Fakeons, quantum gravity and the classical limit

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- How the idea of fake particle ("fakeon") leads to what I claim to be the right theory of quantum gravity
- Physical implications and predictions
- Classical limit
- FLRW solution
- Scale invariance? Wait for it ...

Unitarity: $$S^\dagger S = 1$$

$S = 1 + iT$

$$2\mathrm{Im}T = T^\dagger T$$

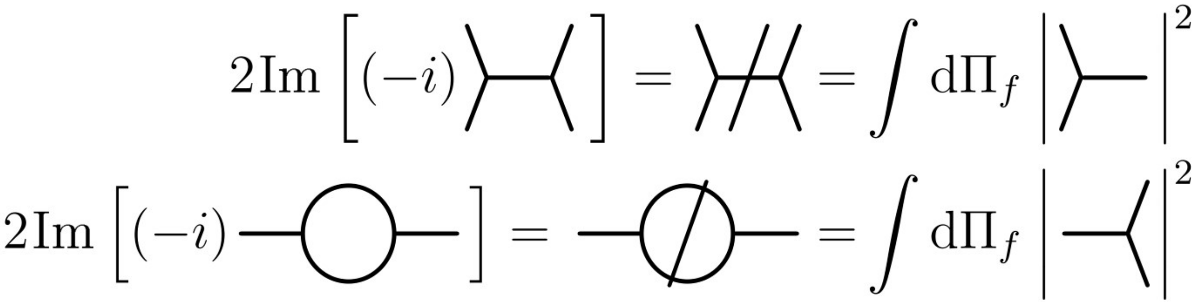
Optical theorem

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \langle b|T^\dagger|n\rangle \underbrace{}^{(-1)^{\sigma_n}} \langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in W$$

$\sigma_n = 0, 1$

In general, W may contain unphysical states

Diagrammatic version of the optical theorem :



cutting equations

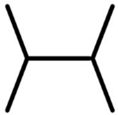
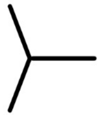
Propagator : $G(p,m) = \dfrac{1}{p^2 - m^2}$

A prescription is needed

The Feynman prescription gives

$$G_+(p,m,\epsilon) = \frac{1}{p^2 - m^2 + i\epsilon}$$

The optical theorem is ok:

$2\,\mathrm{Im}\left[(-i)\right.$  $\left.\right] =$  $= \int d\Pi_f \left|\right.$  $\left.\right|^2 \geq 0$

Indeed:

$$\mathrm{Im}\left[-\frac{1}{p^2-m^2+i\epsilon}\right] = \pi\delta(p^2-m^2) \qquad \geq 0$$

Ghost: opposite residue $\quad -\dfrac{1}{p^2-m^2+i\epsilon}$

The optical theorem is violated

Note that the prescription is crucial

If we flip the sign of $i\epsilon$ and the overall sign, the optical theorem is ok:

$$\mathrm{Im}\left[\frac{1}{p^2 - m^2 - i\epsilon}\right] = \pi\delta(p^2 - m^2) \geq 0$$

Problem:

$$G_\pm(p, m, \epsilon) = \pm\frac{1}{p^2 - m^2 \pm i\epsilon}$$

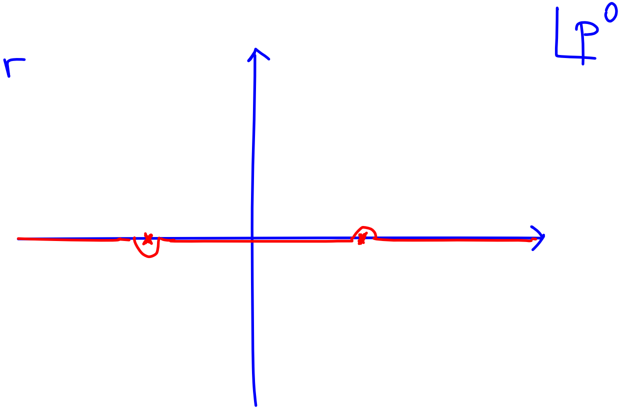
cannot coexist

[Bad divergent behaviors]   So?

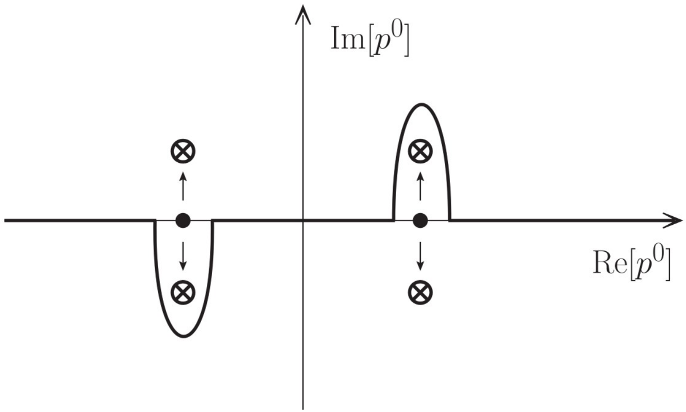
U.G. Aglietti and D. Anselmi, Inconsistency of Minkowski higher-derivative theories, Eur. Phys. J. C 77 (2017) 84, 16A2 Renormalization.com and arXiv:1612.06510 [hep-th]

The fakeon is the answer

$$G(p,m)=\frac{1}{p^2-m^2}$$



Write $\pm\dfrac{p^2-m^2}{(p^2-m^2)^2}$ and split the poles into pairs



This is achieved by inserting an infinitesimal width as follows :

$$\mathbb{G}_{\pm}(p, m, \mathcal{E}^2) = \pm \frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \pm \frac{1}{2} \left[G_+(p, m, \mathcal{E}^2) - G_-(p, m, \mathcal{E}^2) \right]$$

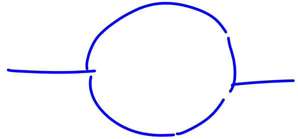
Note that the residue is zero on shell :

$\Rightarrow$ NO PARTICLE

BUT : what about the bad divergences ?

Those occur in Minkowski space

which is not what we are doing here

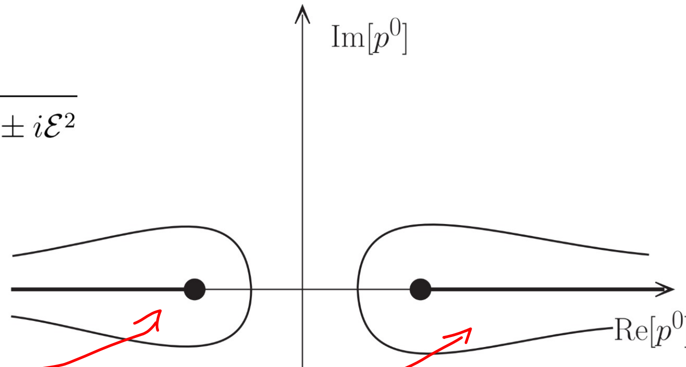
Example: bubble diagram 

$$i\mathcal{M} \propto \int \frac{dk^0}{2\pi} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \mathbb{G}_\pm(p-k, m_1, \mathcal{E}^2)\,\mathbb{G}_\pm(k, m_2, \mathcal{E}^2)$$

$$= \int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p, \mathbf{k})$$

$w(p, \vec{k})$ is singular for

$$|p^0| = \sqrt{(\mathbf{p}-\mathbf{k})^2 + m_1^2 \pm i\mathcal{E}^2} + \sqrt{\mathbf{k}^2 + m_2^2 \pm i\mathcal{E}^2}$$



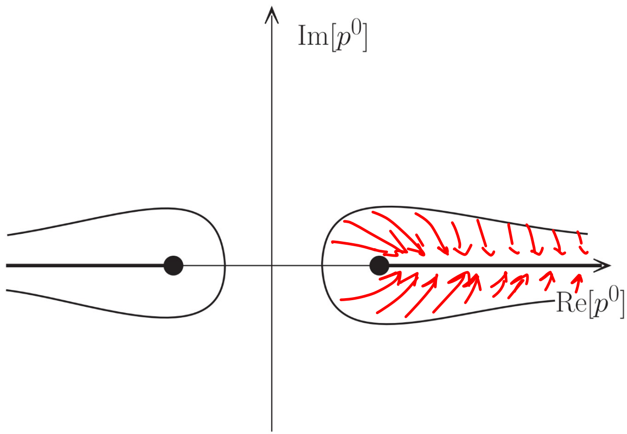
Here Lorentz invariance & analiticity are violated

The integration domain on the loop space momentum must be deformed

$$\int_{\mathbb{R}^3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p, \mathbf{k})$$

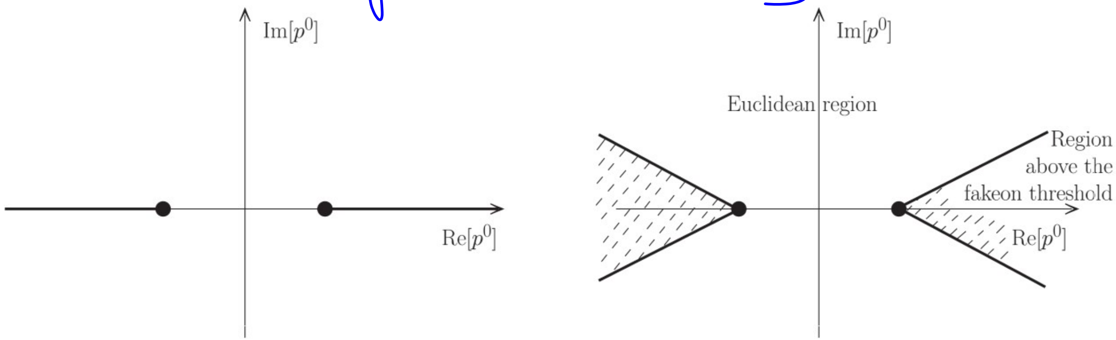
$$\downarrow$$

$$\int_{\mathcal{D}_3} \frac{d^3\mathbf{k}}{(2\pi)^3} w(p, \mathbf{k})$$

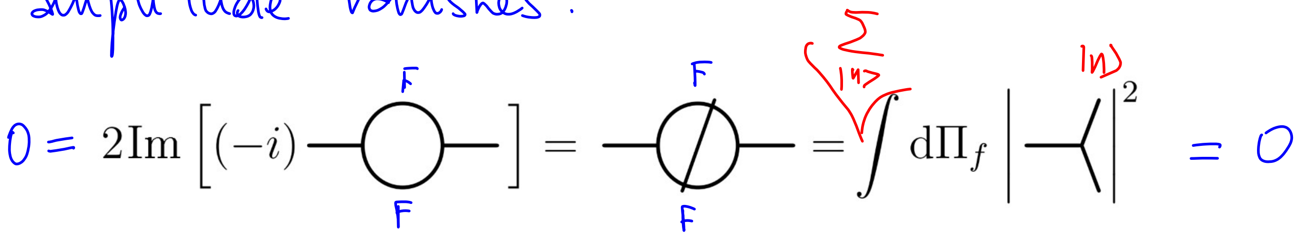


Lorentz invariance and analitycity are recovered in the limit where the areas are shrinked to branch cuts

Regionwise analyticity



Since the prescription is symmetric w.r.t. the real axis, the imaginary part of the amplitude vanishes:



$\Rightarrow$ the fakeon F MUST be projected away :

From

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in W} \langle b|T^\dagger|n\rangle \, \overset{1,0}{\cancel{(-1)^{\sigma_n}}} \, \langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in W$$

to

$$\frac{1}{2i}\left[\langle b|T|a\rangle - \langle b|T^\dagger|a\rangle\right] = \sum_{|n\rangle\in V} \langle b|T^\dagger|n\rangle\langle n|T|a\rangle, \qquad |a\rangle, |b\rangle \in V$$

with $V \subset W$, $|F\rangle \in W$, $|F\rangle \notin V$

$$\ln(q^2 - m^2 + i\varepsilon) \rightarrow \frac{1}{2}\ln(q^2-m^2)^2$$

To all orders:
D. Anselmi (2018), Fakeons & Lee-Wick models, JHEP

Quantum gravity

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, 17A3 Renormalization.com and arXiv: 1704.07728 [hep-th].

Consider the (renormalizable) higher-derivative action

$$S_{\text{QG}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right] + S_{\mathfrak{m}}(g,\Phi)$$

Eliminate the higher derivatives by means of extra fields:

D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, 18A3 Renormalization.com and arXiv:1806.03605 [hep-th].

$$\mathcal{S}_{\text{QG}}(g,\phi,\chi,\Phi) = S_{\text{H}}(g) + S_\chi(g,\chi) + S_\phi(\tilde{g},\phi) + S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

where $\tilde{g}_{\mu\nu} = g_{\mu\nu} + 2\chi_{\mu\nu}$

$$S_{\text{H}}(g) = -\frac{\zeta}{2\kappa^2}\int\sqrt{-g}R, \qquad S_\phi(g,\phi) = \frac{3\zeta}{4}\int\sqrt{-g}\left[\nabla_\mu\phi\nabla^\mu\phi - \frac{m_\phi^2}{\kappa^2}\left(1-\mathrm{e}^{\kappa\phi}\right)^2\right]$$

$$S_\chi(g,\chi) = S_{\text{H}}(\tilde{g}) - S_{\text{H}}(g) - 2\int\chi_{\mu\nu}\frac{\delta S_{\text{H}}(\tilde{g})}{\delta g_{\mu\nu}} + \frac{\zeta^2}{2\alpha\kappa^2}\int\sqrt{-g}(\chi_{\mu\nu}\chi^{\mu\nu} - \chi^2)\Big|_{g\to\tilde{g}}\,.$$

Now, the $\chi_{\mu\nu}$ action has the wrong overall sign:

$$S_\chi(g,\chi) = -\frac{\zeta}{\kappa^2} S_{\mathrm{PF}}(g,\chi,m_\chi^2) - \frac{\zeta}{2\kappa^2}\int \sqrt{-g} R^{\mu\nu}(\chi\chi_{\mu\nu} - 2\chi_{\mu\rho}\chi_\nu^\rho) + S_\chi^{(>2)}(g,\chi)$$

where S_{PF} is the covariantized Pauli-Fierz action

This means that $\chi_{\mu\nu}$ MUST be quantized as a fakeon. This way, we have both renormalizability and unitarity.

Instead, ϕ can be quantized either as a fakeon or as a physical particle

Classical limit

The action

$$\mathcal{S}_{\rm QG}(g,\phi,\chi,\Phi) = S_{\rm H}(g) + S_\chi(g,\chi) + S_\phi(\tilde{g},\phi) + S_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)$$

is NOT the classical limit, because it is unprojected

Unprojected field equations :

$g_{\mu\nu}$: $$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \frac{\kappa^2}{\zeta}\left[\mathrm{e}^{3\kappa\phi} f T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + f T^{\mu\nu}_\phi(\tilde{g},\phi) + T^{\mu\nu}_\chi(g,\chi)\right]$$

ϕ : $$-\frac{1}{\sqrt{-\tilde{g}}}\partial_\mu\left(\sqrt{-\tilde{g}}\tilde{g}^{\mu\nu}\partial_\nu\phi\right) - \frac{m_\phi^2}{\kappa}\left(\mathrm{e}^{\kappa\phi}-1\right)\mathrm{e}^{\kappa\phi} = \frac{\kappa\mathrm{e}^{3\kappa\phi}}{3\zeta}T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi)\tilde{g}_{\mu\nu},$$

$\chi_{\mu\nu}$: $$\frac{1}{\sqrt{-g}}\frac{\delta S_\chi(g,\chi)}{\delta\chi_{\mu\nu}} = \mathrm{e}^{3\kappa\phi} f T^{\mu\nu}_{\mathfrak{m}}(\tilde{g}\mathrm{e}^{\kappa\phi},\Phi) + f T^{\mu\nu}_\phi(\tilde{g},\phi),$$

At the tree level, the subtleties about integration paths and integration domains are not important, so we can take

$$\frac{p^2 - m^2}{(p^2 - m^2)^2 + \mathcal{E}^4} = \mathcal{P}\frac{1}{p^2 - m^2}$$

Use it to solve the fakeon equations

The projected classical action is then

$$S_{\mathrm{QG}}\ (g, \phi, \Phi) = S_{\mathrm{H}}(g) + S_{\chi}(g, \langle\chi\rangle) + S_{\phi}(\bar{g}, \phi) + S_{\mathrm{m}}(\bar{g}\mathrm{e}^{\kappa\phi}, \Phi)$$

where $\langle \chi \rangle$ is the solution $\quad \bar{g}_{\mu\nu} = g_{\mu\nu} + 2\langle\chi_{\mu\nu}\rangle$

Example :

$$\mathcal{L}_{\mathrm{HD}} = \frac{m}{2}(\dot{x}^2 - \tau^2 \ddot{x}^2) + x F_{\mathrm{ext}}(t)$$

$$m \frac{d^2}{dt^2}\left(1 + \tau^2 \frac{d^2}{dt^2}\right) x = F_{ext}$$

The projected equation is

$$m\ddot{x} = \int_{-\infty}^{\infty} \mathrm{d}u \, \frac{\sin(|u|/\tau)}{2\tau} F_{\mathrm{ext}}(t-u)$$

$\cancel{F = ma}$ $\qquad \langle F \rangle = ma$!!

The FLRW metric

Unprojected equations $\left(\frac{\mathcal{L}}{\sqrt{-g}} \sim R + R^2 \right)$

$$\Sigma\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)=\frac{4\pi G}{3}(\rho-3p), \qquad \Upsilon\left(\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}-\frac{k}{a^2}\right)=-4\pi G(\rho+p),$$

$$\Sigma(1-\mathrm{e}^{-\kappa\phi})=-\frac{8\pi G}{3m_\phi^2}(\rho-3p)$$

where

$$\Sigma=1+\frac{1}{m_\phi^2}\left(3\frac{\dot{a}}{a}+\frac{\mathrm{d}}{\mathrm{d}t}\right)\frac{\mathrm{d}}{\mathrm{d}t}, \qquad \Upsilon=\Sigma+\frac{2}{m_\phi^2}\left[\frac{k}{a^2}+3\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\dot{a}}{a}\right)\right].$$

Projection. Define $\langle A\rangle_X \equiv \frac{1}{2}\left[\left.\frac{1}{X}\right|_{\mathrm{rit}}+\left.\frac{1}{X}\right|_{\mathrm{adv}}\right]A$

and use it to define $\frac{1}{\Sigma}$ and $\frac{1}{\Upsilon}$

$$\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}=\frac{4\pi G}{3}\langle\rho-3p\rangle_\Sigma$$

$$\frac{\ddot{a}}{a}-\frac{\dot{a}^2}{a^2}-\frac{k}{a^2}=-4\pi G\langle\rho+p\rangle_\Upsilon$$

it is NOT over yet ....!!

$$1-\mathrm{e}^{-\kappa\phi}=-\frac{8\pi G}{3m_\phi^2}\langle\rho-3p\rangle_\Sigma$$

The projection can be handled exactly for radiation combined with the vacuum energy:

$$p=\frac{\rho}{3}+p_0$$

p_0 = constant

Conclusions

Quantum field theory of particles and fakeons offers new opportunities to high-energy physics and quantum gravity

Fakeons cannot be observed directly. Their presence can however be detected by means of precision measurements, starting from those that probe the contributions coming from the imaginary parts of loop diagrams, above the fakeon thresholds

Higgs?

Fakeons allow us to quantize gravity and lead to the violation of causality at energies larger than their masses

The classical action is just an interim, local action that must be projected to obtain the true classical limit.

The classicization is perturbative: asymptotic series

Final comment : scale invariance ?

Or radial irreversibility ?

Well ..

large distances

Determinism

Quantum uncertainty

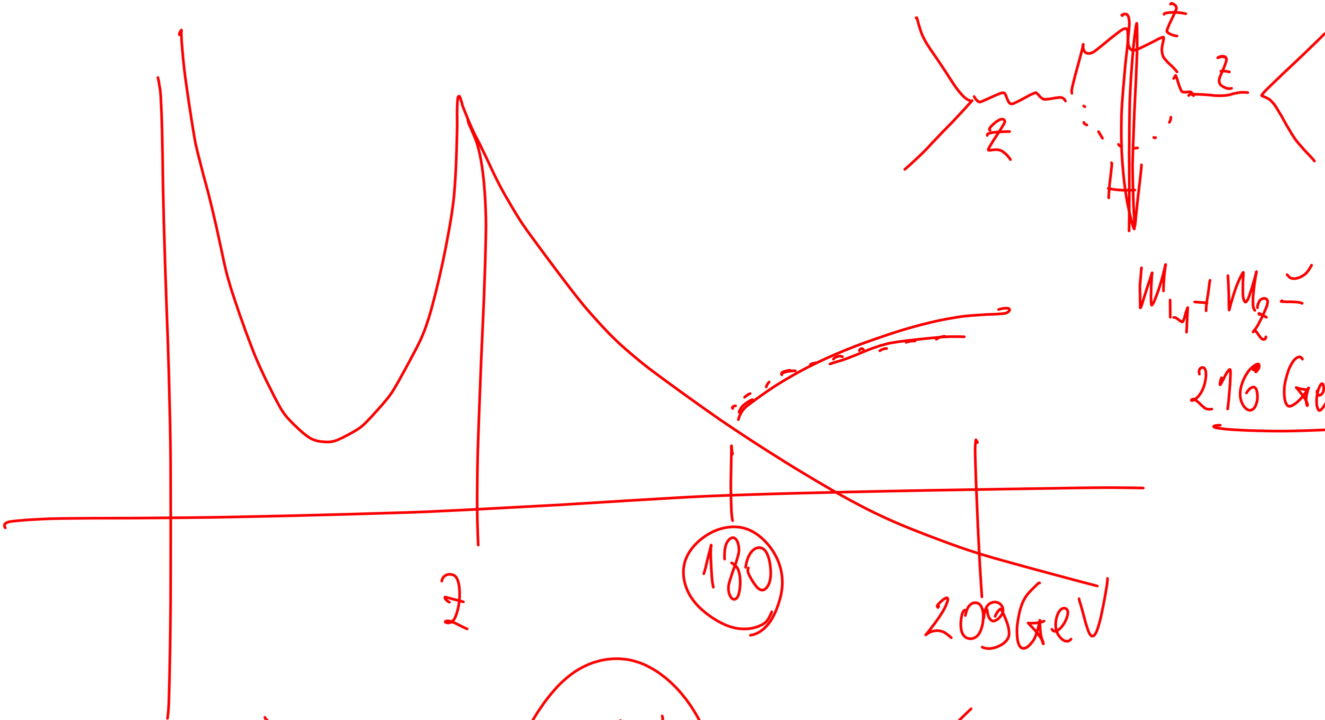
Violation of causality

small distances

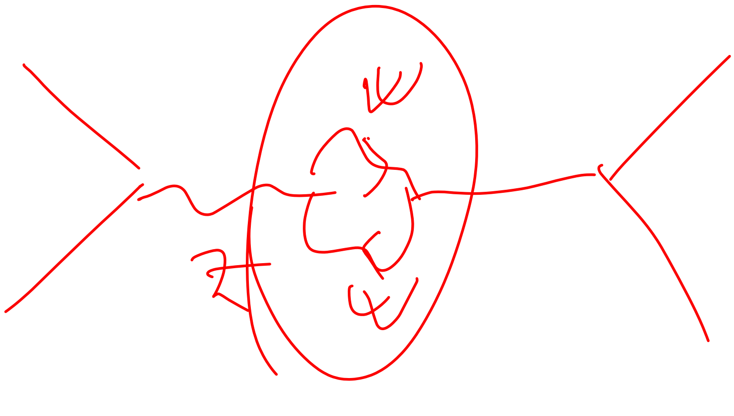
It appears that there is a pattern:

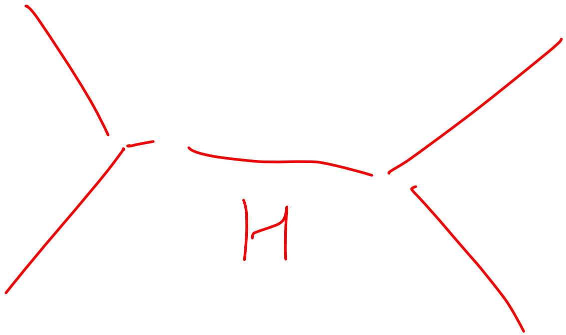
Chains up here, freedom (anarchy?) down there

The universe is **... un-inside-out-able**



$m_1 + m_2 \simeq 216$ GeV

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