxed{\text{with the fakeon prescription}}$ for the diagrams

At the level of generating functionals: $\Gamma(\varphi, \chi)$

φ = physical fields $\qquad$ χ = fakeons

Solve $\delta\Gamma(\varphi, \chi)/\delta\chi = 0$ by means of the fakeon prescription

Let $\langle\chi\rangle$ denote the solution

Projected functionals: $\Gamma_{\mathrm{pr}}(\varphi) = \Gamma(\varphi, \langle\chi\rangle)$

Classical limit : $\underline{\text{Set of tree diagrams with}}$
$\underline{\text{NO fakeon external legs}}$

The starting action

$$S_{\rm QG}(g,\phi,\chi,\Phi) = S_{\rm H}(g) + S_{\chi}(g,\chi) + S_{\phi}(\tilde{g},\phi) + S_{\rm m}(\tilde{g}e^{\kappa\phi},\Phi)$$

is NOT the classical limit, because it is unprojected

INTERIM classical action : LOCAL

E.g. : $$\mathcal{L}_{gf} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\lambda}(\partial\cdot A)^2 + \bar{C}\,\partial D C$$

gauge-fixed Lagrangian : unprojected, but <u>LOCAL</u>

Unprojected field equations :

$g_{\mu\nu}$: $$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{\kappa^2}{\zeta} \left[\mathrm{e}^{3\kappa\phi} f T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) + f T_{\phi}^{\mu\nu}(\tilde{g}, \phi) + T_{\chi}^{\mu\nu}(g, \chi) \right]$$

ϕ : $$-\frac{1}{\sqrt{-\tilde{g}}} \partial_\mu \left(\sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi \right) - \frac{m_\phi^2}{\kappa} \left(\mathrm{e}^{\kappa\phi} - 1 \right) \mathrm{e}^{\kappa\phi} = \frac{\kappa \mathrm{e}^{3\kappa\phi}}{3\zeta} T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) \tilde{g}_{\mu\nu},$$

$\chi_{\mu\nu}$: $$\frac{1}{\sqrt{-g}} \frac{\delta S_\chi(g, \chi)}{\delta \chi_{\mu\nu}} = \mathrm{e}^{3\kappa\phi} f T_{\mathfrak{m}}^{\mu\nu}(\tilde{g}\mathrm{e}^{\kappa\phi}, \Phi) + f T_{\phi}^{\mu\nu}(\tilde{g}, \phi),$$

Projection : solve the χ field equation (with the fakeon prescription) and insert the solution into the other equations

At the tree level, the subtleties about integration paths and average continuations are not important, so we can take

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P} \frac{1}{p^2 - m^2}$$

Example : $\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) + x F_{\mathrm{ext}}(t)$

$$m \frac{d^2}{dt^2} \underbrace{\left(1 + \tau^2 \frac{d^2}{dt^2}\right)} x = F_{ext}$$

$\longrightarrow$ invert with $\mathcal{P} \dfrac{1}{1+\tau^2 \frac{d^2}{dt^2}}$

The projected equation is

$$m\ddot{x} = \int_{-\infty}^{\infty} \mathrm{d}u \, \frac{\sin(|u|/\tau)}{2\tau} F_{\mathrm{ext}}(t-u)$$

$\tau \sim \frac{1}{m_\chi}$

$\longrightarrow$ violation of microcausality

$\cancel{F = ma}$ $\qquad \langle F \rangle = ma$!!

$|u| \sim \tau$

But in general the projection is defined perturbatively (since it comes from quantum gravity, which is defined perturbatively)

→ The classical equations are defined perturbatively: one may have to face asymptotic series and nonperturbative effects (just to write the equations)

Example:

$$\mathcal{L}_{\rm HD} = \frac{m}{2}(\dot{x}^2 - \tau^2\ddot{x}^2) - V(x,t) \qquad V = \frac{\lambda}{4!}x^4$$

Unprojected equations of motion:

$$m\frac{d^2}{dt^2}\left(1+\tau^2\frac{d^2}{dt^2}\right)x = -\frac{\lambda}{3!}x^3$$

Projected equations of motion:

$$m\ddot{x} = -\frac{\lambda}{3!}\langle x^3\rangle$$

$\langle\dots\rangle$ = same average as before

By iterating the projection, we get

$\omega = 0 \quad \tilde{\lambda} \equiv \lambda/m$

$$\ddot{x} = -\frac{\tilde{\lambda} x}{6}\left(x^2 - 6\tau^2\dot{x}^2\right) - \frac{\tilde{\lambda}^2\tau^2 x}{12}\left(x^4 - 48\tau^2x^2\dot{x}^2 + 372\tau^4\dot{x}^4\right)$$
$$-\frac{\tilde{\lambda}^3\tau^4 x}{6}\left(x^6 - 156\tau^2x^4\dot{x}^2 + 4572\tau^4x^2\dot{x}^4 - 31152\tau^6\dot{x}^6\right) + \mathcal{O}(\tilde{\lambda}^4)$$

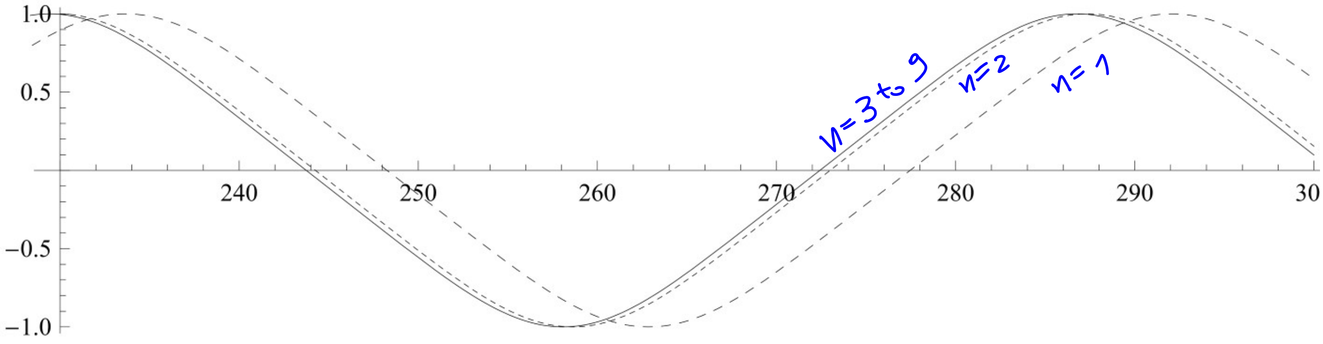
$$\frac{\mathcal{L}}{m} = \frac{\dot{x}^2}{2} - \frac{\tilde{\lambda}x^2}{4!}\left(x^2 + 12\tau^2\dot{x}^2\right) + \frac{\tau^2\tilde{\lambda}^2x^2}{72}\left(x^4 - 54\tau^2x^2\dot{x}^2 + 372\tau^4\dot{x}^4\right) + \mathcal{O}(\tilde{\lambda}^3)$$

Growth of the coefficients:

n	5	10	15	20	25
$c_{n,0}$	10^0	10^6	10^{13}	10^{22}	10^{32}
$c_{n,n}$	10^9	10^{28}	10^{52}	10^{78}	10^{107}

!!

Solution with $x(0) = 1$, $\dot{x}(0) = 0$, $m = \tau = 1$, $\lambda = \frac{1}{10}$



$n=1$: fairly good

$n=2$: good

$n=3$ to $n=9$: excellent

$n>9$: meaningless

't Hooft - Veltman, Diagrammar, CERN 73-09, § 6.1

causality

Every diagram, when multiplied by the appropriate source functions and integrated over all x contributes to the S-matrix. The contribution to the T-matrix, defined by

$$S = 1 + iT \tag{6.7}$$

is obtained by multiplying by a factor -i. Unitarity of the S-matrix implies an equation for the imaginary part of the so defined T matrix

$$T - T^{\dagger} = iT^{\dagger}T \,. \tag{6.8}$$

The T-matrix, or rather the diagrams, are also constrained by the requirement of causality. As yet nobody has found a definition of causality that corresponds directly to the intuitive notions; instead formulations have been proposed involving the off-mass-shell Green's functions. We will employ the causality requirement in the form proposed by Bogoliubov that has at least some intuitive appeal and is most suitable in connection with a diagrammatic analysis. Roughly speaking Bogoliubov's condition can be put as follows: if a space-time point x_1 is in the future with respect to some other space-time point x_2, then the diagrams involving x_1 and x_2 can be rewritten in terms of functions that involve positive energy flow from x_2 to x_1 only.

The trouble with this definition is that space-time points cannot be accurately pin-pointed with relativistic wave packets corresponding to particles on mass-shell. Therefore this definition cannot be formulated as an S-matrix constraint. It can only be used for the Green's functions.

Other definitions refer to the properties of the fields. In particular there is the proposal of Lehmann, Symanzik and Zimmermann that the fields commute outside the light cone. Defining fields in terms of diagrams, this definition can be shown to reduce to Bogoliubov's definition. The formulation of Bogoliubov causality in terms of cutting rules for diagrams will be given in Section 6.4.

Comments on alternative approaches to the problem of quantum gravity

--- **string theory** is criticized for being nonpredictive. Moreover, its calculations often require mathematics that is not completely understood

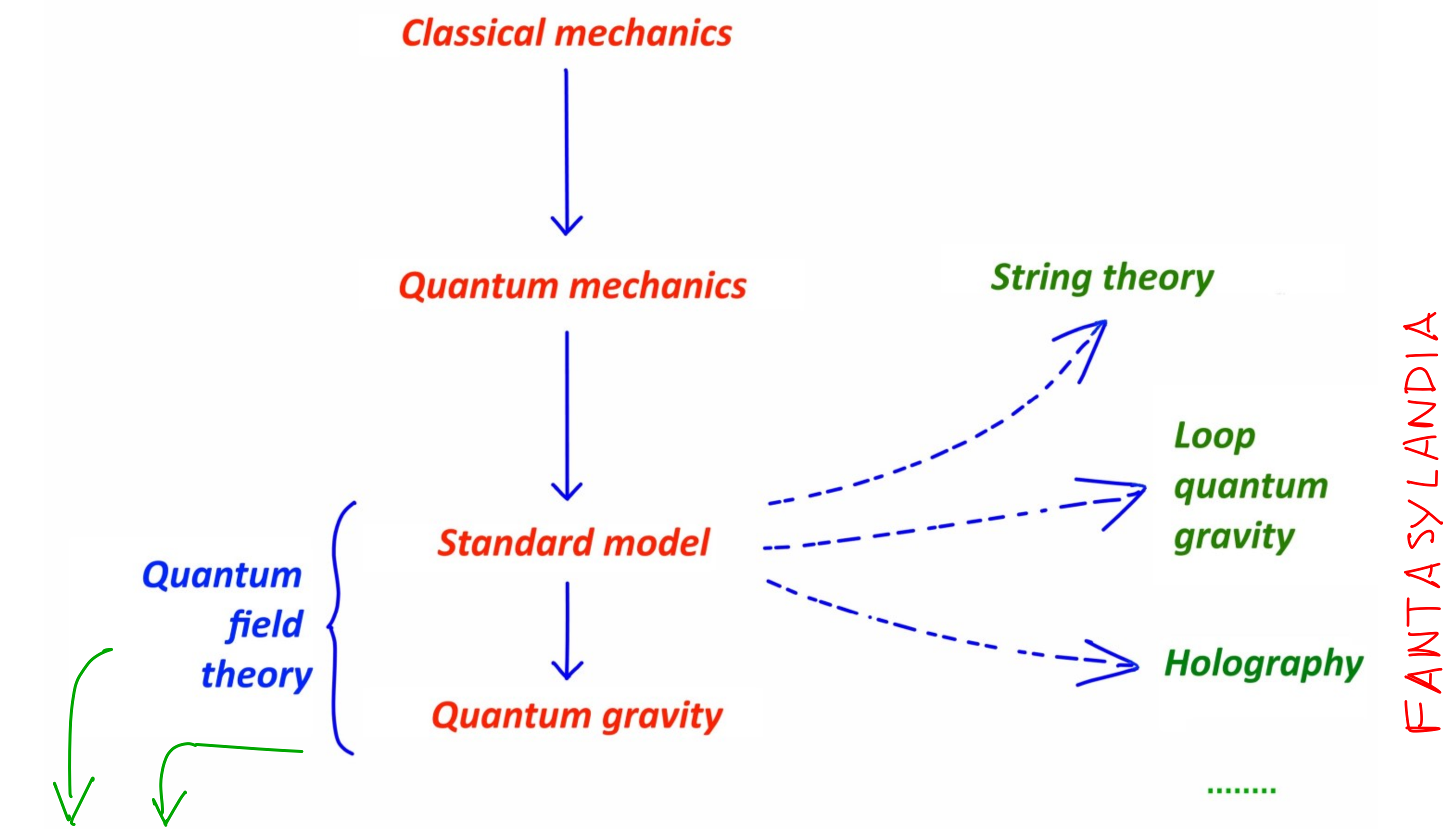
--- **loop quantum gravity** is even more challenging, because it is at an earlier stage of development

--- **holography** (AdS/CFT correspondence) do not admit a weakly coupled expansion

--- **asymptotic safety** in nonperturbative as well.

None is as close to the standard model as the solution based on the fakeon idea, which is a quantum field theory, admits a perturbative expansion in terms of Feynman diagrams and allows us to make calculations with a comparable effort. It can be coupled to th standard model straightforwardly.

Our solution bests its competitors in calculatibity, predictivity and falsifiability. It is also rather rigid, because it contains only two new parameters. It could turn out to be the most predictive theory ever.



QG is the "last mile" of the path to the infinitesimally small, right after the SM