

# Weighted Power Counting And Chiral Dimensional Regularization

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## Abstract

We define a modified dimensional-regularization technique that overcomes several difficulties of the ordinary technique, and is specially designed to work efficiently in chiral and parity violating quantum field theories, in arbitrary dimensions greater than 2. When the dimension of spacetime is continued to complex values, spinors, vectors and tensors keep the components they have in the physical dimension, therefore the  $\gamma$  matrices are the standard ones. Propagators are regularized with the help of evanescent higher-derivative kinetic terms, which are of the Majorana type in the case of chiral fermions. If the new terms are organized in a clever way, weighted power counting provides an efficient control on the renormalization of the theory, and allows us to show that the resulting *chiral dimensional regularization* is consistent to all orders. The new technique considerably simplifies the proofs of properties that hold to all orders, and makes them suitable to be generalized to wider classes of models. Typical examples are the renormalizability of chiral gauge theories and the Adler-Bardeen theorem. The difficulty of explicit computations, on the other hand, may increase.

## 1 Introduction

The dimensional regularization [1, 2] is very convenient to make calculations in quantum field theory and prove properties to all orders, such as renormalizability, when gauge anomalies are manifestly absent. It has some annoying features in chiral, parity violating and supersymmetric theories, where the matrix  $\gamma_5$  and the tensor  $\varepsilon_{\mu\nu\rho\sigma}$  play important roles. There, manifest gauge invariance is lost, and calculations are involved. More importantly, the proofs of all-order properties are quite demanding, because several common arguments do not work and some classifications of counterterms are ambiguous.

According to the standard definition of dimensional regularization for chiral theories in four dimensions [1, 3], calling  $D = 4 - \varepsilon$  the continued, complex dimension of spacetime, the  $D$ -dimensional spacetime manifold  $\mathbb{R}^D$  is broken into the product  $\mathbb{R}^4 \times \mathbb{R}^{-\varepsilon}$  of the physical spacetime manifold  $\mathbb{R}^4$  times an “evanescent” space,  $\mathbb{R}^{-\varepsilon}$ . The  $D$ -dimensional  $\gamma$  matrices are formal objects that satisfy the continued Dirac algebra  $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} = 2\text{diag}(1, -1, \dots, -1)$ . Then, the matrix  $\gamma_5$  is formally defined as the product  $i\gamma^0\gamma^1\gamma^2\gamma^3$  [1]. This approach, due to 't Hooft and Veltman, is consistent to all orders, gives the right one-loop anomalies and can be easily generalized to arbitrary even dimensions. Although it is widely used, it has some undesirable features. The most important one, in our viewpoint, is that it is responsible for various obstacles that obstruct the proofs of theorems to all orders in perturbation theory. Many alternative approaches have been proposed in the literature (see ref. [4] for a nonexhaustive list of recent contributions to this subject), mainly focused on the definition of  $\gamma_5$ . However, most of those proposals do not simplify the proofs of all-order theorems.

The true origin of the difficulties we are concerned with is not  $\gamma_5$  *per se* but the continued Dirac algebra. In this paper we show that it is possible to avoid most inconveniences by working with the usual  $d$ -dimensional  $\gamma$  matrices in arbitrary  $D$  dimensions, where  $d$  denotes the dimension of physical spacetime. To achieve this and other goals, it is necessary to upgrade the whole dimensional regularization to a new technique, which we call *chiral dimensional (CD) regularization*.

The first problem we must face is the regularization of chiral fermions. The naïve  $D$ -dimensional continuation of their action is gauge invariant, but does not provide good propagators. In the CD regularization this problem is solved by adding evanescent kinetic terms of the Majorana type. Since no evanescent  $\gamma$  matrices are allowed, the evanescent kinetic terms must be *higher derivative*, which means that they are multiplied by parameters of negative dimensions in units of mass. In general, those parameters may propagate into the physical sector of the theory, turn nonrenormalizable vertices on and cause all other sorts of troubles. To keep the evanescent sector under control and prove that the CD regularization is consistent to all orders, we arrange the regularization technique so that it satisfies the requirements of weighted power counting [5]. Doing

so, we obtain an effective control over the locality of counterterms and the renormalization to all orders.

The propagators of all fields must be corrected similarly, to make their structure match the structure of fermionic propagators. Since gauge invariance is not (and does not need to be) preserved away from  $d$  dimensions, we show that it is consistent to require that all fields have strictly  $d$ -dimensional components. We use this property to simplify the regularization technique as much as we can.

Summarizing, in the CD regularization all fields (scalars, spinors, vectors, tensors) have exactly the same components in  $D$  dimensions, as they have in  $d$  dimensions, and the  $\gamma$  matrices, as well as the  $\varepsilon_{\mu_1 \dots \mu_d}$  tensors, are just the usual  $d$ -dimensional ones. Evanescent terms are added, guided by weighted power counting, to make propagators fall off with appropriate velocities in all directions of integration.

The CD regularization is particularly convenient to treat general gauge theories that cannot be regularized preserving their gauge symmetries in a manifest way. Examples are chiral theories and parity violating theories, such as the standard model, Lorentz violating extensions of the standard model [6, 7, 8], Chern-Simons theories in three dimensions, and so on. In the realm of nonrenormalizable theories we mention the standard model coupled with quantum gravity, as well as supergravity.

The CD regularization keeps the good properties of the dimensional regularization, among which the fact that local perturbative changes of field variables have Jacobian determinants identically equal to one, which simplifies the Batalin-Vilkovisky master equation [9] and several derivations. At the same time, it overcomes known and less-known inconveniences of the usual dimensional regularization, such as problems due to the dimensional continuation of Fierz identities and the propagation of evanescences through the Batalin-Vilkovisky antiparentheses [9]. Ultimately, the new technique provides a powerful tool to make systematic proofs to all orders with a relatively small effort in perturbative quantum field theory, in arbitrary dimensions  $d > 2$ .

One situation where the problems of the common dimensional regularization can be appreciated is the proof of the Adler-Bardeen theorem [10, 11], which is a realm where the most advanced techniques of perturbative quantum field theory must be used altogether. The original proof given by Adler and Bardeen [10] works only in QED. Most generalizations to non-Abelian gauge theories use arguments based on the renormalization group [12, 13, 14, 15]. However, those arguments have some limitations, because they do not apply to conformal field theories, finite theories or theories where the first coefficients of the beta functions vanish [15]. Algebraic/geometric derivations [16] based on the Wess-Zumino consistency conditions [17] and the quantization of the Wess-Zumino-Witten action also do not seem suitable to be generalized. Recently, a more powerful proof of the cancellation of gauge anomalies to all orders, when they vanish at one loop, was given in ref. [18], elaborating on arguments that first appeared in ref. [7]. That approach has the virtue of

identifying the subtraction scheme where the cancellation to all orders is manifest. Nevertheless, due to the difficulties of the dimensional regularization, it only covers particular classes of models, namely four dimensional, perturbatively unitary power counting renormalizable gauge theories coupled to matter, where it is possible to handle the inconveniences in *ad hoc* ways. It would be important to extend those results to all perturbative quantum field theories, renormalizable or not. Going through the arguments of ref. [18], it is easy to spot several steps that do not generalize to wider classes of models in a straightforward way, and understand that the main obstacle is indeed the dimensional regularization as we normally understand it. On the other hand, when it comes to proving properties to all orders in perturbation theory, no known regularization technique is more powerful than the dimensional one, for a variety of reasons. Thus, to move forward it is necessary to formulate a more versatile regularization technique that overcomes the difficulties, but keeps the benefits of the ordinary dimensional one. The CD regularization provides a satisfactory answer to this problem.

It is known that in parity violating theories algebraic manipulations and evaluations of Feynman diagrams are more demanding than in parity invariant theories. Using the CD regularization the difficulty of explicit computations may increase. Nevertheless, we do not worry about this problem here, because we think that a certain complexity is unavoidable and a reasonable price to pay, if we want to simplify the proofs of all-order theorems. If we are interested in simplifying calculations, instead, we must deform the dimensional regularization in a different way [19, 20], by making propagators have  $SO(1, D - 1)$ -scalar denominators. In an explicit example we show that the effort to compute one-loop divergent parts and anomalies with the CD regularization is comparable to the usual one.

The paper is organized as follows. In section 2 we define the chiral dimensional regularization. We study chiral fermions, scalar fields, gauge fields, gravity in the metric-tensor formalism, gravity in the vielbein formalism and Chern-Simons theories. In section 3 we use weighted power counting to study the locality of counterterms and renormalization, and prove that the CD regularization is consistent to all orders. In section 4 we calculate the one-loop chiral anomaly with the CD technique. In section 5 we show how to use the new technique in several applications. In particular, we simplify the classification of counterterms and contributions to potential anomalies and show how to overcome a number of obstacles that afflict the proofs of all-order theorems. We concentrate in particular on the Adler-Bardeen theorem and the proofs of renormalizability of general gauge theories to all orders, and show that the CD regularization makes the usual derivations suitable to be generalized to wider classes of models. Section 6 contains our conclusions.

## 2 Chiral dimensional regularization

In this section we formulate the new regularization technique, in arbitrary dimensions  $d > 2$ . As usual, we split the  $D$ -dimensional spacetime manifold  $\mathbb{R}^D$  into the product  $\mathbb{R}^d \times \mathbb{R}^{-\varepsilon}$  of the ordinary  $d$ -dimensional spacetime  $\mathbb{R}^d$  times a residual  $(-\varepsilon)$ -dimensional evanescent space,  $\mathbb{R}^{-\varepsilon}$ . The spacetime indices  $\mu, \nu, \dots$  of vectors and tensors are split into the bar indices  $\bar{\mu}, \bar{\nu}, \dots$ , which take the values of  $0, 1, \dots, d-1$ , and the formal hat indices  $\hat{\mu}, \hat{\nu}, \dots$ , which denote the  $\mathbb{R}^{-\varepsilon}$  components. For example, the momenta  $p^\mu$  are split into the pairs  $p^{\bar{\mu}}, p^{\hat{\mu}}$ , also written as  $\bar{p}^{\bar{\mu}}, \hat{p}^{\hat{\mu}}$ , and the coordinates  $x^\mu$  are split into  $\bar{x}^{\bar{\mu}}, \hat{x}^{\hat{\mu}}$ . To evaluate a  $D$ -dimensional integral we must first integrate over the hat components of the momenta or coordinates, using the common formulas of the dimensional regularization, and then integrate over the bar components. The formal flat-space metric  $\eta_{\mu\nu}$  is split into the usual  $d \times d$  flat-space metric  $\eta_{\bar{\mu}\bar{\nu}} = \text{diag}(1, -1, \dots, -1)$  and the formal evanescent metric  $\eta_{\hat{\mu}\hat{\nu}} = -\delta_{\hat{\mu}\hat{\nu}}$  (the off-diagonal components  $\eta_{\bar{\mu}\hat{\nu}}$  being equal to zero). When we contract evanescent components we use the metric  $\eta_{\hat{\mu}\hat{\nu}}$ , so for example  $\hat{p}^2 = p^{\hat{\mu}}\eta_{\hat{\mu}\hat{\nu}}p^{\hat{\nu}}$ .

We assume that all of the fields  $\Phi(x)$  have strictly  $d$ -dimensional components, each of which is a function of  $\bar{x}$  and  $\hat{x}$ . The spinors  $\psi^\alpha$  have  $2^{\lfloor d/2 \rfloor}$  components, where  $\lfloor d/2 \rfloor$  is the integral part of  $d/2$ . Vectors have the  $d$  components  $A_{\bar{\mu}}$ , with no evanescent component  $A_{\hat{\mu}}$  being turned on. Symmetric tensors have  $d(d+1)/2$  components. In particular, the metric tensor  $g_{\mu\nu}$  is made of the diagonal blocks  $g_{\bar{\mu}\bar{\nu}}$  and  $\eta_{\hat{\mu}\hat{\nu}}$ , while the off-diagonal components  $g_{\bar{\mu}\hat{\nu}}$  vanish. Antisymmetric tensors have  $d(d-1)/2$  components, and so on.

As anticipated in the introduction, we take the  $\gamma$  matrices to be strictly  $d$  dimensional, and satisfy the usual Dirac algebra  $\{\gamma^{\bar{\mu}}, \gamma^{\bar{\nu}}\} = 2\eta^{\bar{\mu}\bar{\nu}}$ . When we write  $\gamma^\mu$  we mean  $\delta_\varepsilon^\mu \gamma^{\bar{\nu}}$ . If  $d = 2k$  is even, we define the  $d$ -dimensional generalization of  $\gamma_5$  as

$$\tilde{\gamma} = -i^{k+1} \gamma^0 \gamma^1 \dots \gamma^{2k-1},$$

which satisfies  $\tilde{\gamma}^\dagger = \tilde{\gamma}$ ,  $\tilde{\gamma}^2 = 1$ . Then we have the left and right projectors  $P_L = (1 - \tilde{\gamma})/2$ ,  $P_R = (1 + \tilde{\gamma})/2$  in the usual fashion. The tensor  $\varepsilon^{\bar{\mu}_1 \dots \bar{\mu}_d}$  and the charge-conjugation matrix  $\mathcal{C}$  also coincide with the usual ones. Full  $SO(1, D-1)$  invariance is lost in most expressions, replaced by  $SO(1, d-1) \times SO(-\varepsilon)$  invariance.

These rules are not the end of the story, because once we apply them, we realize that the so modified dimensional regularization does not equip the fields with good propagators. For example, the free action

$$\int d^D x \bar{\psi}_L i \not{\partial} \psi_L \quad (2.1)$$

of the left-handed fermions  $\psi_L$  gives the propagator

$$P_L \frac{i}{\not{p}} P_R, \quad (2.2)$$

which does not fall off in all directions of integration, because it does not depend on the evanescent components  $\hat{p}$  of the momenta. At the same time, we cannot modify the action (2.1) in a local way so that the propagators of chiral fermions get denominators of the form  $\bar{p}^2 + \eta\hat{p}^2$ , where  $\eta$  is a positive constant, because the  $\gamma$  matrices do not have evanescent components,  $\gamma^{\hat{\mu}}$ . These difficulties do not appear with bosonic fields, in general (with the exception of Chern-Simons gauge fields in three dimensions). However, we cannot let different fields have propagators with different structures, because if we did we would not be able to use weighted power counting.

The best we can do is equip all of the fields with propagators that have denominators equal to products of polynomials

$$D(\bar{p}, \hat{p}, m, \varsigma, \eta) = \bar{p}^2 - m^2 - \varsigma \frac{(\hat{p}^2)^2}{M^2} + \eta \frac{\hat{p}^2}{M} + i0, \quad (2.3)$$

where  $\varsigma$  is a positive constant of order one and  $M$  is a mass scale. This property ensures that propagators behave appropriately; in particular they fall off in all directions of integration, even when  $m = \eta = 0$  (which is the case of chiral fermions).

However, formula (2.3) shows that propagators fall off more rapidly in the evanescent directions  $\hat{p}$  of the momenta than in the physical directions  $\bar{p}$ . The right tool to manage different behaviors in different directions of integration is the *weighted power counting* introduced in ref. [5]. Here we recall its fundamental properties, leaving other details to section 4. Basically, the structure (2.3) suggests that in the ultraviolet limit  $\bar{p}$  and  $\hat{p}^2$  should be regarded as equally important. Ordinary power counting, which is based on the dimensions in units of mass, instead states that  $\bar{p}$  and  $\hat{p}$  are equally important in the ultraviolet limit. Thus, we must replace the dimensions in units of mass with suitable “weights”, defined in such a way that  $\bar{p}$  and  $\hat{p}^2$  are equally weighted.

We conventionally take  $\bar{p}$  to have weight 1, so the evanescent components  $\hat{p}$  of the momenta have weight 1/2. The action is obviously weightless, as well as the scale  $M$  appearing in formula (2.3). Writing the dominant kinetic terms (i.e. the kinetic terms with the largest number of derivatives  $\bar{\partial}$ ) of the field  $\Phi$  as

$$\frac{1}{2} \int \Phi \bar{\partial}^{N_\Phi} \Phi, \quad \text{or} \quad \int \bar{\Phi} \bar{\partial}^{N_\Phi} \Phi, \quad (2.4)$$

depending on the case, the weight of  $\Phi$  is equal to  $(d - N_\Phi)/2$  and coincides with its dimension in units of mass.

To ensure that propagators are well behaved in all the directions of integration, we proceed as follows. Consider a polynomial  $Q(\bar{p}, \hat{p})$  that is also a  $SO(1, d-1) \times SO(-\varepsilon)$  scalar. Define its “weighted degree” as its ordinary degree once  $Q$  is rewritten as a polynomial  $\tilde{Q}(\bar{p}, \hat{p}^2)$  of  $\bar{p}$  and  $\hat{p}^2$ . We require that propagators be rational functions of the momenta of the form

$$\frac{P'_{2w-N_\Phi}(\bar{p}, \hat{p})}{P_{2w}(\bar{p}, \hat{p})}, \quad (2.5)$$

where  $P'_{2w-N_\Phi}$  and  $P_{2w}$  are  $SO(1, d-1) \times SO(-\varepsilon)$ -scalar polynomials of weighted degrees  $2w - N_\Phi$  and  $2w$ , respectively, such that the monomials  $(\bar{p}^2)^w$  and  $(\hat{p}^2)^{2w}$  belonging to the denominators  $P_{2w}(\bar{p}, \hat{p})$  are both multiplied by nonvanishing coefficients. In the next subsections we show that it is possible to arrange the regularized action so that all of the fields fulfill these requirements. The total action is the one that contains all of the monomials compatible with weighted power counting and the nonanomalous symmetries of the theory, multiplied by the maximum number of independent coefficients.

Demanding that the action and the scale  $M$  be weightless, we can assign weights to all of the other parameters. The theories that contain only parameters of non-negative weights (and are such that propagators fall off with the correct behaviors in the ultraviolet limit) are renormalizable by weighted power counting. The theories that contain some parameters of strictly negative weights are nonrenormalizable. In that case, the propagators (2.5) must contain only parameters of non-negative weights.

Weighted power counting gives us an efficient control on the renormalization of the CD-regularized theory, including the evanescent sector. It also ensures that the scale  $M$  does not propagate into the physical sector. Precisely,  $M$  is an arbitrary, renormalization-group invariant parameter that belongs to the evanescent sector from the beginning to the end. In particular, there is no need to take the limit  $M \rightarrow \infty$  at any stage.

To summarize, to obtain a regularization that effectively works, we must modify the evanescent sector of the action according to the observations just made. We begin by showing how this is done in the case of chiral fermions.

## 2.1 Chiral fermions

The action of the (left-handed) chiral fermions  $\psi_L$  coupled to gauge fields in even  $d$  dimensions is

$$S_{c\psi} = \int \bar{\psi}_L i \gamma^{\bar{\mu}} D_{\bar{\mu}} \psi_L,$$

where  $D_{\bar{\mu}} = \partial_{\bar{\mu}} + g T^a A_{\bar{\mu}}^a$  is the covariant derivative and  $T^a$  are anti-Hermitian matrices associated with some representation of the gauge group  $G$ . The propagators (2.2) of this action do not fall off in all directions of integration, because they are independent of the evanescent components  $\hat{p}$  of the momenta. To overcome this difficulty, we complete the action  $S_{c\psi}$  by adding higher-derivative evanescent kinetic terms of the Majorana type.

For example, in four dimensions, using the standard basis of  $\gamma$  matrices (see below) the classical action reads

$$S_{c\psi} = \int \psi_L^\dagger i (\partial_{\bar{\mu}} + g T^a A_{\bar{\mu}}^a) \bar{\sigma}^{\bar{\mu}} \psi_L, \quad (2.6)$$

where  $\bar{\sigma}^{\bar{\mu}} = (1, -\vec{\sigma})$ ,  $\vec{\sigma}$  being the Pauli matrices. We regularize it by adding the evanescent

correction

$$S_{\text{ev}\psi} = \frac{i}{2M} \int (\varsigma_\psi \psi_L^\alpha \varepsilon_{\alpha\beta} \hat{\partial}^2 \psi_L^\beta + \varsigma_\psi^* \psi_L^{*\alpha} \varepsilon_{\alpha\beta} \hat{\partial}^2 \psi_L^{*\beta}), \quad (2.7)$$

where  $\varepsilon_{12} = 1$ . Right-handed chiral fermions can be treated similarly. Throughout this paper, the constants  $\varsigma_I$  are dimensionless and of order one, and multiply the dominant evanescent quadratic terms. We need a different constant  $\varsigma_I$  for every field, because fields renormalize independently of one another, even in the evanescent sector.

Defining  $\Psi = (\psi_L, \psi_L^*)$ , the free propagator of the action  $S_{c\psi} + S_{\text{ev}\psi}$  reads

$$\langle \Psi(p) \Psi^T(-p) \rangle_0 = \frac{i}{\bar{p}^2 - \frac{|\varsigma_\psi|^2}{M^2} (\hat{p}^2)^2 + i0} \begin{pmatrix} i\epsilon \frac{\varsigma_\psi^*}{M} \hat{p}^2 & p_{\bar{\mu}} \sigma^{\bar{\mu}} \\ p_{\bar{\mu}} (\sigma^{\bar{\mu}})^T & i\epsilon \frac{\varsigma_\psi}{M} \hat{p}^2 \end{pmatrix}, \quad (2.8)$$

where  $\epsilon$  is the matrix with entries  $\varepsilon_{\alpha\beta}$ ,  $\sigma^{\bar{\mu}} = (1, \vec{\sigma})$  and the superscript  $T$  denotes the transpose.

As promised, the regularized propagators have denominators of the form (2.3), therefore they properly fall off in all directions of integration. Note that without the regularizing terms (2.7), or, equivalently, at  $\varsigma_\psi = 0$ , the propagators of (2.8) would be  $\hat{p}$ -independent, and fermionic loops would integrate to zero in dimensional regularization.

Now we generalize the construction to arbitrary spacetime dimensions  $d > 2$ . First we need to choose a basis of  $\gamma$  matrices. We start from the usual basis of Pauli matrices  $\sigma^i$ ,  $i = 1, 2, 3$  and take  $\gamma^0 = \sigma^1$ ,  $\gamma^1 = i\sigma^2$  in two dimensions. Then we proceed by induction. Let  $\gamma^{\bar{\mu}}$ ,  $\bar{\mu} = 0, 1, \dots, 2k-1$ , denote the  $\gamma$  matrices in  $d = 2k$  dimensions, where  $k = 1, 2, \dots$ . Define  $\gamma^{\bar{\mu}}, \gamma^{2k}$  as the  $\gamma$  matrices in  $2k+1$  dimensions, where

$$\gamma^{2k} = -i^k \gamma^0 \gamma^1 \dots \gamma^{2k-1},$$

and

$$\Gamma^0 = 1 \otimes \sigma^1, \quad \Gamma^1 = i\gamma^0 \otimes \sigma^2, \quad \Gamma^{j+1} = \gamma^j \otimes \sigma^2 \quad \text{for } j = 1, \dots, 2k,$$

as the  $\gamma$  matrices in  $2k+2$  dimensions.

In this basis,  $\gamma^{\bar{\mu}}$  is always symmetric if  $\bar{\mu}$  is even, and always antisymmetric if  $\bar{\mu}$  is odd. The charge-conjugation matrix

$$\mathcal{C} = i^{k(k+1)/2} \gamma^0 \gamma^2 \dots \gamma^{2k-2}$$

in even dimensions  $d = 2k$  is proportional to the product of the  $\gamma$  matrices with even indices, and satisfies

$$\mathcal{C}^\dagger = -\mathcal{C}, \quad \mathcal{C}^2 = -1, \quad \mathcal{C}^T = (-1)^{k(k-1)/2} \mathcal{C}, \quad \mathcal{C} \gamma^{\bar{\mu}} \mathcal{C} = (-1)^k (\gamma^{\bar{\mu}})^T, \quad \mathcal{C} \tilde{\gamma} = (-1)^k \tilde{\gamma} \mathcal{C}.$$

We generalize the evanescent terms (2.7) to even dimensions by choosing

$$S_{\text{ev}\psi} = \frac{i}{2M} \int (\varsigma_\psi \psi_L^T \tilde{\mathcal{C}} \hat{\partial}^2 \psi_L - \varsigma_\psi^* \bar{\psi}_L \tilde{\mathcal{C}} \hat{\partial}^2 \bar{\psi}_L^T), \quad (2.9)$$

where  $\tilde{\mathcal{C}}$  is a suitable matrix that we now identify. It must be antisymmetric, otherwise (2.9) vanishes, and invertible, to give well-behaved propagators. We take a  $\tilde{\mathcal{C}}$  that commutes with  $\tilde{\gamma}$ , because a  $\tilde{\mathcal{C}}$  that anticommutes with  $\tilde{\gamma}$  gives again zero. We also choose  $\tilde{\mathcal{C}}$  such that  $\gamma^0 \tilde{\mathcal{C}}^\dagger = \tilde{\mathcal{C}} \gamma^0$ , so that the expression  $S_{\text{ev}\psi}$  given in (2.9) is Hermitian. Finally, to simplify a number of manipulations, we demand that the square of  $\tilde{\mathcal{C}}$  be proportional to the identity. Precisely, we choose  $\tilde{\mathcal{C}}^2 = -1$ .

If  $d = 4 \pmod 8$  chiral fermions admit Majorana masses, because  $[\mathcal{C}, \tilde{\gamma}] = 0$  and  $\mathcal{C}$  is antisymmetric. There we can take  $\tilde{\mathcal{C}} = \mathcal{C}$ , which ensures that (2.9) is Lorentz invariant in the physical portion of spacetime (therefore global Lorentz symmetry is manifestly nonanomalous). Instead, in even dimensions  $d \neq 4 \pmod 8$ , chiral fermions cannot have Majorana masses, either because the charge-conjugation matrix  $\mathcal{C}$  anticommutes with  $\tilde{\gamma}$ , or because it is symmetric, or both. In even  $d > 2$ ,  $d \neq 4 \pmod 8$ , we choose  $\tilde{\mathcal{C}} = -i\gamma^0 \gamma^2$ , which indeed satisfies  $\tilde{\mathcal{C}}^T = -\tilde{\mathcal{C}}$ ,  $[\tilde{\mathcal{C}}, \tilde{\gamma}] = 0$ ,  $\tilde{\mathcal{C}}^\dagger = -\tilde{\mathcal{C}}$ ,  $\{\tilde{\mathcal{C}}, \gamma^0\} = 0$  and  $\tilde{\mathcal{C}}^2 = -1$ . In this case the regularization explicitly breaks Lorentz symmetry in the evanescent sector.

In  $d = 2$  no matrix  $\tilde{\mathcal{C}}$  exists, which is why we cannot make our regularization work there, in general. However, so far we have just considered single Weyl flavors. With more flavors there are more options and, depending on the case, it may be possible to build a matrix  $\tilde{\mathcal{C}}$  that satisfies our requirements even in  $d = 2$ . With more Weyl flavors Lorentz symmetric evanescent kinetic terms of the Majorana type may also exist in even dimensions  $d \neq 4 \pmod 8$ .

Besides Lorentz symmetry in  $d \neq 4 \pmod 8$ , the Majorana terms (2.9) may also break other global symmetries, such as those associated with the gauge groups  $G$ . However, in most cases such symmetries are nonanomalous, therefore they can be recovered by adding suitable local counterterms.

Defining  $\Psi = (\psi_L, \bar{\psi}_L^T)$ , the propagator of the action  $S_{c\psi} + S_{\text{ev}\psi}$ , with  $S_{\text{ev}\psi}$  given by (2.9), can be worked out with a small effort and reads

$$\langle \Psi(p) \bar{\Psi}(-p) \rangle_0 = \frac{i}{D_p^2 + 4\tilde{p}^2 |h_{\tilde{p}}|^2} \begin{pmatrix} D_p & 2h_{\tilde{p}}^* \tilde{\mathcal{C}} \tilde{\tilde{p}} \\ 2h_{\tilde{p}} \tilde{\mathcal{C}} \tilde{\tilde{p}} & D_p \end{pmatrix} \begin{pmatrix} \tilde{\tilde{p}} & -h_{\tilde{p}}^* \tilde{\mathcal{C}} \\ -h_{\tilde{p}} \tilde{\mathcal{C}} & \tilde{\tilde{p}}^T \end{pmatrix}, \quad (2.10)$$

where

$$D_p = \tilde{p}^2 - |h_{\tilde{p}}|^2, \quad h_{\tilde{p}} \equiv i\zeta_\psi \frac{\tilde{p}^2}{M},$$

and the vector  $\tilde{p}^\mu$  is defined as follows. In even dimensions  $d \neq 4 \pmod 8$   $\tilde{p}^\mu$  is obtained from  $p^\mu$  by suppressing  $p^0$ ,  $p^2$  and  $p^{\text{odd}}$ , which gives  $\tilde{p}^\mu = (0, 0, 0, 0, p^4, 0, p^6, 0, p^8, \dots)$ . In  $d = 4 \pmod 8$ , instead, we take  $\tilde{p}^\mu = 0$ . Finally, in all cases  $\tilde{p}^\mu = 0$ . It is easy to prove the identities

$$\tilde{\tilde{p}} = \tilde{\tilde{p}}^T, \quad [\tilde{\tilde{p}}, \tilde{\mathcal{C}}] = 0.$$

Using these relations it is straightforward to verify that (2.10) is indeed the propagator of the action  $S_{c\psi} + S_{\text{ev}\psi}$  for arbitrary  $d > 2$ .

Formula (2.9) shows that the propagators are well-behaved in all directions of integrations, according to the requirements formulated in the previous subsection. Moreover, their denominators are regular everywhere. This fact can be proved, for example, by checking that the denominators become positive definite when we turn to Euclidean space.

In odd dimensions or with Dirac fermions  $\psi$  we can just use Dirac-type evanescent kinetic terms and write

$$S_{c\psi} + S_{\text{ev}\psi} = \int \bar{\psi} (i\gamma^{\bar{\mu}} D_{\bar{\mu}} - m) \psi - \frac{\varsigma_{\psi}}{M} \int (\partial_{\bar{\mu}} \bar{\psi})(\partial^{\hat{\mu}} \psi),$$

where now  $\varsigma_{\psi}$  is real. The propagator is

$$\frac{i(\bar{p} + m + \frac{\varsigma_{\psi}}{M}\hat{p}^2)}{\bar{p}^2 - (m + \frac{\varsigma_{\psi}}{M}\hat{p}^2)^2}.$$

Ultimately, the modified regularization we consider in this paper can always be used in dimensions  $> 2$ , and sometimes also in  $d = 2$ .

## 2.2 Scalars

For definiteness, consider the theory

$$S_{c\varphi} = \int (D_{\bar{\mu}}\varphi)^{\dagger}(D^{\bar{\mu}}\varphi) - V(\varphi^{\dagger}\varphi) + S_Y, \quad (2.11)$$

where  $V$  is a potential and  $S_Y$  denotes the Yukawa couplings and any other types of perturbative corrections. The propagator of (2.11) is  $i/(\bar{p}^2 - m^2)$ , therefore it does not behave correctly. We add the quadratic evanescent terms

$$S_{\text{ev}\varphi} = -\frac{\varsigma_{\varphi}}{M^2} \int (\hat{\partial}^2 \varphi)^{\dagger}(\hat{\partial}^2 \varphi) + \frac{\eta_{\varphi}}{M} \int (\partial_{\bar{\mu}}\varphi)^{\dagger}(\partial^{\hat{\mu}}\varphi). \quad (2.12)$$

The propagator of the total action  $S_{c\varphi} + S_{\text{ev}\varphi}$  is then

$$\frac{i}{D(\bar{p}, \hat{p}, m, \varsigma_{\varphi}, \eta_{\varphi})}$$

and therefore has the right type of denominator. In some cases evanescent vertices of the form

$$\int (\hat{\partial}^2 \varphi)^{\dagger} \varphi^2, \quad \int (\hat{\partial}^2 \varphi)^{\dagger} \varphi^3, \quad (2.13)$$

etc., in  $d = 3, 4$  might be allowed and should be added to the action. If no parameters of negative weights are around, no other evanescent terms compatible with weighted power counting and the global symmetries of the theory can be constructed. Then  $S_{c\varphi} + S_{\text{ev}\varphi}$  plus the terms of type (2.13) is the total regularized action.

### 2.3 Gauge fields

Now we switch to gauge fields. For definiteness, we take non-Abelian Yang-Mills theory with a simple gauge group  $G$ , coupled to left-handed fermions  $\psi_L$ . The gauge-invariant action is  $S_c = S_{cA} + S_{c\psi}$ , with

$$S_{cA} = -\frac{1}{4} \int F_{\bar{\mu}\bar{\nu}}^a F^{\bar{\mu}\bar{\nu}a}, \quad F_{\bar{\mu}\bar{\nu}}^a = \partial_{\bar{\mu}} A_{\bar{\nu}}^a - \partial_{\bar{\nu}} A_{\bar{\mu}}^a + g f^{abc} A_{\bar{\mu}}^b A_{\bar{\nu}}^c.$$

To keep track of gauge invariance during the renormalization algorithm, it is convenient to use the Batalin-Vilkovisky formalism [9]. A set of fields  $\Phi^\alpha = \{A_{\bar{\mu}}^a, C^a, \bar{C}^a, B^a, \psi_L, \bar{\psi}_L\}$  is defined, to collect the classical fields  $\phi = \{A_{\bar{\mu}}^a, \psi_L, \bar{\psi}_L\}$ , the ghosts  $C$ , the antighosts  $\bar{C}$  and the Lagrange multipliers  $B$  for the gauge fixing. An external source  $K_\alpha$  with opposite statistics is associated with each  $\Phi^\alpha$ , and coupled to the  $\Phi^\alpha$  symmetry transformations  $R^\alpha(\Phi)$ . We have  $K_\alpha = \{K^{\bar{\mu}a}, K_C^a, K_{\bar{C}}^a, K_B^a, K_\psi, \bar{K}_\psi\}$ . If  $X$  and  $Y$  are functionals of  $\Phi$  and  $K$  their *antiparentheses* are defined as

$$(X, Y) \equiv \int \left( \frac{\delta_r X}{\delta \Phi^\alpha} \frac{\delta_l Y}{\delta K_\alpha} - \frac{\delta_r X}{\delta K_\alpha} \frac{\delta_l Y}{\delta \Phi^\alpha} \right), \quad (2.14)$$

where the integral is over spacetime points associated with repeated indices. The *master equation*  $(S, S) = 0$  must be solved for  $S = S(\Phi, K)$  in  $D = d$  with the “boundary condition”  $S = S_c$  at  $C = \bar{C} = B = K_\alpha = 0$ . The solution

$$S_c + S_K \quad (2.15)$$

is the action we start with to quantize the theory, where

$$\begin{aligned} S_K(\Phi, K) = & - \int R^\alpha(\Phi) K_\alpha = - \int (D_{\bar{\mu}} C^a) K^{\bar{\mu}a} + \frac{g}{2} \int f^{abc} C^b C^c K_C^a - \int B^a K_{\bar{C}}^a \\ & + g \int (\bar{\psi}_L T^a C^a K_\psi + \bar{K}_\psi T^a C^a \psi_L), \end{aligned}$$

encodes the symmetry transformations  $R^\alpha(\Phi)$  of the fields,  $D_{\bar{\mu}} C^a = \partial_{\bar{\mu}} C^a + g f^{abc} A_{\bar{\mu}}^b C^c$  being the covariant derivative of the ghosts. We can easily check that  $(S_c, S_c) = (S_c, S_K) = (S_K, S_K) = 0$  in arbitrary  $D$  dimensions.

We gauge fix the theory with the gauge fermion

$$\Psi(\Phi) = \int \bar{C}^a \left( \partial^{\bar{\mu}} A_{\bar{\mu}}^a + \frac{\xi}{2} B^a \right),$$

which means that we add

$$(S_K, \Psi) = \int B^a \left( \partial^{\bar{\mu}} A_{\bar{\mu}}^a + \frac{\xi}{2} B^a \right) - \int \bar{C}^a \partial^{\bar{\mu}} D_{\bar{\mu}} C^a$$

to the action (2.15), and obtain the gauge-fixed action

$$S_d(\Phi, K) = S_c + (S_K, \Psi) + S_K, \quad (2.16)$$

which still satisfies  $(S_d, S_d) = 0$  exactly in arbitrary  $D$  dimensions.

As in the cases of fermions and scalars, the action  $S_d$  is not well regularized, since its propagators depend only on the  $d$ -dimensional components of the momenta. Thus, we add evanescent corrections  $S_{ev}$ , guided by weighted power counting. We have learned from the treatment of fermions that the weights of the fields and the physical components of the momenta coincide with their dimensions in units of mass, and two derivatives  $\hat{\partial}$  count as one derivative  $\bar{\partial}$ . Denoting weights with square brackets, we have

$$[A_{\bar{\mu}}^a] = [\bar{C}^a] = [C^a] = \frac{d}{2} - 1, \quad [B^a] = \frac{d}{2}, \quad [\bar{\partial}] = 1, \quad [\hat{\partial}] = \frac{1}{2}.$$

Then we take

$$S_{evA} = \frac{\varsigma_A}{2M^2} \int A_{\bar{\mu}}^a (\hat{\partial}^2)^2 A^{\bar{\mu}a} + \frac{\eta_A}{2M} \int A_{\bar{\mu}}^a (\hat{\partial}^2) A^{\bar{\mu}a}, \quad (2.17)$$

$$S_{evC} = -\frac{\varsigma_C}{M^2} \int \bar{C}^a (\hat{\partial}^2)^2 C^a - \frac{\eta_C}{M} \int \bar{C}^a (\hat{\partial}^2) C^a, \quad (2.18)$$

for gauge fields and ghosts, respectively. Note that no evanescent terms can be constructed with  $B^a$ .

The final step is to include all of the other terms allowed by weighted power counting and ghost number conservation. We can distinguish nonevanescant additional terms  $\Delta S_c$  and evanescent additional terms  $\Delta S_{ev}$ . The total classical action

$$S_c = S_{cA} + S_{c\psi} + \Delta S_c$$

must be such that the gauge-fixed action  $S_d$  defined by formula (2.16) still satisfies  $(S_d, S_d) = 0$  exactly in arbitrary  $D$  dimensions. For example, if the theory is nonrenormalizable (like the standard model coupled to quantum gravity, or a low-energy effective field theory)  $\Delta S_c$  collects infinitely many corrections of higher dimensions (such as two scalar-two fermion vertices, Pauli terms, four-fermion vertices, etc.). Recall that in  $d > 4$  all gauge theories are nonrenormalizable.

The total evanescent action reads

$$S_{ev} = S_{evA} + S_{ev\psi} + S_{evC} + \Delta S_{ev}.$$

For example, in  $d = 3$  we have  $\Delta S_{ev} = \Delta S_{evA}$ , where

$$\Delta S_{evA} = \frac{1}{M} \int \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}} \left( \zeta'_A A_{\bar{\mu}}^a (\hat{\partial}^2) \partial_{\bar{\nu}} A_{\bar{\rho}}^a + \zeta_A g f^{abc} A_{\bar{\mu}}^a A_{\bar{\nu}}^b \hat{\partial}^2 A_{\bar{\rho}}^c \right),$$

$\zeta'_A$  and  $\zeta_A$  being constants.

Finally, the gauge-fixed regularized action of chiral gauge theories reads

$$S(\Phi, K) = S_c + (S_K, \Psi) + S_K + S_{ev} = S_d + S_{ev}. \quad (2.19)$$

Since  $(S_d, S_d) = 0$ , the action (2.19) satisfies the deformed master equation

$$(S, S) = \mathcal{O}(\varepsilon). \quad (2.20)$$

To prove that the CD regularization is consistent, we need to focus on the propagators, which contain only parameters of non-negative weights. Thus, even when the theory is nonrenormalizable, it is enough to study the subsector where the parameters of negative weights are switched off. In this subsector no new terms are allowed besides those listed so far. In particular, no evanescent terms depending on the sources  $K$  can be constructed. Moreover,  $\Delta S_{\text{ev}} = 0$  in  $d > 3$ .

Collecting all pieces together (and switching off the parameters of negative dimensions), we find

$$\begin{aligned} S(\Phi, K) = & -\frac{1}{4} \int F_{\bar{\mu}\bar{\nu}}^a F^{\bar{\mu}\bar{\nu}a} + \int B^a \left( \partial^{\bar{\mu}} A_{\bar{\mu}}^a + \frac{\xi}{2} B^a \right) + \frac{1}{2} \int A_{\bar{\mu}}^a \frac{\hat{\partial}^2}{M} \left( \varsigma_A \frac{\hat{\partial}^2}{M} + \eta_A \right) A^{\bar{\mu}a} \\ & + \int \bar{\psi}_L i \gamma^{\bar{\mu}} D_{\bar{\mu}} \psi_L + \frac{i}{2M} \int \left( \varsigma_\psi \psi_L^T \tilde{C} \hat{\partial}^2 \psi_L - \varsigma_\psi^* \bar{\psi}_L \tilde{C} \hat{\partial}^2 \bar{\psi}_L^T \right) \\ & - \int \bar{C}^a \left( \partial^{\bar{\mu}} D_{\bar{\mu}} + \frac{\varsigma_C (\hat{\partial}^2)^2}{M^2} + \eta_C \frac{\hat{\partial}^2}{M} \right) C^a + \Delta S_{\text{ev}A} + S_K. \end{aligned} \quad (2.21)$$

Observe that the ghosts still decouple in the Abelian case. The ghost propagators are

$$\langle C^a(p) \bar{C}^b(-p) \rangle_0 = \frac{i \delta^{ab}}{D(\bar{p}, \hat{p}, 0, \varsigma_C, \eta_C)}. \quad (2.22)$$

Ignoring  $\Delta S_{\text{ev}A}$  for a moment, the propagators of the multiplet made of  $A_{\bar{\mu}}^a$  and  $B^a$  are

$$\begin{aligned} \langle A_{\bar{\mu}}^a(p) A_{\bar{\nu}}^b(-p) \rangle_0 &= \frac{-i \delta^{ab}}{D(\bar{p}, \hat{p}, 0, \varsigma_A, \eta_A)} \left( \eta_{\bar{\mu}\bar{\nu}} + \frac{(\xi - 1) p_{\bar{\mu}} p_{\bar{\nu}}}{D(\bar{p}, \hat{p}, 0, \xi \varsigma_A, \xi \eta_A)} \right), \\ \langle A_{\bar{\mu}}^a(p) B^b(-p) \rangle_0 &= \frac{-p_{\bar{\mu}} \delta^{ab}}{D(\bar{p}, \hat{p}, 0, \xi \varsigma_A, \xi \eta_A)}, \quad \langle B^a(p) B^b(-p) \rangle_0 = i \delta^{ab} \frac{\hat{p}^2}{M} \frac{\frac{\varsigma_A}{M} \hat{p}^2 - \eta_A}{D(\bar{p}, \hat{p}, 0, \xi \varsigma_A, \xi \eta_A)}. \end{aligned}$$

All of them have correct denominators and correct structures to ensure the locality of counterterms, according to weighted power counting. Note that we must keep  $\xi \neq 0$ , which means that the Landau gauge is not available in the CD regularization.

When we switch  $\Delta S_{\text{ev}A}$  on, in  $d = 3$ , the propagators remain regular. Indeed, towards the end of subsection 2.5 we prove a theorem stating that if the propagators of some action  $S$  are regular, they remain regular when  $S$  is extended by adding new terms compatible with weighted power counting, multiplied by independent parameters.

## 2.4 Gravity

Now we move to quantum gravity. We recall that  $x^{\bar{\mu}}$  are the coordinates of the physical portion of spacetime, and  $x^{\hat{\mu}}$  are those of the evanescent portion. The metric tensor depends on both,

like every other field, but its nontrivial components are just the usual  $d$ -dimensional ones  $g_{\bar{\mu}\bar{\nu}}$ . Precisely, in the evanescent sector we take the flat-space metric  $g_{\bar{\mu}\bar{\nu}} = \eta_{\bar{\mu}\bar{\nu}}$ , while the off-diagonal components  $g_{\bar{\mu}\hat{\nu}}$  identically vanish, so we have

$$g_{\mu\nu}(\bar{x}, \hat{x}) = \begin{pmatrix} g_{\bar{\mu}\bar{\nu}}(\bar{x}, \hat{x}) & 0 \\ 0 & \eta_{\hat{\mu}\hat{\nu}} \end{pmatrix}. \quad (2.23)$$

General changes of coordinates affect only  $x^{\bar{\mu}}$ , and leave  $x^{\hat{\mu}}$  unmodified:

$$x^{\bar{\mu}'} = \xi^{\bar{\mu}}(\bar{x}, \hat{x}), \quad x^{\hat{\mu}'} = x^{\hat{\mu}}. \quad (2.24)$$

Under these transformations  $g_{\bar{\mu}\bar{\nu}}$  and  $\partial_{\bar{\mu}}$  transform as usual, and  $d^D x \sqrt{|g|}$  is invariant, where  $g$  is the determinant of  $g_{\bar{\mu}\bar{\nu}}$ . Actually,  $g_{\hat{\mu}\hat{\nu}}$ ,  $g_{\bar{\mu}\hat{\nu}}$  and  $\hat{\partial}$  also change, so  $g_{\mu\nu}$  does not keep the form (2.23). However, this is not a problem, since we do not require that the evanescent sector of the theory be invariant under the general coordinate transformations (2.24).

We start from the ordinary  $d$ -dimensional Hilbert action

$$S_{cG} = S_H = -\frac{1}{2\kappa^2} \int d^D x \sqrt{|g(\bar{x}, \hat{x})|} R(\bar{x}, \hat{x}), \quad (2.25)$$

the constant  $\kappa$  having the dimension of an energy to the power  $(2 - D)/2$ . In  $D$  dimensions covariant derivatives, as well as the Christoffel symbols, the Riemann and Ricci tensors and the Ricci scalar are defined by exactly the same formulas that hold in  $d$  dimensions, therefore they transform as usual:

$$\begin{aligned} \Gamma_{\bar{\nu}\bar{\rho}}^{\bar{\mu}} &= \frac{1}{2} g^{\bar{\mu}\bar{\sigma}} (\partial_{\bar{\nu}} g_{\bar{\rho}\bar{\sigma}} + \partial_{\bar{\rho}} g_{\bar{\nu}\bar{\sigma}} - \partial_{\bar{\sigma}} g_{\bar{\nu}\bar{\rho}}), & R_{\bar{\nu}\bar{\rho}\bar{\sigma}}^{\bar{\mu}} &= \partial_{\bar{\rho}} \Gamma_{\bar{\nu}\bar{\sigma}}^{\bar{\mu}} - \partial_{\bar{\sigma}} \Gamma_{\bar{\nu}\bar{\rho}}^{\bar{\mu}} + \Gamma_{\bar{\alpha}\bar{\rho}}^{\bar{\mu}} \Gamma_{\bar{\nu}\bar{\sigma}}^{\bar{\alpha}} - \Gamma_{\bar{\alpha}\bar{\sigma}}^{\bar{\mu}} \Gamma_{\bar{\nu}\bar{\rho}}^{\bar{\alpha}}, \\ R_{\bar{\mu}\bar{\nu}} &= R_{\bar{\mu}\bar{\rho}\bar{\nu}}^{\bar{\rho}}, & R &= g^{\bar{\mu}\bar{\nu}} R_{\bar{\mu}\bar{\nu}}. \end{aligned}$$

Basically, the evanescent components  $x^{\hat{\mu}}$  of the coordinates are treated as external parameters. Clearly, (2.25) is invariant under (2.24) in arbitrary  $D$  dimensions.

Now we introduce the ghosts of diffeomorphisms  $C^{\bar{\mu}}$ , as well as the antighosts  $\bar{C}_{\bar{\mu}}$  and the Lagrange multipliers  $B_{\bar{\mu}}$ , and an external source  $K$  for every field. All of them depend on  $\bar{x}, \hat{x}$ . The infinitesimal gauge transformations are collected into the functional

$$S_K = \int (g_{\bar{\mu}\bar{\rho}} \partial_{\bar{\nu}} C^{\bar{\rho}} + g_{\bar{\nu}\bar{\rho}} \partial_{\bar{\mu}} C^{\bar{\rho}} + C^{\bar{\rho}} \partial_{\bar{\rho}} g_{\bar{\mu}\bar{\nu}}) K^{\bar{\mu}\bar{\nu}} + \int C^{\bar{\rho}} (\partial_{\bar{\rho}} C^{\bar{\mu}}) K_{\bar{\mu}}^C - \int B_{\bar{\mu}} K_{\bar{C}}^{\bar{\mu}}. \quad (2.26)$$

As before, it is easy to check that  $(S_{cG}, S_{cG}) = (S_{cG}, S_K) = (S_K, S_K) = 0$  in  $D$  dimensions.

For several applications it is important to preserve invariance under *rigid diffeomorphisms*, which are the coordinate transformations

$$x^{\bar{\mu}'} = M_{\bar{\nu}}^{\bar{\mu}} x^{\bar{\nu}}, \quad x^{\hat{\mu}'} = x^{\hat{\mu}}, \quad (2.27)$$

where  $M_{\bar{\nu}}^{\bar{\mu}}$  is an arbitrary invertible constant matrix. The action  $S_{cG}$  is obviously invariant under rigid diffeomorphisms, but also  $S_K$  is, if we declare that the sources  $K$  are, according to the case, scalar densities, vector densities or tensor densities of weight 1. That means, in practice, that they carry a hidden  $\sqrt{|g|}$ .

To ensure invariance under rigid diffeomorphisms, it is sufficient to express all fields and derivatives  $\bar{\partial}$  using lower spacetime indices, contract those indices with the inverse metric tensor  $g^{\bar{\mu}\bar{\nu}}$  everywhere, and finally multiply by an appropriate power of  $\sqrt{|g|}$ , to obtain scalar densities of weight 1. Derivatives  $\hat{\partial}$ , instead, must be contracted with  $\eta^{\hat{\mu}\hat{\nu}}$ , to ensure  $SO(-\varepsilon)$  invariance. Preserving invariance under rigid diffeomorphisms is convenient for some applications (see section 5), because, among other things, it constrains the forms of counterterms and allows us to work without introducing “second metrics”. By that we mean any additional metrics (including the flat-space metric  $\eta_{\bar{\mu}\bar{\nu}}$ ) that are often used for gauge-fixing and regularization purposes.

We want to show that the CD regularization is fully compatible with invariance under rigid diffeomorphisms. First, it is possible to choose gauge-fixing conditions that preserve this global symmetry. For example, we can take the gauge fermion

$$\Psi = - \int \sqrt{|g|} \bar{C}_{\bar{\mu}} \left[ \frac{1}{2\kappa} (\partial_{\bar{\nu}} g^{\bar{\mu}\bar{\nu}} + \lambda g^{\bar{\mu}\bar{\nu}} g_{\bar{\rho}\bar{\sigma}} \partial_{\bar{\nu}} g^{\bar{\rho}\bar{\sigma}}) + \frac{\xi}{2} g^{\bar{\mu}\bar{\nu}} B_{\bar{\nu}} \right], \quad (2.28)$$

where  $\lambda$  and  $\xi$  are gauge-fixing parameters. As usual, the action is gauge fixed by adding  $(S_K, \Psi)$ :

$$S_d = S_{cG} + (S_K, \Psi) + S_K. \quad (2.29)$$

We clearly have  $(S_d, S_d) = 0$  in  $D$  dimensions.

At this point we observe that the action (2.29) is not equipped with well-regularized propagators, so we must add evanescent terms consistent with weighted power counting. From (2.29) we derive the weight assignments

$$[g_{\bar{\mu}\bar{\nu}}] = [C^{\bar{\mu}}] = 0, \quad [\bar{C}_{\bar{\mu}}] = \frac{d}{2} - 1, \quad [B_{\bar{\mu}}] = \frac{d}{2}, \quad [K^{\bar{\mu}\bar{\nu}}] = [K_{\bar{\mu}}^C] = d - 1, \quad [K_{\bar{C}}^{\bar{\mu}}] = \frac{d}{2}.$$

We have used  $[\Phi^\alpha] + [K_\alpha] = d - 1$  for every  $\alpha$ . We also want to arrange the regularizing terms so that the full gauge-fixed CD-regularized action is invariant under rigid diffeomorphisms.

We start adding the evanescent quadratic terms

$$S_{evG} = \frac{1}{8\kappa^2 M^2} \int \sqrt{|g|} \left( \varsigma_G (\hat{\partial}^2 g_{\bar{\mu}\bar{\nu}}) (\hat{\partial}^2 g^{\bar{\mu}\bar{\nu}}) + \varsigma'_G (g_{\bar{\alpha}\bar{\beta}} \hat{\partial}^2 g^{\bar{\alpha}\bar{\beta}})^2 \right) + \frac{\varsigma_{cG}}{2\kappa M^2} \int \sqrt{|g|} \bar{C}_{\bar{\mu}} (\hat{\partial}^2)^2 C^{\bar{\mu}}, \quad (2.30)$$

which are the key ones to make propagators well behaved. Other evanescent terms can be included in  $S_{evG}$ , such as: (i) quadratic terms similar to those of (2.30), but with just two  $\hat{\partial}$ 's instead of four, contracted in various ways, and (ii) terms that contribute only to vertices when the metric tensor is expanded around flat space, for example

$$\frac{1}{\kappa^4 M^2} \int \sqrt{|g|} (\partial_{\hat{\alpha}} g_{\bar{\mu}\bar{\nu}}) (\partial^{\hat{\alpha}} g^{\bar{\nu}\bar{\rho}}) (\partial_{\hat{\beta}} g_{\bar{\rho}\bar{\sigma}}) (\partial^{\hat{\beta}} g^{\bar{\sigma}\bar{\mu}}).$$

The total gauge-fixed action is then

$$S(\Phi, K) = S_{cG} + (S_K, \Psi) + S_K + S_{\text{ev}G} = S_d + S_{\text{ev}G}, \quad (2.31)$$

and is clearly such that  $(S, S) = \mathcal{O}(\varepsilon)$ . Indeed, the nonevanescant part  $S_d$  satisfies the master equation exactly, while the evanescent part violates the master equation, since the derivatives  $\hat{\partial}$  are noncovariant.

Now we show that the propagators of (2.31) are indeed well behaved. We expand around flat spacetime by writing

$$g_{\bar{\mu}\bar{\nu}} = \eta_{\bar{\mu}\bar{\nu}} + 2\kappa\phi_{\bar{\mu}\bar{\nu}}, \quad C^{\bar{\mu}} = \kappa\tilde{C}^{\bar{\mu}}, \quad (2.32)$$

and work out the expansion to the quadratic order. From  $S_{cG}$  we obtain

$$\frac{1}{2} \int ((\partial_{\bar{\alpha}}\phi^{\bar{\mu}\bar{\nu}})(\partial^{\bar{\alpha}}\phi_{\bar{\mu}\bar{\nu}}) - (\partial_{\bar{\alpha}}\phi)(\partial^{\bar{\alpha}}\phi) - 2(\partial_{\bar{\mu}}\phi^{\bar{\mu}\bar{\nu}})(\partial^{\bar{\rho}}\phi_{\bar{\rho}\bar{\nu}}) - 2\phi(\partial_{\bar{\mu}}\partial_{\bar{\nu}}\phi^{\bar{\mu}\bar{\nu}})), \quad (2.33)$$

where  $\phi = \phi_{\bar{\mu}}^{\bar{\mu}}$  and indices are raised and lowered with the flat metric  $\eta_{\bar{\mu}\bar{\nu}}$ . The gauge fixing  $(S_K, \Psi)$  contributes with

$$\int B_{\bar{\mu}} \left( \partial_{\bar{\nu}}\phi^{\bar{\mu}\bar{\nu}} + \lambda\partial^{\bar{\mu}}\phi - \frac{\xi}{2}B^{\bar{\mu}} \right) + \frac{1}{2} \int \bar{C}_{\bar{\mu}} \left( \partial_{\bar{\nu}}\partial^{\bar{\nu}}\tilde{C}^{\bar{\mu}} + (1 + 2\lambda)\partial^{\bar{\mu}}\partial_{\bar{\nu}}\tilde{C}^{\bar{\nu}} \right).$$

Finally, the quadratic part of the evanescent sector  $S_{\text{ev}G}$  is

$$-\frac{1}{2M^2} \int \left( \varsigma_G\phi_{\bar{\mu}\bar{\nu}}(\hat{\partial}^2)^2\phi^{\bar{\mu}\bar{\nu}} - \varsigma'_G\phi(\hat{\partial}^2)^2\phi \right) + \frac{\varsigma_{cG}}{2M^2} \int \bar{C}_{\bar{\mu}}(\hat{\partial}^2)^2\tilde{C}^{\bar{\mu}}$$

plus similar terms obtained making the substitutions  $(\hat{\partial}^2)^2/M^2 \rightarrow \hat{\partial}^2/M$  and  $\varsigma \rightarrow \eta$ .

The propagators of the multiplet  $\phi_{\bar{\mu}\bar{\nu}}, B_{\bar{\rho}}$  are very involved. We have worked them out with the help of a computer program. We do not give the result here, but just report that they have the right structure to make the CD regularization work, as long as  $\xi \neq 0$ ,  $\lambda \neq -1$ ,  $\varsigma_G \neq d\varsigma'_G$  and  $d > 2$ . The denominators are polynomials  $P_{2w}(\bar{p}, \hat{p})$  of even weights  $2w$  such that both monomials  $(\bar{p}^2)^w$  and  $(\hat{p}^2)^{2w}$  are multiplied by nonvanishing coefficients. Moreover, the propagators fall off with the correct velocities in all directions of integration.

The ghost propagator is

$$\langle \tilde{C}^{\bar{\mu}}(p)\bar{C}_{\bar{\nu}}(-p) \rangle_0 = -\frac{2i}{D(\bar{p}, \hat{p}, 0, \varsigma_{cG}, \eta_{cG})} \left( \delta_{\bar{\nu}}^{\bar{\mu}} - \frac{(1 + 2\lambda)p^{\bar{\mu}}p_{\bar{\nu}}}{2(1 + \lambda)D\left(\bar{p}, \hat{p}, 0, \frac{\varsigma_{cG}}{2(1+\lambda)}, \frac{\eta_{cG}}{2(1+\lambda)}\right)} \right)$$

and also has the right structure.

When the cosmological constant  $\Lambda$  is turned on, we must treat it nonperturbatively, as if it were the squared mass of a bosonic particle. For the purposes of renormalization, since counterterms are polynomial in  $\Lambda$  we can still expand around flat space, although flat space is no longer an extreme of the classical action.

## 2.5 Gravity in the vielbein formalism

When gravity is coupled to matter, the actions of chiral fermions, scalars and gauge fields must be covariantized, possibly adding nonminimal terms. The covariantization of nonevanescant terms, such as  $S_{cA}$ , proceeds as in  $d$  dimensions, while the covariantization of the evanescent corrections, such as  $S_{evA}$ , is made only with respect to rigid diffeomorphisms. For example, (2.17) and (2.18) become

$$\begin{aligned} S_{evA} &= \frac{S_A}{2M^2} \int \sqrt{|g|} g^{\bar{\mu}\bar{\nu}} A_{\bar{\mu}}^a (\hat{\partial}^2)^2 A_{\bar{\nu}}^a + \frac{\eta_A}{2M} \int \sqrt{|g|} g^{\bar{\mu}\bar{\nu}} A_{\bar{\mu}}^a (\hat{\partial}^2) A_{\bar{\nu}}^a, \\ S_{evC} &= -\frac{S_C}{M^2} \int \sqrt{|g|} \bar{C}^a (\hat{\partial}^2)^2 C^a - \frac{\eta_C}{M} \int \sqrt{|g|} \bar{C}^a (\hat{\partial}^2) C^a, \end{aligned}$$

respectively.

When fermions are present, we must switch to the vielbein formalism. Then it is necessary to distinguish spacetime indices  $\mu, \nu, \dots$  from Lorentz indices  $a, b, \dots$ , and split both into bar indices and hat indices. The vielbein has physical components  $e_{\bar{\mu}}^{\hat{a}}$  and evanescent components  $e_{\bar{\mu}}^{\hat{a}} = e_{\hat{\mu}}^{\bar{a}} = 0$  and  $e_{\hat{\mu}}^{\bar{a}} = \delta_{\hat{\mu}}^{\bar{a}}$ . The spin connection  $\omega_{\bar{\mu}}^{\bar{a}\bar{b}}$  and the Riemann and Ricci curvature tensors  $R_{\bar{\mu}\bar{\nu}}^{\bar{a}\bar{b}}, R_{\bar{\mu}}^{\bar{a}}$  in  $D$  dimensions are defined by the same formulas that hold in  $d$  dimensions, the evanescent components  $x^{\hat{\mu}}$  of the coordinates being treated as external parameters.

The starting classical action of gravity coupled to (left-handed) chiral fermions is

$$S'_{cG} = S_{cG} + \int e \bar{\psi}_L i e_{\bar{a}}^{\bar{\mu}} \gamma^{\bar{a}} D_{\bar{\mu}} \psi_L, \quad (2.34)$$

where  $e$  is the determinant of the vielbein and  $D_{\bar{\mu}}$  is the gravitational covariant derivative. The functional (2.26) is replaced by

$$\begin{aligned} S'_K &= \int (e_{\bar{\rho}}^{\bar{a}} \partial_{\bar{\mu}} C^{\bar{\rho}} + C^{\bar{\rho}} \partial_{\bar{\rho}} e_{\bar{\mu}}^{\bar{a}} + C^{\bar{a}\bar{b}} e_{\bar{\mu}\bar{b}}) K_{\bar{a}}^{\bar{\mu}} + \int C^{\bar{\rho}} (\partial_{\bar{\rho}} C^{\bar{\mu}}) K_{\bar{\mu}}^C \\ &\quad + \int (C^{\bar{a}\bar{c}} \eta_{\bar{c}\bar{d}} C^{\bar{d}\bar{b}} + C^{\bar{\rho}} \partial_{\bar{\rho}} C^{\bar{a}\bar{b}}) K_{\bar{a}\bar{b}}^C - \int B_{\bar{\mu}} K_{\bar{C}}^{\bar{\mu}} - \int B_{\bar{a}\bar{b}} K_{\bar{C}}^{\bar{a}\bar{b}} \\ &\quad + \int C^{\bar{\rho}} (\partial_{\bar{\rho}} \bar{\psi}_L) K_{\psi} - \frac{i}{4} \int \bar{\psi}_L \sigma^{\bar{a}\bar{b}} C_{\bar{a}\bar{b}} K_{\psi} + \int K_{\bar{\psi}} C^{\bar{\rho}} (\partial_{\bar{\rho}} \psi_L) - \frac{i}{4} \int K_{\bar{\psi}} \sigma^{\bar{a}\bar{b}} C_{\bar{a}\bar{b}} \psi_L, \end{aligned} \quad (2.35)$$

where  $\sigma^{\bar{a}\bar{b}} = i[\gamma^{\bar{a}}, \gamma^{\bar{b}}]/2$  and  $C^{\bar{a}\bar{b}}$  are the ghosts of local Lorentz symmetry. Obviously, the identities  $(S'_{cG}, S'_{cG}) = (S'_{cG}, S'_K) = (S'_K, S'_K) = 0$  hold in  $D$  dimensions.

The gauge fermion must be corrected to include gauge-fixing conditions for local Lorentz symmetry. The common symmetric condition  $e_{\bar{\mu}}^a = e_{\bar{\nu}}^b \eta_{b\mu} \eta^{\nu a}$  cannot be used, since it violates invariance under rigid diffeomorphisms. It is better to start from the less common gauge-fixing condition  $\partial^{\bar{\mu}} \omega_{\bar{\mu}}^{\bar{a}\bar{b}} = 0$ , write it in a form that is compatible with rigid diffeomorphisms, and then include every term allowed by weighted power counting, ghost number conservation and invariance under rigid diffeomorphisms.

The gauge-fixing sector and the evanescent sector must also include parity violating terms constructed with the tensor  $\varepsilon^{a_1 \dots a_d}$ . Those terms are specific of every  $d$  and in general introduce a large number of new parameters. To prove that the propagators of gravity in the vielbein formalism are well defined in arbitrary  $d$ , we proceed in two steps. We first ignore the parity violating terms belonging to the gravitational sector and prove that the propagators are well behaved in that particular case. Later, we prove that they remain well behaved when the parity violating terms are turned on.

From (2.34), (2.35) and the gauge-fixing condition, we find the weight assignments of fields and sources, which are

$$\begin{aligned} [e_{\bar{\mu}}^{\bar{a}}] = [C^{\bar{\mu}}] = 0, \quad [C^{\bar{a}\bar{b}}] = [\omega_{\bar{\mu}}^{\bar{a}\bar{b}}] = 1, \quad [\bar{C}_{\bar{\mu}}] = [B^{\bar{a}\bar{b}}] = \frac{d}{2} - 1, \quad [\bar{C}_{\bar{a}\bar{b}}] = \frac{d}{2} - 2, \\ [B_{\bar{\mu}}] = \frac{d}{2}, \quad [K_{\bar{a}}^{\bar{\mu}}] = [K_{\bar{\mu}}^C] = d - 1, \quad [K_{\bar{a}\bar{b}}^C] = d - 2, \quad [K_C^{\bar{\mu}}] = \frac{d}{2}, \quad [K_C^{\bar{a}\bar{b}}] = \frac{d}{2} + 1. \end{aligned}$$

The parity invariant sector of the new gauge fermion is equal to

$$\begin{aligned} \Psi' = \Psi + \frac{1}{\kappa} \int \bar{C}_{\bar{a}\bar{b}} \partial_{\bar{\mu}} \left( e g^{\bar{\mu}\bar{\nu}} e^{\bar{\rho}\bar{a}} \partial_{\bar{\nu}} e_{\bar{\rho}}^{\bar{b}} + \lambda_1 e e^{\bar{\mu}\bar{a}} g^{\bar{\rho}\bar{\nu}} \partial_{\bar{\rho}} e_{\bar{\nu}}^{\bar{b}} + \lambda_2 e e^{\bar{\mu}\bar{a}} e^{\bar{\nu}\bar{b}} e_{\bar{c}}^{\bar{\rho}} \partial_{\bar{\rho}} e_{\bar{\nu}}^{\bar{c}} \right) \\ + \lambda_3 \int e \bar{C}_{\bar{\mu}} (e^{\bar{\nu}\bar{a}} \partial_{\bar{\nu}} e_{\bar{a}}^{\bar{\mu}} - e^{\bar{\mu}\bar{a}} \partial_{\bar{\nu}} e_{\bar{a}}^{\bar{\nu}}) + \int e e^{\bar{\mu}\bar{a}} e^{\bar{\nu}\bar{b}} (\lambda_4 B_{\bar{\nu}} \partial_{\bar{\mu}} \bar{C}_{\bar{a}\bar{b}} + \lambda_5 \bar{C}_{\bar{\mu}} \partial_{\bar{\nu}} B_{\bar{a}\bar{b}}) \\ + \frac{1}{2} \int \bar{C}_{\bar{a}\bar{b}} \left[ e \xi_1 B^{\bar{a}\bar{b}} + \xi_2 \partial_{\bar{\mu}} (e g^{\bar{\mu}\bar{\nu}} \partial_{\bar{\nu}} B^{\bar{a}\bar{b}}) + \xi_3 \partial_{\bar{\mu}} (e e^{\bar{\mu}\bar{a}} e_{\bar{c}}^{\bar{\nu}} \partial_{\bar{\nu}} B^{\bar{b}\bar{c}}) \right], \end{aligned} \quad (2.36)$$

where  $\Psi$  is the same as in formula (2.28), plus terms that contribute only to vertices in the expansion (2.38).

The nonevanescant sector of the total gauge-fixed action is then

$$S'_d(\Phi, K) = S'_{eG} + (S'_K, \Psi') + S'_K,$$

and satisfies  $(S'_d, S'_d) = 0$  in  $D$  dimensions.

The evanescent sector (2.30) is turned into

$$\begin{aligned} S'_{evG} = S_{evG} + \frac{i}{2M} \int e \left( \varsigma_{\psi} \psi_L^T \tilde{C} \hat{\partial}^2 \psi_L - \varsigma_{\psi}^* \bar{\psi}_L \tilde{C} \hat{\partial}^2 \bar{\psi}_L^T \right) \\ + \frac{\varsigma_1}{4\kappa^2 M^2} \int e (\hat{\partial}^2 e_{\bar{\mu}\bar{a}}) \left( \hat{\partial}^2 e^{\bar{\mu}\bar{a}} - e_{\bar{\nu}}^{\bar{a}} e_{\bar{b}}^{\bar{\mu}} \hat{\partial}^2 e^{\bar{\nu}\bar{b}} \right) \\ + \frac{\varsigma_2}{\kappa M^2} \int e (\hat{\partial}^2 B^{\bar{a}\bar{b}}) e_{\bar{a}}^{\bar{\mu}} (\hat{\partial}^2 e_{\bar{\mu}\bar{b}}) + \frac{\varsigma_3}{2M^2} \int e (\hat{\partial}^2 B^{\bar{a}\bar{b}}) (\hat{\partial}^2 B_{\bar{a}\bar{b}}) \\ - \frac{1}{\kappa M^2} \int e \bar{C}_{\bar{a}\bar{b}} (\hat{\partial}^2)^2 \left( \varsigma_4 C^{\bar{a}\bar{b}} + \varsigma_5 e^{\bar{\mu}\bar{a}} e_{\bar{\rho}}^{\bar{b}} \partial_{\bar{\mu}} C^{\bar{\rho}} \right). \end{aligned} \quad (2.37)$$

Again, we can also add evanescent quadratic terms with just two  $\hat{\partial}$ 's instead of four, and one power of  $M$  in the denominator instead of two, contracted in various ways, plus evanescent terms that contribute only to vertices in the expansion (2.38).

Finally, the total gauge-fixed action is

$$S'(\Phi, K) = S'_{cG} + (S'_K, \Psi') + S'_K + S'_{\text{ev}G} = S'_d(\Phi, K) + S'_{\text{ev}G},$$

and satisfies  $(S', S') = \mathcal{O}(\varepsilon)$ .

To study the propagators, we expand around flat space by writing

$$e_{\bar{\mu}}^{\bar{a}} = (e^{\kappa\phi})_{\bar{\mu}}^{\bar{b}} (e^{\kappa\chi})_{\bar{b}}^{\bar{a}}, \quad e_{\bar{\mu}}^{\hat{a}} = e_{\bar{\mu}}^{\bar{a}} = 0, \quad e_{\bar{\mu}}^{\hat{a}} = \delta_{\bar{\mu}}^{\hat{a}}, \quad C^{\bar{\mu}} = \kappa\tilde{C}^{\bar{\mu}}, \quad C^{\bar{a}\bar{b}} = \kappa\tilde{C}^{\bar{a}\bar{b}}, \quad (2.38)$$

where  $\phi$  and  $\chi$  are matrices with entries  $\phi_{\bar{\mu}}^{\bar{b}}$  and  $\chi_{\bar{b}}^{\bar{a}}$ , such that  $\phi_{\bar{\mu}\bar{a}} \equiv \phi_{\bar{\mu}}^{\bar{b}}\eta_{\bar{b}\bar{a}}$  is symmetric and  $\chi_{\bar{\mu}\bar{a}} \equiv \chi_{\bar{\mu}}^{\bar{b}}\eta_{\bar{b}\bar{a}}$  is antisymmetric. Then we concentrate on the terms that are quadratic in the fields. We write the quadratic part of the gauge-fixed regularized gravitational action in compact form as

$$\frac{1}{2} \int \phi_i \tilde{Q}^{ij} \phi_j,$$

where  $\phi_i = \{\phi_{\bar{\mu}\bar{a}}, B_{\bar{\mu}}, \chi_{\bar{\mu}\bar{a}}, B^{\bar{a}\bar{b}}\}$  is a multiplet collecting the fluctuations  $\phi_{\bar{\mu}\bar{a}}$  and  $\chi_{\bar{\mu}\bar{a}}$  of the vielbein around flat space, as well as the Lagrange multipliers  $B_{\bar{\mu}}$  and  $B^{\bar{a}\bar{b}}$ . Switching to momentum space,  $\tilde{Q}^{ij}$  turns into a matrix  $Q^{ij}$  whose entries depend polynomially on the momentum  $p$  and the various parameters it contains.

The gravitational propagators  $P_{ij} = i(Q^{-1})_{ij}$  are much more involved than in the metric-tensor formalism. To simplify the proof that they are indeed well behaved, we first establish a useful property. Let  $\lambda$  denote a subset of the parameters contained in  $Q$ . If the decomposition

$$Q^{ij} = Q_0^{ij} + R_{\lambda}^{ij},$$

where  $Q_0 = Q|_{\lambda=0}$ , is such that  $(P_0)_{ij} = i(Q_0^{-1})_{ij}$  are well behaved, then  $P_{ij} = i(Q^{-1})_{ij}$  are also well behaved.

To prove this fact, we define the parameters  $\lambda$  so that  $R_{\lambda}^{ij}$  is a linear combination of terms multiplied by  $\lambda$ . Since  $(Q_0^{-1})_{ij}$  exists, the eigenvalues of  $Q_0$  are nonvanishing for generic values of  $p$  and the parameters contained in  $Q_0$ . Then, within a certain nonvanishing radius of convergence for the parameters  $\lambda$ , the eigenvalues of  $Q_0^{-1}R_{\lambda}$  have absolute values smaller than one, therefore the eigenvalues of  $Q_0 + R_{\lambda}$  are also nonvanishing, the inverse of  $Q_0 + R_{\lambda}$  exists and the series

$$\frac{1}{Q_0 + R_{\lambda}} = P_0 \sum_{n=0}^{\infty} (-1)^n (R_{\lambda} P_0)^n \quad (2.39)$$

is convergent. Write

$$\frac{1}{Q_0} = \frac{N_0}{\det Q_0},$$

where  $N_0$  is a polynomial matrix defined by this same equation. Within the convergence radius, we also have

$$\frac{1}{Q_0 + R_{\lambda}} = \frac{N_{\lambda}}{\det(Q_0 + R_{\lambda})}, \quad (2.40)$$

where  $N_\lambda$  is a polynomial matrix and  $N_0 = N|_{\lambda=0}$ . Formula (2.40) tells us that in the domain of convergence the entries of  $(Q + R_\lambda)^{-1}$  are rational functions of  $p$  and the parameters. But then formula (2.40) also holds outside the domain of convergence, for generic values of  $p$  and the parameters, because the algebraic operations that give

$$\frac{(Q_0 + R_\lambda) N_\lambda}{\det(Q_0 + R_\lambda)} = 1$$

are exactly the same. Thus formula (2.40) gives the propagators whenever  $p$  and the parameters have nonexceptional values.

Now we study the ultraviolet behaviors of (2.40). We can focus on the denominator  $\det(Q_0 + R_\lambda)$ . When  $\bar{p}$  and/or  $\hat{p}$  tend to infinity the corrections due to  $R_\lambda$  cannot ruin the ultraviolet behavior due to  $\det(Q_0)$ . To see this, let  $(\bar{p}^2)^w$  and  $(\hat{p}^2)^{2w}$  denote the dominant monomials of  $\det(Q_0)$  for  $\bar{p} \rightarrow \infty$  and  $\hat{p} \rightarrow \infty$ , respectively. They are multiplied by nonvanishing coefficients, because, by assumption, the denominators of  $(Q_0)^{-1}$  have dominant terms multiplied by nonvanishing coefficients. Since propagators contain only parameters of non-negative weights, the corrections brought by  $R_\lambda$  can at most change the coefficients of the dominant terms  $(\bar{p}^2)^w$  and  $(\hat{p}^2)^{2w}$  inside  $\det(Q)$ , but not their powers, which are still  $w$  and  $2w$ . In other words, the dominant terms of the denominators continue to have nonvanishing coefficients for generic values of the parameters, so the propagators are well behaved.

Thanks to this result we do not need to work out the most general propagators  $Q^{-1}$ . It is sufficient to identify a particular case  $Q_0$  such that the inverse  $Q_0^{-1}$  is well behaved. It is convenient to choose  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \xi_3 = 0$ , in (2.36),  $\varsigma_5 = -\varsigma_4$  in (2.37), and turn off the coefficients of all parity violating terms. Then, the multiplet  $\phi_i$  splits into the two submultiplets  $\{\phi_{\bar{\mu}\bar{a}}, B_{\bar{\nu}}\}$  and  $\{\chi_{\bar{\mu}\bar{a}}, B^{\bar{b}\bar{c}}\}$ , in the sense that the matrix  $Q_0$  becomes block-diagonal in those submultiplets. The ghost action also diagonalizes in the pairs  $\bar{C}_{\bar{\mu}}-C^{\bar{\nu}}$  and  $\bar{C}_{\bar{a}\bar{b}}-C^{\bar{c}\bar{d}}$ , where

$$C^{\bar{a}\bar{b}} = \bar{C}^{\bar{a}\bar{b}} - \frac{1}{2} \left( e^{\bar{\mu}\bar{a}} e_{\bar{\nu}}^{\bar{b}} - e^{\bar{\mu}\bar{b}} e_{\bar{\nu}}^{\bar{a}} \right) \partial_{\bar{\mu}} C^{\bar{\nu}}.$$

It is easy to check that the propagators of the Lorentz ghosts are well behaved. Moreover, the propagators of the submultiplet  $\{\phi_{\bar{\mu}\bar{a}}, B_{\bar{\nu}}\}$  and those of the ghosts of diffeomorphisms are also well behaved, because they coincide with the ones of the previous subsection. It remains to study the propagators of the submultiplet  $\{\chi_{\bar{\mu}\bar{a}}, B^{\bar{b}\bar{c}}\}$ . This can be done immediately, since the relevant quadratic part is just

$$-\frac{\varsigma_1}{2} \int \chi^{\bar{a}\bar{b}} \frac{(\hat{\partial}^2)^2}{M^2} \chi_{\bar{a}\bar{b}} + \int B^{\bar{a}\bar{b}} \left( \bar{\partial}^2 \chi_{\bar{a}\bar{b}} + \varsigma_2 \frac{(\hat{\partial}^2)^2}{M^2} \chi_{\bar{a}\bar{b}} \right) + \frac{1}{2} \int B^{\bar{a}\bar{b}} \left( \xi_2 \bar{\partial}^2 B_{\bar{a}\bar{b}} + \varsigma_3 \frac{(\hat{\partial}^2)^2}{M^2} B_{\bar{a}\bar{b}} \right),$$

plus terms that are subdominant in the ultraviolet limit.

Having shown that the propagators are well behaved in the particular case we have identified, the missing parameters can be turned on using the property proved above, so we conclude that the most general propagators are also well behaved.

The parity violating terms can be included with the same procedure. We do not list all of them here, because they are too many. We just mention that they can appear in the gauge fermion, such as

$$\frac{1}{\kappa} \int \bar{C}_{\bar{a}\bar{b}} \varepsilon^{\bar{a}\bar{b}\bar{c}\bar{d}} \partial_{\bar{\mu}} (e g^{\bar{\mu}\bar{\nu}} \omega_{\bar{\nu}\bar{c}\bar{d}}), \quad \int \bar{C}_{\bar{a}\bar{b}} \varepsilon^{\bar{a}\bar{b}\bar{c}\bar{d}} \partial_{\bar{\mu}} (e \partial_{\bar{\mu}} B_{\bar{c}\bar{d}}),$$

and in the evanescent sector, such as

$$\frac{1}{\kappa^2 M^2} \int e(\hat{\partial}^2 e_{\bar{\mu}\bar{a}}) \varepsilon^{\bar{a}\bar{b}\bar{c}\bar{d}} e_{\bar{b}}^{\bar{\mu}} e_{\bar{c}}^{\bar{\nu}} (\hat{\partial}^2 e_{\bar{\nu}\bar{d}}), \quad \frac{1}{M^2} \int e(\hat{\partial}^2 B_{\bar{a}\bar{b}}) \varepsilon^{\bar{a}\bar{b}\bar{c}\bar{d}} (\hat{\partial}^2 B_{\bar{c}\bar{d}}), \quad \frac{1}{M^2} \int e(\hat{\partial}^2 \bar{C}_{\bar{a}\bar{b}}) \varepsilon^{\bar{a}\bar{b}\bar{c}\bar{d}} (\hat{\partial}^2 C_{\bar{c}\bar{d}}).$$

## 2.6 Chern-Simons theories

Parity violating theories in odd dimensions  $d$  may contain Chern-Simons terms, which are built with the tensor  $\varepsilon^{\mu_1 \dots \mu_d}$ . The dimensional regularization of such theories raises issues that are in some respects similar to those raised by the matrix  $\gamma_5$  in four dimensions. We start from three dimensional Chern-Simons Yang-Mills theories, where

$$S_{cA} = \frac{1}{2} \int \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}} A_{\bar{\mu}}^a \left( \partial_{\bar{\nu}} A_{\bar{\rho}}^a + \frac{g}{3} f^{abc} A_{\bar{\nu}}^b A_{\bar{\rho}}^c \right),$$

$$S_K = - \int (D_{\bar{\mu}} C^a) K^{\bar{\mu}a} + \frac{g}{2} \int f^{abc} C^b C^c K_C^a - \int B^a K_C^a.$$

We choose the gauge fermion

$$\Psi(\Phi) = \int \bar{C}^a \left( \partial^{\bar{\mu}} A_{\bar{\mu}}^a + \frac{h(i\bar{\partial})}{2} B^a \right),$$

where  $h(i\bar{\partial})$  is an unspecified derivative operator. The weight assignments are

$$[A] = [C] = [B] = 1, \quad [\bar{C}] = 0, \quad [g] = 0, \quad [K_A] = [K_C] = 1, \quad [K_{\bar{C}}] = 2, \quad [h] = 1.$$

However,  $[h] = 1$  implies that  $h$  is not a polynomial, so we are forced to set  $h = 0$ .

The evanescent terms we can add compatibly with weighted power counting are

$$S_{\text{ev}} = -\frac{\varsigma_A}{2M} \int A_{\bar{\mu}}^a (\hat{\partial}^2) A^{\bar{\mu}a} + S_{\text{ev}C} - \frac{\varsigma_B}{2M} \int B^a \hat{\partial}^2 B^a, \quad (2.41)$$

where  $S_{\text{ev}C}$  is still given by (2.18). Note the last term, which is crucial to make the propagators well behaved even if  $h = 0$ . The ghost propagators coincide with (2.22), while the  $A$  and  $B$  propagators are

$$\langle A_{\bar{\mu}}^a(p) A_{\bar{\nu}}^b(-p) \rangle_0 = \frac{\delta^{ab}}{D(\bar{p}, \hat{p}, 0, \varsigma_A^2, 0)} \left[ \varepsilon_{\bar{\mu}\bar{\rho}\bar{\nu}} p^{\bar{\rho}} - i \frac{\hat{p}^2}{M} \left( \varsigma_A \eta_{\bar{\mu}\bar{\nu}} + \frac{(\varsigma_B - \varsigma_A) p_{\bar{\mu}} p_{\bar{\nu}}}{D(\bar{p}, \hat{p}, 0, \varsigma_A \varsigma_B, 0)} \right) \right],$$

$$\langle A_{\bar{\mu}}^a(p) B^b(-p) \rangle_0 = -\frac{p_{\bar{\mu}} \delta^{ab}}{D(\bar{p}, \hat{p}, 0, \varsigma_A \varsigma_B, 0)}, \quad \langle B^a(p) B^b(-p) \rangle_0 = -i \varsigma_A \frac{\hat{p}^2}{M} \frac{\delta^{ab}}{D(\bar{p}, \hat{p}, 0, \varsigma_A \varsigma_B, 0)}.$$

We see that all of them fall off with the appropriate weights in the ultraviolet limit, in all directions of integration, as long as  $\varsigma_A$  and  $\varsigma_B$  do not vanish.

There is no difficulty in studying Chern-Simons–Maxwell theory along the same lines, that is to say include the term  $F_{\bar{\mu}\bar{\nu}}^a F^{a\bar{\mu}\bar{\nu}}$  in  $S_{cA}$ . Since the Maxwell term prevails over the Chern-Simons one in the ultraviolet limit, this model works like the Yang-Mills theories studied in subsection 2.3, as if the Chern-Simons term were absent.

The coupling to matter is straightforward. Instead, the coupling to gravity must be discussed in detail, because when parity is violated the gravitational Chern-Simons term

$$S_{GCS} = \frac{1}{2\alpha^2} \int \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}} \Gamma_{\bar{\mu}\bar{\beta}}^{\bar{\alpha}} \left( \partial_{\bar{\nu}} \Gamma_{\bar{\rho}\bar{\alpha}}^{\bar{\beta}} + \frac{2}{3} \Gamma_{\bar{\nu}\bar{\gamma}}^{\bar{\beta}} \Gamma_{\bar{\rho}\bar{\alpha}}^{\bar{\gamma}} \right)$$

must be added to the Hilbert action  $S_H$  of (2.25).

Due to the large number of terms involved, it is convenient to block-diagonalize the propagators. We expand around flat space by writing  $\alpha = \kappa m^{1/2}$  and

$$e_{\bar{\mu}}^{\bar{a}} = e^{\kappa\phi} (e^{\alpha\tilde{\phi}})_{\bar{\mu}}^{\bar{b}} (e^{\kappa\chi})_{\bar{b}}^{\bar{a}}, \quad e_{\bar{\mu}}^{\hat{a}} = e_{\bar{\mu}}^{\bar{a}} = 0, \quad e_{\bar{\mu}}^{\hat{a}} = \delta_{\bar{\mu}}^{\hat{a}}, \quad C^{\bar{\mu}} = \alpha \tilde{C}^{\bar{\mu}}, \quad C^{\bar{a}\bar{b}} = \kappa \tilde{C}^{\bar{a}\bar{b}},$$

where  $\tilde{\phi}$  and  $\chi$  are matrices with entries  $\tilde{\phi}_{\bar{\mu}}^{\bar{b}}$  and  $\chi_{\bar{b}}^{\bar{a}}$ , such that  $\tilde{\phi}_{\bar{\mu}\bar{a}} \equiv \tilde{\phi}_{\bar{\mu}}^{\bar{b}} \eta_{\bar{b}\bar{a}}$  is symmetric and traceless and  $\chi_{\bar{\mu}\bar{a}} = \chi_{\bar{\mu}}^{\bar{b}} \eta_{\bar{b}\bar{a}}$  is antisymmetric. Moreover,  $m$  is a parameter of dimension 1 that must be treated nonperturbatively (in this sense, it behaves like an ordinary mass). Since  $S_{GCS}$  is conformally invariant, it does not depend on the conformal factor  $e^{\kappa\phi}$ . Moreover, both  $S_{GCS}$  and  $S_H$  obviously do not depend on  $\chi_{\bar{\mu}\bar{a}}$ , since they do not contain the vielbein, but just the metric tensor. Precisely, we have

$$S_{GCS}(\tilde{\phi}, \alpha) = \frac{1}{\alpha^2} S'_{GCS}(\alpha\tilde{\phi}), \quad S_H(\phi, \tilde{\phi}, \kappa, m) = \frac{1}{\kappa^2} S'_H(\kappa\phi, \alpha\tilde{\phi}),$$

which are perturbative expansions in powers of  $\alpha$  and  $\kappa$ , respectively.

To avoid unnecessary complications, we search for a special case where it is simpler to prove that the propagators are well defined. Using the trick explained in the previous subsection we know that when we turn on the other parameters the propagators remain well defined.

Since the Chern-Simons term is higher derivative, we need a higher-derivative gauge fixing for  $\tilde{\phi}_{\bar{\mu}\bar{a}}$  to obtain well-behaved propagators. It is convenient to make the new gauge fermion independent of the conformal factor to the lowest order around flat space, using

$$\frac{1}{2\alpha} \left( \partial_{\bar{\nu}} g^{\bar{\mu}\bar{\nu}} - \frac{1}{3} g^{\bar{\mu}\bar{\nu}} g_{\bar{\rho}\bar{\sigma}} \partial_{\bar{\nu}} g^{\bar{\rho}\bar{\sigma}} \right) = -g^{\bar{\mu}\bar{\nu}} \partial_{\bar{\rho}} \tilde{\phi}_{\bar{\nu}}^{\bar{\rho}}. \quad (2.42)$$

Moreover, it is not necessary to include  $B_{\bar{\mu}}$ -dependent terms. In the sector  $\chi_{\bar{\mu}\bar{a}} B^{\bar{b}\bar{c}}$  we can take the gauge fermion  $\Psi' - \Psi$  of formula (2.36) with  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \xi_3 = 0$ . We thus have

$$\begin{aligned} \Psi' &= \frac{1}{2\alpha} \int e \bar{C}_{\bar{\mu}} g^{\bar{\alpha}\bar{\beta}} \bar{\partial}_{\bar{\alpha}} \bar{\partial}_{\bar{\beta}} \left( \partial_{\bar{\nu}} g^{\bar{\mu}\bar{\nu}} - \frac{1}{3} g^{\bar{\mu}\bar{\nu}} g_{\bar{\rho}\bar{\sigma}} \partial_{\bar{\nu}} g^{\bar{\rho}\bar{\sigma}} \right) \\ &\quad - \frac{1}{\kappa} \int e (\partial_{\bar{\mu}} \bar{C}_{\bar{a}\bar{b}}) g^{\bar{\mu}\bar{\nu}} e^{\bar{\rho}\bar{a}} \partial_{\bar{\nu}} e_{\bar{\rho}}^{\bar{b}} - \frac{\xi_2}{2} \int e (\partial_{\bar{\mu}} \bar{C}_{\bar{a}\bar{b}}) g^{\bar{\mu}\bar{\nu}} \partial_{\bar{\nu}} B^{\bar{a}\bar{b}}, \end{aligned}$$

plus subdominant terms. The weight assignments read

$$\begin{aligned} [\tilde{\phi}_{\bar{\mu}\bar{a}}] &= 0, & [\phi] &= [\chi_{\bar{\mu}\bar{a}}] = \frac{1}{2}, & [C^{\bar{\mu}}] &= 0, & [C^{\bar{a}\bar{b}}] &= 1, & [\bar{C}_{\bar{\mu}}] &= -1, \\ [\bar{C}^{\bar{a}\bar{b}}] &= -\frac{1}{2}, & [B_{\bar{\mu}}] &= 0, & [B^{\bar{a}\bar{b}}] &= \frac{1}{2}, & [\kappa] &= -\frac{1}{2}, & [\alpha] &= 0. \end{aligned}$$

Summing

$$S_{\text{H}} + S_{\text{GCS}} + (S'_K, \Psi') + S'_K$$

and expanding around flat space, we find a quadratic part of the form

$$\int \phi \bar{\partial}^2 \phi + \tilde{\phi} \bar{\partial}^3 \tilde{\phi} + m^{1/2} \tilde{\phi} \bar{\partial}^2 \phi + m \tilde{\phi} \bar{\partial}^2 \tilde{\phi} + B \bar{\partial}^3 \tilde{\phi} + B' \bar{\partial}^2 \chi + B' \bar{\partial}^2 B' + \bar{C} \bar{\partial}^4 \bar{C} + \bar{C}' \bar{\partial}^2 \bar{C}',$$

where  $B, \bar{C}, \bar{C}'$  stand for  $B_{\bar{\mu}}, \bar{C}_{\bar{\mu}}, \bar{C}^{\bar{\mu}}$  and  $B', \bar{C}', \bar{C}'$  stand for  $B_{\bar{a}\bar{b}}, \bar{C}_{\bar{a}\bar{b}}, \bar{C}^{\bar{a}\bar{b}}$ .

The kinetic terms diagonalize in the blocks  $\{\tilde{\phi}_{\bar{\mu}\bar{a}}, B_{\bar{\nu}}\}$ ,  $\{\chi_{\bar{\mu}\bar{a}}, B^{\bar{b}\bar{c}}\}$  and  $\{\phi\}$  at  $m = 0$ . Thus, it is convenient to switch  $m$  off, prove that the propagators are well defined in that case, and then use the trick of the previous subsection to conclude that they remain well behaved when  $m$  is turned on again.

The evanescent kinetic terms can be arranged to preserve the diagonal structure just outlined. In particular, we can separate the conformal factor  $\phi$  from  $\tilde{\phi}_{\bar{\mu}\bar{a}}$  using the formulas

$$\frac{1}{6\kappa} g^{\bar{\mu}\bar{\nu}} \hat{\partial} g_{\bar{\mu}\bar{\nu}} = \hat{\partial} \phi, \quad \frac{1}{2\alpha} \left( \hat{\partial} g_{\bar{\mu}\bar{\nu}} - \frac{1}{3} g_{\bar{\mu}\bar{\nu}} g^{\bar{\alpha}\bar{\beta}} \hat{\partial} g_{\bar{\alpha}\bar{\beta}} \right) = \hat{\partial} \tilde{\phi}_{\bar{\mu}\bar{\nu}}. \quad (2.43)$$

At  $m = 0$  the conformal factor  $\phi$  behaves as an ordinary scalar field, so its regularized propagator is straightforward, the evanescent kinetic terms being

$$S_{\text{ev}\phi} = -\frac{\varsigma_{\phi}}{2\kappa^2 M^2} \int \sqrt{|g|} (g^{\bar{\mu}\bar{\nu}} \hat{\partial} g_{\bar{\mu}\bar{\nu}}) \hat{\partial}^2 (g^{\bar{\alpha}\bar{\beta}} \hat{\partial} g_{\bar{\alpha}\bar{\beta}}).$$

The block  $\{\tilde{\phi}_{\bar{\mu}\bar{a}}, B_{\bar{\nu}}\}$  can be regularized by means of the evanescent terms

$$\begin{aligned} S_{\text{ev}\tilde{\phi}} &= -\frac{1}{2M} \int \sqrt{|g|} (\hat{\partial}_{\bar{\tau}} \tilde{\phi}_{\bar{\mu}\bar{\nu}}) g^{\bar{\mu}\bar{\rho}} \left( \bar{\varsigma}_{\tilde{\phi}} g^{\bar{\alpha}\bar{\beta}} \bar{\partial}_{\bar{\alpha}} \bar{\partial}_{\bar{\beta}} + \varsigma_{\tilde{\phi}} \frac{(\hat{\partial}^2)^2}{M^2} \right) g^{\bar{\nu}\bar{\sigma}} (\hat{\partial}^{\bar{\tau}} \tilde{\phi}_{\bar{\rho}\bar{\sigma}}) \\ &\quad - \varsigma \int \sqrt{|g|} B_{\bar{\mu}} \frac{(\hat{\partial}^2)^2}{M^2} g^{\bar{\mu}\bar{\nu}} \bar{\partial}_{\bar{\rho}} \tilde{\phi}_{\bar{\nu}}^{\bar{\rho}} - \frac{1}{2} \int \sqrt{|g|} g^{\bar{\mu}\bar{\nu}} B_{\bar{\mu}} \frac{\hat{\partial}^2}{M} \left( \varsigma'_B g^{\bar{\alpha}\bar{\beta}} \bar{\partial}_{\bar{\alpha}} \bar{\partial}_{\bar{\beta}} + \varsigma_B \frac{(\hat{\partial}^2)^2}{M^2} \right) B_{\bar{\nu}}, \end{aligned}$$

where  $\partial_{\bar{\rho}} \tilde{\phi}_{\bar{\nu}}^{\bar{\rho}}$  and  $\hat{\partial} \tilde{\phi}_{\bar{\mu}\bar{\nu}}$  are shortcuts for the expressions of formulas (2.42) and (2.43). The propagators of this block are too involved to be reported here, but we have checked that they are well behaved by means of a computer program.

The propagators of the block  $\{\chi_{\bar{\mu}\bar{a}}, B^{\bar{b}\bar{c}}\}$  and those of the Lorentz ghosts coincide with the ones studied in the previous subsection, the evanescent terms being the last three lines of formula (2.37). Note that we can set  $\varsigma_5 = 0$  at  $m = 0$ .

Finally, to make the propagators of  $\langle C^{\bar{\mu}}(p)\bar{C}_{\bar{\nu}}(-p)\rangle_0$  well behaved it is sufficient to add the evanescent kinetic terms

$$-\frac{\varsigma_C}{2\alpha} \int \sqrt{|g|} \bar{C}_{\bar{\mu}} \frac{(\hat{\partial}^2)^2}{M^2} \left( 2g^{\bar{\alpha}\bar{\beta}} \bar{\partial}_{\bar{\alpha}} \bar{\partial}_{\bar{\beta}} + \varsigma_C \frac{(\hat{\partial}^2)^2}{M^2} \right) C^{\bar{\mu}}$$

Since the propagators are well behaved in the particular case just examined, we know that they are also well behaved when we turn on the missing terms, therefore we conclude that they are well behaved in the most general case.

### 3 Weighted power counting, locality of counterterms and renormalization

Because propagators have the form (2.5), the locality of counterterms and renormalization are controlled by weighted power counting [5], instead of ordinary power counting. Weighted power counting was introduced for Lorentz violating theories, where quadratic terms contain the usual numbers of time derivatives (two for bosons, one for fermions, in unitary theories), but are allowed to contain higher-space derivatives. So far, we have only considered theories that are Lorentz symmetric in the physical spacetime  $\mathbb{R}^d$ , but our treatment can be easily generalized to include models where Lorentz symmetry is explicitly violated in  $\mathbb{R}^d$ . What is important for our discussion is that Lorentz symmetry is certainly violated in the continued spacetime  $\mathbb{R}^D = \mathbb{R}^d \times \mathbb{R}^{-\varepsilon}$ , so the results of refs. [5, 7] apply to our case. For comparison with those references, it may be useful to take into account that the scale  $M$  appearing here plays the role of the “scale of Lorentz violation”  $\Lambda_L$  appearing there. In this section we explain how weighted counting works within the CD regularization.

The denominators of (2.5) have dominant powers  $(\bar{p}^2)^w$  and  $(\hat{p}^2)^{2w}$  for  $\bar{p} \rightarrow \infty$  and  $\hat{p} \rightarrow \infty$ , respectively. This tells us that  $\bar{p}^2$  and  $(\hat{p}^2)^2$  are equally important in the ultraviolet limit. Weights of fields, momenta and parameters have to be assigned so that  $\bar{p}^2$  and  $(\hat{p}^2)^2$  have the same weights, and the action  $S$  and the scale  $M$  are weightless. For convenience, we take the energy to have weight equal to 1, which coincides with its dimension in units of mass. Then  $\bar{p}$  and  $\hat{p}$  have weights 1 and 1/2, respectively, and the polynomials (2.3) have weight equal to 2. The weights of fields and parameters then follow from the requirement that  $M$  and the action be weightless. Note that for the purposes of renormalization what are important are the values of the weights (and dimensions) at  $\varepsilon = 0$ , so we define them as such.

Denoting weights with square brackets, we find

$$\begin{aligned} [\bar{\partial}] &= 1, & [\hat{\partial}] &= \frac{1}{2}, & [\hat{x}] &= -1, & [\hat{x}] &= -\frac{1}{2}, & [M] &= 0, \\ [\Phi] &= \frac{d - N_{\Phi}}{2}, & [gA_{\bar{\mu}}] &= 1, & [g_{\bar{\mu}\bar{\nu}}] &= 0, \end{aligned}$$

where  $N_\Phi$  are the numbers of derivatives  $\bar{\partial}$  contained in the dominant quadratic terms (2.4) of the fields  $\Phi$ . In the ultraviolet limit the propagator of each field  $\Phi$  must have the form (2.5) and fall off with weight  $N_\Phi$ , at least, in all directions of integration.

The weights of fields, sources and derivatives  $\bar{\partial}$  coincide with their dimensions in units of mass. The weights of  $\hat{\partial}$ ,  $\hat{x}$  and  $M$  are different from their dimensions in units of mass. However, the combinations  $M^{-1/2}\hat{\partial}$  and  $M^{1/2}\hat{x}$  have weights equal to their dimensions. Local actions are sums of spacetime integrals of monomials constructed with the fields, the sources, and their derivatives  $\bar{\partial}$  and  $\hat{\partial}$ . If  $n_2$  denotes the number of derivatives  $\hat{\partial}$ , we write the coefficient in front of each monomial as  $\lambda/M^{n_2/2}$ , to factorize an appropriate power of  $M$ . Symbolically, the monomial reads

$$\lambda \int d^D x \bar{\partial}^{n_1} \left( M^{-1/2} \hat{\partial} \right)^{n_2} \Phi^{n_3} K^{n_4}. \quad (3.1)$$

Using this convention, which we have tacitly adopted in the previous section and maintain throughout the paper, all parameters  $\lambda$  have weights equal to their dimensions in units of mass.

At this point, we need to recall a few facts about the dimensional regularization. Divergences are poles in  $\varepsilon$ , but the terms that disappear when  $D \rightarrow d$ , called “evanescences” can be of two types: *formal* evanescences or *analytic* evanescences. Analytically evanescent terms, briefly denoted as “aev”, are those that factorize at least one  $\varepsilon$ , such as  $\varepsilon F_{\bar{\mu}\bar{\nu}} F^{\bar{\mu}\bar{\nu}}$ ,  $\varepsilon \bar{\psi}_L i \not{D} \psi_L$ , etc. Formally evanescent terms, briefly denoted as “fev”, are those that formally disappear when  $D \rightarrow d$ , but do not factorize powers of  $\varepsilon$ . An example is  $\psi_L^T \hat{\partial}^2 \psi_L$ . Because the fields have no evanescent components, in the CD regularization there are fewer formal evanescences than usual. They are built with the tensor  $\eta_{\hat{\mu}\hat{\nu}}$  and the extra components of the coordinates  $\hat{x}$ , momenta  $\hat{p}$  and derivatives  $\hat{\partial}$ . The poles in  $\varepsilon$  can multiply either nonevanescents terms or formally evanescent terms. In the latter case we speak of *divergent evanescences*, also denoted as “divev”. An example is  $\psi_L^T \hat{\partial}^2 \psi_L / \varepsilon$ .

When we differentiate propagators with respect to any components of momenta, their ultraviolet behaviors improve by an amount equal to the weight of the derivatives. In particular, each derivative  $\partial/\partial\bar{p}$  lowers the weight of the ultraviolet behavior by one unit, in all directions of integration, and each derivative  $\partial/\partial\hat{p}$  lowers the weight by 1/2. For example,

$$\frac{\partial}{\partial p^{\bar{\mu}}} \frac{1}{D(\bar{p}, \hat{p}, \varsigma)} = -\frac{2p^{\bar{\mu}}}{D(\bar{p}, \hat{p}, \varsigma)^2}, \quad \frac{\partial}{\partial p^{\hat{\mu}}} \frac{1}{D(\bar{p}, \hat{p}, \varsigma)} = \frac{4\varsigma \hat{p}^2 p^{\hat{\mu}}}{M^2 D(\bar{p}, \hat{p}, \varsigma)^2}. \quad (3.2)$$

When we differentiate a diagram  $G$  a sufficient number of times with respect to any components  $\bar{k}$ ,  $\hat{k}$  of its external momenta  $k$ , we obtain an overall convergent integral.

It is convenient to subtract away divergent evanescences like any other divergences. Indeed, it is simple to show that this prescription ensures the locality of all types of divergences and is consistent to all orders. Consider a diagram  $G$ , subtract its subdivergences according this rule, and call  $G_{\text{sub}}$  the subtracted diagram. When  $G_{\text{sub}}$  is differentiated with respect to any components  $\bar{k}$ ,  $\hat{k}$  of its external momenta, a completely convergent integral is obtained. Thus, both the

nonevanescant and formally evanescent divergences of  $G_{\text{sub}}$  are local, and can be subtracted away. Iterating this procedure, both types of divergences are local to arbitrarily high orders.

In the end, locality ensures that the divergences of  $G_{\text{sub}}$  are polynomials in  $\bar{k}$  and  $\hat{k}$ . The weight  $\omega(G)$  of each such polynomial is equal to the weight of  $G$  minus the weights of the parameters  $\lambda$  that multiply the vertices belonging to  $G$ . Since the weight of the loop integration measure  $d^D p = d^d \bar{p} d^{-\varepsilon} \hat{p}$  is equal to  $d$  (at  $\varepsilon = 0$ ), the overall degree of divergence  $\omega(G)$  coincides with the dimension of the integral associated with it.

All counterterms have the structure (3.1) with  $\lambda$  replaced by a product of  $\lambda$ 's. The generating functional  $\Gamma$  of one-particle irreducible correlation functions is nonlocal and can be expressed using only the quantities  $\Phi, K, \bar{x}, M^{1/2}\hat{x}, \bar{\partial}, M^{-1/2}\hat{\partial}$  and  $\lambda$ , which have weights equal to their dimensions. Symbolically, we write

$$\Gamma = \Gamma(\Phi, K, \bar{x}, M^{1/2}\hat{x}, \bar{\partial}, M^{-1/2}\hat{\partial}, \lambda). \quad (3.3)$$

This result shows that the scale  $M$  is always attached to the formally evanescent quantities  $\hat{x}$  and  $\hat{\partial}$ . The  $\Gamma$  functional keeps the structure (3.3) throughout the renormalization algorithm.

When divergences are removed, it is possible to take the limit  $\varepsilon \rightarrow 0$ . This is done by first letting  $\varepsilon$  tend to 0 inside correlation functions, without affecting the formally evanescent quantities  $M^{1/2}\hat{x}$  and  $M^{-1/2}\hat{\partial}$ , and then dropping the formally evanescent quantities. The combination of the two operations gives the physical correlation functions. Formula (3.3) shows that the scale  $M$  drops out of physical correlation functions. Thanks to weighted power counting,  $M$  does not propagate into the physical sector of the theory, and there is no need to take the limit  $M \rightarrow \infty$ .

The settings of the previous section show that all the requirements of weighted power counting, in particular those concerning the correct behaviors of propagators, can be satisfied in perturbatively unitary quantum fields theories, where bosons have  $N_{\Phi} = 2$  and fermions have  $N_{\Phi} = 1$ . The CD regularization can be generalized to nonunitary theories, in particular higher-derivative theories, where  $N_{\Phi}$  can exceed those values. For example, we may consider higher-derivative gravity [21] or even nonlocal theories [22] coupled with chiral fermions. We do not give details here, because the generalization is straightforward.

The parameters of positive, vanishing and negative weights are superrenormalizable, strictly renormalizable and nonrenormalizable by weighted power counting, respectively. The action of weighted power counting renormalizable theories contains all the terms, and only those, that are compatible with the nonanomalous symmetries and are multiplied by parameters  $\lambda$  of non-negative weights.

If we ignore symmetries for the moment, all monomials must be multiplied by independent parameters. Then, from the strict point of view of weighted power counting, the counterterms have the same forms as the terms of the classical action, and can be subtracted away by redefining parameters and making field redefinitions.

When gauge and global symmetries are present, on the other hand, the coefficients of various monomials are related to one another, therefore it is necessary to verify which symmetries are nonanomalous and drop those that are anomalous. In general gauge theories, the nonevanescant sector  $S_d$  of the action  $S$  must be exactly gauge invariant, therefore it includes the usual terms and satisfies the master equation  $(S_d, S_d) = 0$ . Instead, the total CD-regularized action  $S = S_d + S_{ev}$  satisfies the deformed master equation

$$(S, S) = \mathcal{O}(\varepsilon), \quad (3.4)$$

where the right-hand side collects both analytically and formally evanescent terms.

Assuming that gauge symmetries are nonanomalous, namely that there exists a subtraction scheme where anomalies cancel to all orders in perturbation theory, we still have the problem of *parametric completeness*. A classical action is parametrically complete if it can be renormalized by means of canonical transformations and parameter redefinitions. One way to ensure that this is true is to start from the most general solution of (3.4). When the theory is renormalizable, that solution contains a finite number of independent terms, therefore it is possible to work it out by means of a direct analysis. When the theory is not renormalizable, it contains an infinite number of independent terms, and the issue is more involved. Moreover, in principle counterterms may deform the gauge symmetry in observable ways and affect the classification of invariants.

In nonrenormalizable theories that are manifestly free of gauge anomalies the problem of parametric completeness can be solved in various ways. If the gauge symmetries satisfy certain linearity assumptions (satisfied among others by Yang-Mills gauge symmetries, general covariance and local Lorentz symmetry), it is convenient to use the background field method [23]. Cohomological classifications of invariants [24] may also solve the problem, if the result is of a suitable form. More generally, the classical action can be algorithmically extended by brute force till it becomes parametrically complete [25]. The CD regularization may be useful to prove these and related results in more economic ways, and generalize them to (possibly nonrenormalizable) theories that are not manifestly free of gauge anomalies (see section 5 for more details).

Obviously, the action (2.31) of quantum gravity is not parametrically complete. The classical action  $S_{cG}$  must be extended to include all the nonevanescant counterterms generated by renormalization, which are infinitely many, multiplied by new independent parameters. A convenient form of the extended nonevanescant action  $S_{cext}$  is the one given in ref. [26], where invariants are organized in an economic way by means of field redefinitions. In particular,  $S_{cext}$  does not need to contain higher-derivative quadratic corrections, since counterterms of that type can be subtracted away by means of canonical transformations. This property ensures that a perturbatively unitary theory is not driven by renormalization into a higher-derivative, perturbatively nonunitary theory [27].

The total extended action  $S_{ext} = S_{cext} + S_{evext}$  must also include an extended version  $S_{evext}$

of the evanescent sector  $S_{\text{ev}}$ , which collects evanescent terms of higher weights compatible with invariance under rigid diffeomorphisms and the other global nonanomalous symmetries. The easiest way to classify the infinitely many terms contained in  $S_{\text{ext}}$  is to organize the  $\kappa$  dependence so that

$$S_{\text{ext}}(\Phi, K, \kappa, \lambda) = \frac{1}{\kappa^2} S'_{\text{ext}}(\kappa\Phi, \kappa K, \lambda), \quad (3.5)$$

where  $\lambda$  denotes any other parameters. Then radiative corrections have the structure (see for example [18])

$$\Gamma_{\text{ext}}(\Phi, K, \kappa, \lambda) = \sum_{L=0}^{\infty} \kappa^{2L-2} \Gamma'_{\text{ext}L}(\kappa\Phi, \kappa K, \lambda)$$

where  $L$  labels the  $L$ -loop contributions.

The extensions just advocated do not affect the theory we perturb around (which is provided by  $S_{cG} + S_{\text{ev}G}$  in the case of pure quantum gravity). Indeed, the additional parameters  $\varsigma_{\text{ext}}$  contained in  $S_{\text{cext}} + S_{\text{evext}}$  have negative weights, therefore they are treated perturbatively. In this respect, observe that the set of  $\varsigma_{\text{ext}}$ s includes the coefficients of quadratic terms of high weights, such as

$$\frac{\varsigma_{nG}}{2\kappa^2 M^2} \int \sqrt{|g|} (\hat{\partial}^2 g_{\bar{\mu}\bar{\nu}}) (g^{\bar{\alpha}\bar{\beta}} \bar{\partial}_{\bar{\alpha}} \bar{\partial}_{\bar{\beta}})^n (\hat{\partial}^2 g^{\bar{\mu}\bar{\nu}}), \quad (3.6)$$

which in principle modify the propagators. However, since terms like this are just introduced for regularization purposes, and do not affect the physical poles, it is convenient to treat the quadratic contributions coming from the extension  $S_{\text{cext}} + S_{\text{evext}}$  as “two-leg vertices” and drop all parameters of negative weights from propagators, as we did in the previous section.

At the practical level, a nonrenormalizable theory must be truncated. The truncation must contain finitely many terms and must correspond to some perturbative expansion. Commonly, the truncated action  $S_T$  of quantum gravity contains the terms that have dimensions smaller than some  $T$  in units of mass. The perturbative expansion is then an expansion in powers of the energy divided by some reference mass scale, typically the Planck mass. In the CD regularization it is sufficient to truncate to the terms that have *weights* smaller than  $T$ . The truncated action  $S_T$  must solve the master equation (2.20) up to corrections that fall outside the truncation. To build  $S_T$ , we can list all monomials that have weights smaller than  $T$ , multiply them by independent parameters, and finally relate the parameters to one another by imposing (2.20), as well as invariance under rigid diffeomorphisms and the other global nonanomalous symmetries.

## 4 One-loop chiral anomalies

In this section we use the CD-regularization technique to calculate the known one-loop anomalies of chiral gauge theories in four dimensions. We recall that the anomaly functional  $\mathcal{A}$  is defined as

the antiparenthesis  $(\Gamma, \Gamma)$ , which is also equal to the average  $\langle (S, S) \rangle$ . This identity can be proved making the change of variables  $\Phi^\alpha \rightarrow \Phi^\alpha + \theta(S, \Phi^\alpha)$  in the functional integral

$$Z(J, K) = \int [d\Phi] \exp \left( iS(\Phi, K) + i \int \Phi^\alpha J_\alpha \right),$$

where  $\theta$  is a constant anticommuting parameter, and using the fact that the Jacobian determinant is equal to one in the (ordinary, as well as chiral) dimensional regularization (for details, see for example [23]). Moreover,  $\mathcal{A}$  satisfies the Wess-Zumino consistency conditions [17], which are expressed by the identity  $(\Gamma, \mathcal{A}) = 0$  using the Batalin-Vilkovisky formalism.

We begin with chiral QED. The action is (2.21) with  $\mathcal{C} = -i\gamma^0\gamma^2$ ,  $T^a \rightarrow i$  and  $\Delta S_{\text{ev}A} = 0$ . We have

$$\begin{aligned} (S, S) &= 2(S_K, S_{\text{ev}A} + S_{\text{ev}\psi} + S_{\text{ev}C}) \\ &= \frac{2g}{M} \int C(\varsigma_\psi \psi_L^\alpha \varepsilon_{\alpha\beta} \hat{\partial}^2 \psi_L^\beta - \varsigma_\psi^* \psi_L^{*\alpha} \varepsilon_{\alpha\beta} \hat{\partial}^2 \psi_L^{*\beta}) + a \\ &= 2 \int C(\partial_{\bar{\mu}} J^{\bar{\mu}}) + 2ig \int C \left( \psi_L^* \frac{\delta_l \bar{S}}{\delta \psi_L^*} - \frac{\delta_r \bar{S}}{\delta \psi_L} \psi_L \right) + a, \end{aligned}$$

where  $J^{\bar{\mu}} = g\psi_L^\dagger \bar{\sigma}^{\bar{\mu}} \psi_L$  is the gauge current,  $\bar{S}(\Phi) = S(\Phi, 0)$  and

$$a = 2 \int A_{\bar{\mu}} \frac{\hat{\partial}^2}{M} \left( \varsigma_A \frac{\hat{\partial}^2}{M} + \eta_A \right) \partial^{\bar{\mu}} C - 2 \int B \frac{\hat{\partial}^2}{M} \left( \varsigma_C \frac{\hat{\partial}^2}{M} + \eta_C \right) C$$

collects evanescent contributions that are independent of the fermions. Since the ghosts decouple, the average  $\langle a \rangle$  does not give nonevanescant one-loop contributions to  $\mathcal{A}$ , so we can concentrate on the rest.

Switching to momentum space, the one-loop anomaly then reads

$$\mathcal{A}^{(1)} = \langle (S, S) \rangle_{1\text{loop}} = \frac{2g}{M} \int \frac{d^D p}{(2\pi)^D} \hat{p}^2 C(-k) \text{tr} \left[ \langle \Psi(p) \Psi^T(-p+k) \rangle \begin{pmatrix} \varsigma_\psi \epsilon & 0 \\ 0 & -\varsigma_\psi^* \epsilon \end{pmatrix} \right],$$

where  $\Psi = (\psi_L, \psi_L^*)$ . Here and below the integrals on momenta  $k$  in  $\mathcal{A}^{(1)}$  are understood. These momenta can be taken to be strictly four dimensional.

Now we expand in powers of the gauge field. By locality, power counting and ghost number conservation, the nonevanescant contribution of the linear term is proportional to

$$\int (\bar{\partial}^2 C)(\partial^{\bar{\mu}} A_{\bar{\mu}}) = (S_K, \chi'), \quad \chi' = \frac{1}{2} \int (\partial_{\bar{\mu}} A_{\bar{\nu}})(\partial^{\bar{\mu}} A^{\bar{\nu}}),$$

therefore it is trivial. For the moment, we neglect the trivial contributions and focus on the terms that are quadratic in the gauge field. Calculating the trace and rotating the integrals to Euclidean space we find

$$\mathcal{A}^{(1)} = -8g^3 \varepsilon_{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}} \int C(-k_1 - k_2) A^{\bar{\nu}}(k_1) A^{\bar{\sigma}}(k_2) (k_1^{\bar{\mu}} k_2^{\bar{\rho}} \bar{I} + 2k_1^{\bar{\mu}} \bar{I}^{\bar{\rho}} - 2k_2^{\bar{\rho}} \bar{I}^{\bar{\mu}}),$$

where  $\varepsilon^{0123} = 1$  and  $\bar{I}$ ,  $\bar{I}^{\bar{\mu}}$  are the finite parts of

$$I = \frac{|\varsigma_\psi|^2}{M^2} \int_{Eucl} \frac{d^D p}{(2\pi)^D} \frac{(\hat{p}^2)^2}{\mathcal{D}(\bar{p}, k_1, k_2)}, \quad I^{\bar{\mu}} = \frac{|\varsigma_\psi|^2}{M^2} \int_{Eucl} \frac{d^D p}{(2\pi)^D} \frac{(\hat{p}^2)^2 \bar{p}^{\bar{\mu}}}{\mathcal{D}(\bar{p}, k_1, k_2)},$$

respectively, and

$$\mathcal{D}(\bar{p}, k_1, k_2) = \tilde{D}(\bar{p})\tilde{D}(\bar{p} - k_1)\tilde{D}(\bar{p} + k_2), \quad \tilde{D}(\bar{q}) = \bar{q}^2 + |\varsigma_\psi|^2 \frac{(\hat{p}^2)^2}{M^2}.$$

Since  $\bar{I}^{\bar{\mu}}$  is proportional to  $k_1^{\bar{\mu}} - k_2^{\bar{\mu}}$  it is sufficient to calculate  $\partial \bar{I}^{\bar{\mu}} / \partial k_1^{\bar{\mu}}$ . The desired finite parts can be worked out by inserting infrared cutoffs  $\delta$  and setting  $k_1 = k_2 = 0$ . The integral over  $\hat{p}$  is done first, using the standard rules of the dimensional regularization, and factorizes an  $\varepsilon = 4 - D$ . Finally, the integral over  $\bar{p}$  is just

$$\int_{\delta} d\bar{p} \bar{p}^{-1-\varepsilon/2} = \frac{2}{\varepsilon} + \text{finite}.$$

We find

$$\bar{I} = -\frac{1}{32\pi^2}, \quad \bar{I}^{\bar{\mu}} = -\frac{1}{96\pi^2} (k_1^{\bar{\mu}} - k_2^{\bar{\mu}}),$$

whence

$$\mathcal{A}^{(1)} = \frac{g^3}{48\pi^2} \int C \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}} F_{\bar{\mu}\bar{\nu}} F_{\bar{\rho}\bar{\sigma}}, \quad \langle \partial_{\bar{\mu}} J^{\bar{\mu}} \rangle = \frac{g^3}{96\pi^2} \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}} F_{\bar{\mu}\bar{\nu}} F_{\bar{\rho}\bar{\sigma}} - ig \left( \psi_L^* \frac{\delta_l \bar{S}}{\delta \psi_L^*} - \frac{\delta_r \bar{S}}{\delta \psi_L} \psi_L \right).$$

These formulas are written at  $\varepsilon = 0$ . Check [13] and the appendix of ref. [18] for comparison with calculations done using the common dimensional-regularization technique.

In gauge theories with generic gauge group  $G$ , we insert the matrices  $T^a$  and the structure constants  $f^{abc}$  where appropriate, and reintroduce the trivial contributions that we have neglected so far. In the end, we obtain the Bardeen formula

$$\mathcal{A}^{(1)} = -\frac{ig^3}{12\pi^2} \int d^D x \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}} \text{Tr} \left[ \partial_{\bar{\mu}} C \left( A_{\bar{\nu}} \partial_{\bar{\rho}} A_{\bar{\sigma}} + \frac{g}{2} A_{\bar{\nu}} A_{\bar{\rho}} A_{\bar{\sigma}} \right) \right] + (S_K, \chi), \quad (4.1)$$

where  $C = C^a T^a$ ,  $A_{\bar{\mu}} = A_{\bar{\mu}}^a T^a$  and  $\chi$  is a local functional.

The example of this section shows that some one-loop explicit calculations of divergent parts and anomalies with the CD regularization exhibit more or less the same difficulties as with the ordinary dimensional technique. The numerator algebra is considerably simplified, because the  $\gamma$  matrices are just the ones of  $d$  dimensions, but denominators are not fully  $SO(1, D-1)$  invariant. Actually, their structure is the typical one of higher-derivative theories. However, because higher derivatives only belong to the evanescent sector, the computational effort does not increase dramatically.

At the same time, in section 2 we have shown that propagators are rather involved, due to the evanescent sector, especially when gravity is present, so in general we expect that calculations

with the CD regularization are more difficult than usual. What is important for the purposes of this paper is that the CD regularization is consistent, its main virtue being that it simplifies the proofs of all-order theorems, not that it makes explicit calculations easier.

## 5 Applications

In this section we outline several applications to exhibit the advantages of the new technique. We focus on the benefits the CD regularization brings to the proofs of all-order properties, in particular the renormalization algorithm for general chiral gauge theories, and the Adler-Bardeen theorem. We begin showing that it removes certain dangerous ambiguities that appear using the common dimensional regularization. Then we prove that the CD technique simplifies the extraction of divergent parts out of antiparentheses of functionals, which is a key step in all renormalization algorithms. Third, we show that it simplifies the classification of counterterms in the presence of gravity, eliminating arbitrary dimensionless functions that otherwise would appear in the gauge-fixing and regularization sectors. Finally, we show that it considerably simplifies the proof of the *manifest* Adler-Bardeen theorem for the cancellation of gauge anomalies in perturbatively unitary gauge theories coupled to matter. By the “manifest” Adler-Bardeen theorem we mean the algorithm that not only proves the cancellation of gauge anomalies to all orders, when they vanish at one loop, but also identifies the subtraction scheme where the cancellation occurs automatically from two loops onwards [18]. Moreover, the simplified proof obtained using the CD regularization is suitable to be generalized to larger classes of theories, while the previous proof is not. We also compare some features of the CD regularization with Siegel’s dimensional continuation [28, 29], to further emphasize the consistency of our technique to all orders.

We stress that, at the same time, the CD technique does preserve the good properties of the dimensional regularization. One of them is that local perturbative changes of field variables have Jacobian determinants identically equal to one, which follows from the fact that the integrals of polynomials  $P(p)$  of the momenta in  $d^D p$  vanish. Thanks to this, the Batalin-Vilkovisky master equation, as well as its deformed version (3.4), are simpler than their general versions [9], and several key arguments proceed more smoothly.

### 5.1 Removal of ambiguities

In the usual dimensional regularization the  $\gamma$  matrices are formal objects that satisfy the dimensionally continued Dirac algebra  $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ . The completely antisymmetric products  $\gamma^{\rho_1 \dots \rho_k}$  of  $\gamma^{\rho_1}, \dots, \gamma^{\rho_k}$  are nonvanishing for arbitrary  $k$ , and evanescent for  $k > d$ . The fermion bilinears  $\bar{\psi}_1 \gamma^{\rho_1 \dots \rho_k} \psi_2$  are all inequivalent and can be used to build infinitely many higher-dimensional

objects with the same dimensions in units of mass, such as

$$(\bar{\psi}_1 \gamma^{\rho_1 \dots \rho_k} \psi_2)(\bar{\psi}_3 \gamma_{\rho_1 \dots \rho_k} \psi_4). \quad (5.1)$$

These properties complicate the procedure of renormalization and the proofs of all-order theorems. For example, the Fierz identities in continued spacetime read [18]

$$\psi_2 \bar{\psi}_3 = -\frac{1}{f(D)} \sum_{k=0}^{\infty} \frac{(-1)^{k(k-1)/2}}{k!} \gamma^{\rho_1 \dots \rho_k} (\bar{\psi}_3 \gamma_{\rho_1 \dots \rho_k} \psi_2), \quad (5.2)$$

where  $f(D) = \text{tr}[\mathbb{1}]$ , therefore we can find relations such as

$$(\bar{\psi}_1 \gamma^{\hat{\mu}} \psi_2)(\bar{\psi}_3 \gamma_{\hat{\mu}} \psi_4) = \frac{\varepsilon}{f(D)} (\bar{\psi}_1 \psi_4)(\bar{\psi}_3 \psi_2) - \frac{2}{f(D)} (\bar{\psi}_1 \gamma^{\hat{\rho}} \psi_4)(\bar{\psi}_3 \gamma_{\hat{\rho}} \psi_2) - \frac{\varepsilon}{f(D)} (\bar{\psi}_1 \gamma^{\rho} \psi_4)(\bar{\psi}_3 \gamma_{\rho} \psi_2) + \dots \quad (5.3)$$

that have the form “fev = aev + fev” and show that the distinction between formally evanescent terms and analytically evanescent terms is ambiguous, starting from monomials of dimension  $2(d-1)$  constructed with the fields, the sources and their derivatives.

In section 3 we have stressed that the theorem of locality of counterterms demands that we renormalize divergent evanescences away, together with ordinary divergences. Clearly, this statement makes sense only if we can define divergent evanescences unambiguously. In the usual framework this is problematic, since if we multiply, for example, both sides of formula (5.3) by  $1/\varepsilon$  we get a relation of the type “divev = finite + divev”.

In unitary, four-dimensional power counting renormalizable theories the problem just mentioned is harmless, because it does not concern counterterms and local contributions to anomalies [18]. On the other hand, in more general situations, such as nonrenormalizable theories, or theories that are renormalizable by power counting, but contain higher-derivative kinetic terms, it poses serious difficulties. A possible way out is to define a basis of evanescent and nonevanescents monomials constructed with the fields, the sources and their derivatives, and then express every counterterm using that basis. This is not an easy task, since the Fierz identities (5.2) relate monomials that may appear to be independent of one another at first sight.

The chiral dimensional regularization technique avoids all such troubles from the start, because the  $\gamma$  matrices and the Fierz identities are just the usual  $d$ -dimensional ones, therefore the fermion bilinears are just those we are accustomed to. In particular, they are nonevanescents and finitely many. This is a huge simplification with respect to the ordinary technique.

## 5.2 Divergent parts of antiparentheses

The CD regularization eludes other inconveniences, thanks to the fact that the fields have strictly  $d$ -dimensional components. In particular, the proofs of renormalizability to all orders and the

manifest Adler-Bardeen theorem [18] require to extract divergent parts out of antiparentheses like  $\mathcal{A} = (\Gamma, \Gamma)$  or  $(\Gamma, \mathcal{A})$ . It is helpful to know whether we can freely take this operation across the sign of the antiparenthesis or not. We must establish, for example, that the divergent part of  $(S, \Gamma^{(1)})$  is equal to  $(S, \Gamma_{\text{div}}^{(1)})$ , where  $\Gamma^{(1)}$  is the one-loop contribution to  $\Gamma$  and  $\Gamma_{\text{div}}^{(1)}$  is the divergent part of  $\Gamma^{(1)}$ . To achieve this and similar goals, we must be sure that the antiparentheses themselves do not generate poles in  $\varepsilon$ , or factors of  $\varepsilon$ , and do not convert formal evanescences into analytic ones.

It is easy to prove, in full generality, that the antiparentheses do not generate poles in  $\varepsilon$ . We do not repeat the derivation here, because it is identical to the one of ref. [18]. On the other hand, using the common dimensional regularization the antiparentheses *can* generate factors of  $\varepsilon$  and convert formal evanescences into analytic ones. Using the CD regularization this can never happen.

More precisely, the formally evanescent quantities that appear using the CD regularization are just  $\eta^{\hat{\mu}\hat{\nu}}$  and the evanescent components of momenta and coordinates. The only way these objects have to generate factors of  $\varepsilon$  is by means of the contraction  $\eta^{\hat{\mu}\hat{\nu}}\eta_{\hat{\mu}\hat{\nu}} = -\varepsilon$ . However, the contractions of Lorentz indices brought by the functional derivatives  $\delta/\delta\Phi^\alpha$  and  $\delta/\delta K_\alpha$  due to the antiparentheses can never generate  $\eta^{\hat{\mu}\hat{\nu}}\eta_{\hat{\mu}\hat{\nu}}$ , because fields and sources have no evanescent components. For the same reason, the antiparentheses cannot convert formal evanescences into analytic ones. Then, we can freely cross the sign of antiparentheses when we extract divergent parts. We can write down useful symbolic identities that summarize these properties, such as  $(\text{fev}, \text{fev}) = \text{fev}$ ,  $(\text{fev}, \text{nonev}) = \text{fev}$  and  $(\text{fev}, \text{div}) = \text{divev}$ , where “nonev” denotes convergent nonevanescence quantities, and “div” denotes poles in  $\varepsilon$ .

Instead, using the common dimensional regularization the metric tensor  $g_{\mu\nu}$  and its source  $K^{\mu\nu}$ , for example, have traceful evanescent components  $g_{\hat{\mu}\hat{\nu}}$  and  $K^{\hat{\mu}\hat{\nu}}$ , therefore the antiparentheses can generate factors of  $\varepsilon$  and convert formal evanescences into analytic ones. Moreover, four-fermion terms can generate both factors of  $\varepsilon$  and the ambiguities mentioned above. These problems are harmless only in unitary power counting renormalizable theories, where gravity is absent and four-fermion terms do not appear in counterterms and local contributions to anomalies [18]. However, in more general theories the CD regularization is definitely more convenient than the usual dimensional regularization.

### 5.3 Classification of counterterms in the presence of gravity

The third application we mention concerns the classification of counterterms and contributions to anomalies in chiral theories coupled to gravity. In several situations, using the ordinary dimensional regularization, we may be forced to introduce an independent metric  $h_{\mu\nu}$  besides the metric tensor  $g_{\mu\nu}$  and the background metric  $\bar{g}_{\mu\nu}$  around which we expand  $g_{\mu\nu}$  perturbatively. Field translations leave the functional integral invariant, therefore correlation functions are inde-

pendent of  $\bar{g}_{\mu\nu}$ . Instead, they may depend on some  $h_{\mu\nu}$ , which we therefore call a “second metric”. A second metric can enter the classical action through the gauge fixing or the regularization itself. Most gauge-fixing functions commonly used (e.g.  $\eta^{\rho\nu}\partial_\rho g_{\mu\nu}$ ) do introduce a second metric, which is typically the flat-space metric  $\eta_{\mu\nu}$ .

When two independent metrics  $g_{\mu\nu}$  and  $h_{\mu\nu}$  are present, arbitrary dimensionless functions can be built, for example functions of  $g_{\mu\nu}h^{\mu\nu}$ ,  $g_{\mu\nu}h^{\nu\rho}g_{\rho\sigma}h^{\sigma\mu}$ , and similar contractions. The classification of counterterms and contributions to anomalies is then plagued with unnecessary complications. The gauge-invariant sector is insensitive to this problem, because general covariance forbids the arbitrary functions just mentioned. In the gauge-fixing sector, as well as in the regularization sector, instead, we can forbid these functions using invariance under rigid diffeomorphisms, if the theory contains a unique metric.

Let us see an explicit example. We know that the naïve continuation of the action (2.6) to  $D$  dimensions is not well regularized. One way to deal with this problem, using the ordinary dimensional regularization, is to introduce right-handed partners  $\psi_R$  that decouple in the physical limit [18]. The regularized action reads

$$\int \bar{\psi}_L i \not{D} \psi_L + \int \bar{\psi}_R i \not{\partial} \psi_L + \int \bar{\psi}_L i \not{\partial} \psi_R + \int \bar{\psi}_R i \not{\partial} \psi_R. \quad (5.4)$$

The propagator of  $\psi = \psi_L + \psi_R$  is  $i/\not{p}$ , which is of course fine. Now, we must guarantee that the right-handed partners  $\psi_R$  decouple from the  $S$  matrix. This is true in flat space, because by formula (5.4)  $\psi_R$  does not appear in any vertices, therefore no one-particle irreducible diagrams with  $\psi_R$  external legs can be constructed. If we couple the theory to quantum gravity, we must covariantize the first term of (5.4), but keep the last three terms of (5.4) in flat space, otherwise the right-handed partners  $\psi_R$  do not decouple from the  $S$  matrix. In this way, we do introduce the flat-space metric  $\eta_{\mu\nu}$  as a second independent metric, besides the metric tensor  $g_{\mu\nu}$ .

Similarly, the most common gauge-fixing conditions for diffeomorphisms, such as  $\eta^{\mu\rho}\partial_\rho g_{\mu\nu} = 0$  introduce a second metric, which is either the flat-space one or a background metric. That is enough to create the problem we are concerned with.

As we know, a nonrenormalizable theory must be truncated so that the truncated action  $S_T$  contains at most a finite number of terms. When the theory contains two metrics, we can construct infinitely many terms with the same dimensions in units of mass, in the gauge-fixing and regularization sectors. Then the truncation is unable to really reduce the arbitrariness to a finite number of parameters, which makes the classification of counterterms much less practical.

No such problems appear using the CD regularization, because it does not introduce decoupling partners of chiral fermions, nor second metrics, and we have proved that it is fully compatible with invariance under rigid diffeomorphisms.

As an example, consider higher-derivative quantum gravity [21] in four dimensions, with or

without chiral fermions. The theory is power counting renormalizable and its action reads

$$S_{cG} = -\frac{1}{2\kappa^2} \int \sqrt{|g|} (2\Lambda + R - aR_{\mu\nu}R^{\mu\nu} - bR^2) + \int e\bar{\psi}_L e_a^\mu \gamma^a iD_\mu \psi_L.$$

If fermions are regularized according to (5.4), or the gauge fixing is not invariant under rigid diffeomorphisms, the classification of counterterms must deal with arbitrary functions that appear in the gauge-fixing sector. In ref. [21] this difficulty was dodged by advocating a generalization of the Kluberg-Stern–Zuber conjecture [30]. Because of this, the proof that higher-derivative gravity is indeed renormalizable was incomplete. It was first completed in ref. [23] with a different approach, using the background field method in the absence of chiral matter. When chiral matter is present, it is much easier to use the CD regularization and invariance under rigid diffeomorphisms, because then it is sufficient to determine the coefficients of a few Lagrangian terms. We see, again, that the CD regularization is more convenient than the ordinary dimensional regularization.

#### 5.4 Proof of the *manifest* Adler-Bardeen theorem

In ref. [18] the Adler-Bardeen theorem was reconsidered, and proved in the most general four-dimensional, unitary, power counting renormalizable theory, without using renormalization-group arguments. Thanks to this, the subtraction scheme where gauge anomalies manifestly cancel to all orders, if they vanish at one loop, was identified. To achieve that goal, the dimensional regularization was merged with a suitable higher-derivative gauge invariant regularization, and the combined technique was called *dimensional/higher-derivative* (DHD) regularization.

The DHD regularization inherits the difficulties of the ordinary dimensional regularization. The ambiguities due to the continued Fierz identities, and the generation of factors  $\varepsilon$  by the antiparentheses make it difficult to generalize the proof of [18] to wider classes of quantum field theories. To overcome this problem, we must first upgrade the DHD regularization by replacing the usual dimensional regularization with the CD regularization. We call the new combined technique *chiral-dimensional/higher-derivative* (CDHD) regularization.

The CDHD regularization has two cutoffs,  $\varepsilon$  for the CD regularization and  $\Lambda$  for the higher-derivative regularization. The cutoffs are removed with a procedure similar to the DHD limit defined in ref. [18]: (i) we first subtract the poles in  $\varepsilon$ , which have nonevanescant or formally evanescent residues; (ii) then we subtract the  $\Lambda$  divergences, which from the  $D$ -dimensional viewpoint are, again, either nonevanescant or formally evanescent; (iii) then we take the analytic limit  $\varepsilon \rightarrow 0$ , followed by the limit  $\Lambda \rightarrow \infty$ , without affecting formally evanescent quantities; (iv) finally, we drop all formally evanescent quantities. We do not give more details about the combination of the two techniques, because everything else works exactly as in ref. [18].

The higher-derivative sector is arranged so that at fixed  $\Lambda$  the regularized theory is superrenormalizable, and has just a few one-loop, matter-independent and source-independent divergences

and potential anomalies. Thus, at fixed  $\Lambda$  the manifest Adler-Bardeen theorem is a consequence of simple power counting arguments. At a second stage it is proved that the theorem survives when the  $\Lambda$ -divergences are renormalized away and  $\Lambda$  is taken to infinity. The basic reason why this happens is that the higher-derivative sector of the regularization is manifestly gauge invariant.

Using the CD regularization instead of the ordinary dimensional one, the complications mentioned in the previous subsections are properly dodged and the upgraded version of the proof given in ref. [18] proceeds much more smoothly. However, the greatest advantage of the CDHD regularization is that the upgraded proof of the manifest Adler-Bardeen theorem is ready to be generalized to a much larger class of theories. For the moment we content ourselves with this remark, because for reasons of space we have to postpone the investigation of this possibility to a future publication.

## 5.5 Comparison with Siegel's dimensional continuation

Years ago, Siegel proposed a regularization technique for supersymmetric theories [28] suggested by dimensional reduction, where the dimension  $d$  of spacetime is analytically continued to the complex value  $D$ , but all fields keep their physical components, and the  $\gamma$  matrices are the usual  $d$ -dimensional ones. Siegel's technique, called *dimensional continuation*, was soon realized to be inconsistent by Siegel himself [29]. Here we point out the differences between the CD regularization and Siegel's dimensional continuation, to emphasize once again how our approach works and why it is consistent to all orders. It should also be said that our technique does not treat supersymmetry in any special way.

If the  $\gamma$  matrices are the usual  $d$ -dimensional ones,  $\not{p}$  cannot depend on all components of the momentum  $p$ , therefore its reciprocal is not a good propagator. Siegel's initial idea was that if we imagine that  $D$  is *smaller* than  $d$ ,  $\not{p}$  actually does not lose any contribution, because, so to speak, "there are more  $\gamma$  matrices than  $p$  components". However, dimensional regularization is based on the analytic continuation of integrals to the complex plane, where no ordering is well defined. Insisting that  $D$  must be smaller than  $d$  forces us to work on the real axis, but then the analytic continuation itself is not well defined. In four dimensions the inconsistency of Siegel's technique can appear from four loops onwards [29]. At low orders computations are safe. There, the dimensional continuation is commonly used together with superfields [31].

It is convenient to recapitulate here how the CD regularization avoids these problems, and solves new ones that appear along the way. First,  $D = d - \varepsilon$  and  $\varepsilon$  are complex numbers, as they should be, so  $1/\not{p}$  is not a good propagator. To overcome this problem, we have corrected the action by adding suitable evanescent kinetic terms. Chiral fermions force those corrections to be higher derivative, which brings a parameter of negative dimension (that is to say  $1/M$ ) into the game, even when the theory is power counting renormalizable. This is dangerous, because an

evanescent nonrenormalizable term is sufficient to turn the theory into a nonrenormalizable one, unless there is some mechanism that prevents this from happening.

For example, take the  $\varphi^4$ -theory in four dimensions, and add an evanescent  $\varphi^6$ -vertex. The action is

$$S = \int \left[ \frac{1}{2} (\partial_\mu \varphi) (\partial^\mu \varphi) - \frac{m^2}{2} \varphi^2 - \frac{\lambda}{4!} \varphi^4 - \frac{\varepsilon \lambda'}{6! M^2} \varphi^6 \right]. \quad (5.5)$$

Next, consider the one-loop diagram  $G$  made with one  $\varphi^4$ -vertex and one  $\varphi^6$ -vertex, and having six external legs. The factor  $\varepsilon$  contained in the  $\varphi^6$ -vertex simplifies the pole  $1/\varepsilon$  of the diagram, and the result is equal to a nonevanescant constant. The diagram  $G$  is thus equivalent to a local nonevanescant “one-loop” vertex with six legs. Using it as a subdiagram, we can easily construct divergent two-loop diagrams with six external legs. To renormalize those we must make the coefficient of  $\varphi^6$  in (5.5) nonevanescant. In the end,  $M$  propagates into the physical sector and the theory becomes nonrenormalizable.

This cannot happen using the CD regularization, because weighted power counting forbids it. For instance, the four-dimensional regularized theory

$$S = \frac{1}{2} \int \left[ (\partial_{\bar{\mu}} \varphi) (\partial^{\bar{\mu}} \varphi) - \frac{\varsigma_\varphi}{M^2} (\hat{\partial}^2 \varphi)^2 + \frac{\eta_\varphi}{M} (\partial_{\bar{\mu}} \varphi) (\partial^{\bar{\mu}} \varphi) - m^2 \varphi^2 - \frac{\lambda}{12} \varphi^4 \right] \quad (5.6)$$

cannot generate nonevanescant counterterms of dimensions greater than four. Indeed, those counterterms would have weights greater than four, so they are excluded by weighted power counting.

By the arguments of section 3, formula (3.3), weighted power counting forbids the propagation of  $M$  into the physical sector also in nonrenormalizable theories. For these reasons, despite a few coincidental similarities, the regularization technique formulated in this paper is rather different from Siegel’s one. It does not suffer from the weaknesses of that technique, and ultimately is fully consistent.

## 6 Conclusions

We have formulated a modified dimensional-regularization technique that avoids several inconveniences of the usual dimensional technique. Fields have exactly the same components as in the physical limit, and the  $\gamma$  matrices are just the usual ones. Only coordinates and momenta are continued to complex  $D$  dimensions. Invariance under rigid diffeomorphisms can be manifestly preserved, and no extra fields are introduced for regularization purposes, in particular no second metrics and no partners of chiral fermions that decouple in the physical limit. We have called the new technique chiral dimensional regularization.

The propagators are cured by means of evanescent kinetic terms. In the case of chiral fermions such terms are higher derivative and of the Majorana type. Weighted power counting gives us control over the locality of counterterms and renormalization in the presence of such corrections.

The evanescent sectors of all fields must agree with one another, to produce propagators with the same types of denominators.

Thanks to these features, the CD regularization is consistent to all orders, and has numerous advantages. Ambiguities due to dimensionally continued Fierz identities never show up. Divergences and evanescences propagate in a straightforward way through the Batalin-Vilkovisky antiparentheses, therefore the classification of counterterms and local contributions to anomalies is considerably simpler. Moreover, the CD regularization is compatible with invariance under rigid diffeomorphisms. Thanks to this property, when gravity is present counterterms are severely constrained even in the gauge-fixing and regularization sectors, and every truncation of the theory contains finitely many free parameters. Finally, the proofs of all-order theorems are less involved and ready to be generalized to wider classes of models.

## References

- [1] G.t Hooft and M.Veltman, Regularization and renormalization of gauge fields, Nucl. Phys. B 44 (1972) 189.
- [2] C.G. Bollini and J.J. Giambiagi, Lowest order divergent graphs in nu-dimensional space, Phys. Lett. B40, 566 (1972);  
G.M. Cicuta and E. Montaldi, Analytic renormalization via continuous space dimension, Lett. Nuovo Cimento 4 (1972) 329.
- [3] P. Breitenlohner and D. Maison, Analytic renormalization and the action principle, Commun. Math. Phys. 52 (1977) 11.
- [4] C. Schubert, The Yukawa model as an example for dimensional renormalization With  $\gamma_5$ , Nucl. Phys. B 323 (1989) 478;  
I. An and H. - S. Song, One way to solve the puzzle of  $\gamma_5$  in the dimensional regularization, Commun. Theor. Phys. 12 (1989) 201;  
J. G. Korner, D. Kreimer and K. Schilcher, A practicable  $\gamma_5$  scheme in dimensional regularization, Z. Phys. C 54 (1992) 503;  
P. A. Baikov and V. A. Ilyin, Status of  $\gamma_5$  in dimensional regularization, Theor. Math. Phys. 88 (1991) 789 [Teor. Mat. Fiz. 88 (1991) 163];  
D. Kreimer, The role of  $\gamma_5$  in dimensional regularization, hep-ph/9401354;  
R. Ferrari, A. Le Yaouanc, L. Oliver and J. C. Raynal, Gauge invariance and dimensional regularization with  $\gamma_5$  in flavor changing neutral processes, Phys. Rev. D 52 (1995) 3036;

- D. Sanchez-Ruiz, BRS symmetry restoration of chiral Abelian Higgs-Kibble theory in dimensional renormalization with nonanticommuting  $\gamma_5$ , Phys. Rev. D 68 (2003) 025009 and hep-th/0209023
- E.C. Tsai, Gauge invariant treatment of  $\gamma_5$  in the scheme of 't Hooft and Veltman", Phys. Rev. D 83 (2011) 025020 and arXiv:0905.1550 [hep-th];
- R. Ferrari, Managing  $\gamma_5$  in dimensional regularization and the ABJ anomaly, arXiv:1403.4212 [hep-th].
- [5] D. Anselmi and M. Halat, Renormalization of Lorentz violating theories, Phys.Rev. D76 (2007) 125011 and arXiv:0707.2480 [hep-th].
- [6] D. Colladay and V.A. Kostelecký, Lorentz-violating extension of the standard model, Phys. Rev. D58 (1998) 116002 and arXiv:hep-ph/9809521.
- [7] D. Anselmi, Weighted power counting, neutrino masses and Lorentz violating extensions of the standard model, Phys. Rev. D79 (2009) 025017, 08A4 Renormalization.com and arXiv:0808.3475 [hep-ph].
- [8] D. Anselmi, Standard model without elementary scalars and high energy Lorentz violation, Eur. Phys. J. C65 (2010) 523, 09A1 Renormalization.com, and arXiv:0904.1849 [hep-ph].
- [9] I.A. Batalin and G.A. Vilkovisky, Gauge algebra and quantization, Phys. Lett. B 102 (1981) 27-31;
- I.A. Batalin and G.A. Vilkovisky, Quantization of gauge theories with linearly dependent generators, Phys. Rev. D 28 (1983) 2567, Erratum-ibid. D 30 (1984) 508;
- see also S. Weinberg, *The quantum theory of fields*, vol. II, Cambridge University Press, Cambridge 1995.
- [10] S.L. Adler and W.A. Bardeen, Absence of higher order corrections in the anomalous axial vector divergence, Phys. Rev. 182 (1969) 1517.
- [11] For a review of existing proofs and references on this subject, see for example S.L. Adler, Anomalies to all orders, in "*Fifty Years of Yang-Mills Theory*", G. 't Hooft ed., World Scientific, Singapore, 2005, p. 187-228, and arXiv:hep-th/0405040.
- [12] A. Zee, Axial-vector anomalies and the scaling property of field theory, Phys. Rev. Lett. 29 (1972) 1198.
- [13] J. Collins, *Renormalization*, Cambridge University Press, 1984, Chapter 13.

- [14] T. Marinucci and M. Tonin, Dimensional regularization and anomalies, *Il Nuovo Cimento A* 31 (1976) 381;  
 G. Costa, J. Julve, T. Marinucci and M. Tonin, Non-Abelian gauge theories and triangle anomalies, *Nuovo Cimento A* 38 (1977) 373;  
 C. Lucchesi, O. Piguet and K. Sibold, The Adler-Bardeen theorem for the axial U(1) anomaly in a general non-Abelian gauge theory, *Int. J. Mod. Phys. A* 02 (1987) 385.
- [15] O. Piguet and S. Sorella, Adler-Bardeen theorem and vanishing of the gauge beta function, *Nucl.Phys. B* 395 (1993) 661 and arXiv:hep-th/9302123.
- [16] E. Witten, Global aspects of current algebra, *Nucl. Phys. B* 223 (1983) 422.
- [17] Wess, J. and B. Zumino, Consequences of anomalous Ward identities, *Phys. Lett. B* 37 (1971) 95.
- [18] D. Anselmi, Adler-Bardeen theorem and manifest anomaly cancellation to all orders in gauge theories, 14A1 Renormalization.com and arXiv:1402.6453 [hep-th].
- [19] D. Anselmi, Deformed dimensional regularization for odd (and even) dimensional theories, *Int.J.Mod.Phys. A*20 (2005) 1389, 04A2 Renormalization.com and arXiv:hep-th/0404053.
- [20] D. Anselmi, A note on the dimensional regularization of the standard model coupled with quantum gravity, *Phys.Lett. B*596 (2004) 90, 04A1 Renormalization.com and arXiv:hep-th/0404032.
- [21] K.S. Stelle, Renormalization of higher derivative quantum gravity, *Phys. Rev. D* 16 (1977) 953.
- [22] E.T. Tomboulis, Superrenormalizable gauge and gravitational theories, arXiv:hep-th/9702146;  
 L. Modesto, Super-renormalizable quantum gravity, *Phys.Rev. D*86 (2012) 044005 and arXiv:1107.2403 [hep-th];  
 T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Towards singularity and ghost free theories of gravity, *Phys.Rev.Lett.* 108 (2012) 031101 and arXiv:1110.5249 [gr-qc];  
 L. Modesto, Finite quantum gravity, arXiv:1305.6741 [hep-th].
- [23] D. Anselmi, Background field method, Batalin-Vilkovisky formalism and parametric completeness of renormalization, *Phys. Rev. D* 89 (2014) 045004, 13A3 Renormalization.com and arXiv:1311.2704 [hep-th].

- [24] G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in the antifield formalism. I. General theorems, *Commun. Math. Phys.* 174 (1995) 57 and arXiv:hep-th/9405109;  
 G. Barnich, F. Brandt, M. Henneaux, Local BRST cohomology in the antifield formalism. II. Application to Yang-Mills theory, *Commun. Math. Phys.* 174 (1995) 116 and arXiv:hep-th/9405194;  
 G. Barnich, F. Brandt, M. Henneaux, General solution of the Wess-Zumino consistency condition for Einstein gravity, *Phys. Rev. D* 51 (1995) R1435 and arXiv:hep-th/9409104;  
 S.D. Joglekar and B.W. Lee, General theory of renormalization of gauge invariant operators, *Ann. Phys. (Amsterdam)* 97 (1976) 160.
- [25] D. Anselmi, Renormalization of gauge theories without cohomology, *Eur. Phys. J. C* 73 (2013) 2508, 13A1 Renormalization.com, arXiv:1301.7577 [hep-th].
- [26] D. Anselmi, Properties of the classical action of quantum gravity, *JHEP* 1305 (2013) 028, 13A2 Renormalization.com and arXiv:1302.7100 [gr-qc].
- [27] D. Anselmi, Absence of higher derivatives in the renormalization of propagators in quantum field theories with infinitely many couplings, *Class. Quant. Grav.* 20 (2003) 2355, 02A4 Renormalization.com and arXiv:hep-th/0212013.
- [28] W. Siegel, Supersymmetric dimensional regularization via dimensional reduction, *Phys.Lett. B* 84 (1979) 193.
- [29] W. Siegel, Inconsistency of supersymmetric dimensional regularization, *Phys.Lett. B* 94 (1980) 37.
- [30] H. Kluberg-Stern and J.B. Zuber, Renormalization of nonabelian gauge theories in a background field gauge. 1. Green functions, *Phys. Rev. D* 12 (1975) 482;  
 H. Kluberg-Stern and J.B. Zuber, Renormalization of nonabelian gauge theories in a background field gauge. 2. Gauge invariant operators, *Phys. Rev. D* 12 (1975) 3159.
- [31] For example, it is extensively used together with superspace techniques. See S.J. Gates, M.T. Grisaru, M. Rocek and W. Siegel, *Superspace or one thousand and one lessons in supersymmetry*, *Front.Phys.* 58 (1983) 1-548, arXiv:hep-th/0108200.